Authors

Edward B. Burger, Ph.D., is Professor of Mathematics at Williams College and is the author of numerous articles, books, and videos. He has won several of the most prestigious writing and teaching awards offered by the Mathematical Association of America. Dr. Burger has made numerous television and radio appearances and has given countless mathematical presentations around the world.

David J. Chard, Ph.D., is the Leon Simmons Dean of the School of Education and Human Development at Southern Methodist University. He is a past president of the Division of Research at the Council for Exceptional Children, a member of the International Academy for Research on Learning Disabilities, and has been the Principal Investigator on numerous research projects for the U.S. Department of Education.

Paul A. Kennedy, Ph.D., is a professor and Distinguished University Teaching Scholar in the Department of Mathematics at Colorado State University. Dr. Kennedy is a leader in mathematics education. His research focuses on developing algebraic thinking by using multiple representations and technology. He is the author of numerous publications.

Freddie L. Renfro, MA, has 35 years of experience in Texas education as a classroom teacher and director/coordination of Mathematics PreK-12 for school districts in the Houston area. She has served as a reviewer and TXTEAM trainer for Texas Math Institutes and has presented at numerous math workshops.

Tom W. Roby, Ph.D., is Associate Professor of Mathematics and Director of the Quantitative Learning Center at the University of Connecticut. He founded and directed the Bay Area-based ACCLAIM professional development program. He also chaired the advisory board of the California Mathematics Project and reviewed content for the California Standards Tests.

Bert K. Waits, Ph.D., is a Professor Emeritus of Mathematics at The Ohio State University and cofounder of T³ (Teachers Teaching with Technology), a national professional development program. Dr. Waits is also a former board member of the NCTM and an author of the original NCTM Standards.

Steven J. Leinwand is a Principal Research Analyst at the American Institutes for Research in Washington, D.C. He was previously, for 22 years, the Mathematics Supervisor with the Connecticut Department of Education.
<table>
<thead>
<tr>
<th>Reviewers</th>
<th>Position</th>
<th>School</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>John Bakelaar</td>
<td>Assistant Principal</td>
<td>Whitten Middle School</td>
<td>Jackson, MS</td>
</tr>
<tr>
<td>Jennifer Bauer</td>
<td>Mathematics Instructional Leader</td>
<td>East Haven High School</td>
<td>East Haven, CT</td>
</tr>
<tr>
<td>Doug Becker</td>
<td>Mathematics Teacher</td>
<td>Gaylord High School</td>
<td>Gaylord, MI</td>
</tr>
<tr>
<td>Joe Brady</td>
<td>Mathematics Department Chair</td>
<td>Ensworth High School</td>
<td>Nashville, TN</td>
</tr>
<tr>
<td>Sharon Butler</td>
<td>Adjunct Faculty</td>
<td>Montgomery College of The Woodlands</td>
<td>Spring, TX</td>
</tr>
<tr>
<td>Kathy Dean Davis</td>
<td>Mathematics Department Chair, retired</td>
<td>Bowling Green Junior High</td>
<td>Bowling Green, KY</td>
</tr>
<tr>
<td>Maureen “Willie” DiLaura</td>
<td>Middle School Math Specialist, retired</td>
<td>Lockman Middle School</td>
<td>Denton, MD</td>
</tr>
<tr>
<td>Arlane Frederick</td>
<td>Curriculum &amp; Learning Specialist in Mathematics, retired</td>
<td>Kenmore-Town of Tonawanda UFSD</td>
<td>Buffalo, NY</td>
</tr>
<tr>
<td>Marieta W. Harris</td>
<td>Mathematics Specialist</td>
<td>Memphis</td>
<td>Memphis, TN</td>
</tr>
<tr>
<td>Connie Johnsen</td>
<td>Mathematics Teacher</td>
<td>Harker Heights High School</td>
<td>Harker Heights, TX</td>
</tr>
<tr>
<td>Mary Jones</td>
<td>Mathematics Supervisor/Teacher</td>
<td>Grand Rapids Public Schools</td>
<td>Grand Rapids, MI</td>
</tr>
<tr>
<td>Lendy Jones</td>
<td>Algebra Teacher</td>
<td>Liberty Hill Middle School</td>
<td>Killeen, TX</td>
</tr>
<tr>
<td>Mary Joy</td>
<td>Algebra Teacher</td>
<td>Mayfield High School</td>
<td>Las Cruces, NM</td>
</tr>
<tr>
<td>Vilma Martinez</td>
<td>Algebra Teacher</td>
<td>Nikki Rowe High School</td>
<td>McAllen, TX</td>
</tr>
<tr>
<td>Mende Mays</td>
<td>Algebra Teacher</td>
<td>Crockett Junior High</td>
<td>Odessa, TX</td>
</tr>
<tr>
<td>Rebecca Newburn</td>
<td>Lead Math Teacher</td>
<td>Davidson Middle School</td>
<td>San Rafael, CA</td>
</tr>
<tr>
<td>Vicki Petty</td>
<td>Mathematics Teacher</td>
<td>Central Middle School</td>
<td>Murfreesboro, TN</td>
</tr>
<tr>
<td>Susan Pippen</td>
<td>Mathematics Department Chair</td>
<td>Hinsdale South High School</td>
<td>Darien, IL</td>
</tr>
<tr>
<td>Elaine Rafferty</td>
<td>Mathematics Learning Specialist</td>
<td>Charleston County SD</td>
<td>Charleston, SC</td>
</tr>
<tr>
<td>Susan Rash</td>
<td>Manager of Secondary Curriculum</td>
<td>Red Clay CSD</td>
<td>Wilmington, DE</td>
</tr>
</tbody>
</table>
Contributing Authors

Carmen Whitman
Pflugerville, TX
Linda Antinone
Fort Worth, TX

Field Test Participants

Len Zigment
Mesa Ridge High School
Colorado Springs, CO

Vicky Petty
Central Middle School
Murfreesboro, TN

John Bakelaar
Peebles Middle School
Jackson, MS

Carey Carter
Alvarado High School
Alvarado, TX
# Data Analysis and Probability

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Mastering the Standards for Mathematical Practice

The topics described in the Standards for Mathematical Content will vary from year to year. However, the *way* in which you learn, study, and think about mathematics will not. The Standards for Mathematical Practice describe skills that you will use in all of your math courses. These pages show some features of your book that will help you gain these skills and use them to master this year’s topics.

1. **Make sense of problems and persevere in solving them.**
   
   Mathematically proficient students start by explaining to themselves the meaning of a problem... They analyze given, constraints, relationships, and goals. They make conjectures about the form... of the solution and plan a solution pathway...

   **In your book**
   
   Focus on Problem Solving describes a four-step plan for problem solving. The plan is introduced at the beginning of your book, and practice appears throughout.

2. **Reason abstractly and quantitatively.**

3. **Construct viable arguments and critique the reasoning of others.**

   Mathematically proficient students... justify their conclusions, [and]... distinguish correct... reasoning from that which is flawed.

   **In your book**
   
   Think and Discuss asks you to evaluate statements, explain relationships, apply mathematical principles, and justify your reasoning.
**Model with mathematics.**

Mathematically proficient students can apply... mathematics... to... problems... in everyday life, society, and the workplace...

**In your book**

Multi-Step Test Prep and Real-World Connections apply mathematics to other disciplines and in real-world scenarios.

---

**Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a... problem... [and] are... able to use technological tools to explore and deepen their understanding...

**In your book**

Algebra Labs and Technology Labs use concrete and technological tools to explore mathematical concepts.

---

**Attend to precision.**

Mathematically proficient students... communicate precisely... with others and in their own reasoning... [They] give carefully formulated explanations...

**In your book**

Reading and Writing Math and Write About It help you learn and use the language of math to communicate mathematics precisely.

---

**Look for and express regularity in repeated reasoning.**

**Look for and make use of structure.**

Mathematically proficient students... look both for general methods and for shortcuts...

**In your book**

Lesson examples group similar types of problems together, and the solutions are carefully stepped out. This allows you to make generalizations about—and notice variations in—the underlying structures.

---

**EXAMPLE 3** Finding Products in the Form \((a + b)(a - b)\)

\[
\begin{align*}
(a + b)(a - b) &= a^2 - b^2 \\
(3 + 8)(3 - 8) &= 3^2 - 8^2 \\
81 - 64 &= 17
\end{align*}
\]

Use the rule for \((a + b)(a - b)\) identify a and \(b\) and simplify.
**WEEK 1**

**DAY 1**
Which expression always represents an odd number when \( n \) is a natural number?
- [ ] \( n^2 + 1 \)
- [ ] \( 2n + 1 \)
- [ ] \( n^2 \)
- [ ] \( n + 1 \)

**DAY 2**

Cell phone bills are based on a flat monthly fee and the number of minutes used. In the equation \( c = 0.07m + 29.99 \), what does the variable \( m \) represent?
- [ ] The number of months billed
- [ ] The total amount of the bill
- [ ] The number of minutes used
- [ ] The phone number

**DAY 3**

Which equation best describes the relationship between the number of students and the number of tables in the cafeteria?

<table>
<thead>
<tr>
<th>Students (( n ))</th>
<th>Tables (( t ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>720</td>
<td>18</td>
</tr>
<tr>
<td>600</td>
<td>15</td>
</tr>
<tr>
<td>960</td>
<td>24</td>
</tr>
</tbody>
</table>

- [ ] \( n = 40t \)
- [ ] \( t = 40n \)
- [ ] \( n = 35t + 90 \)
- [ ] \( n = 45t - 90 \)

**DAY 4**

Which expression represents the verbal phrase “the sum of three times a number and five”?
- [ ] \( 3(n + 5) \)
- [ ] \( 3 + n \cdot 5 \)
- [ ] \( 3n + 5 \)
- [ ] \( 3 + (n + 5) \)

**DAY 5**

What is the next term in the pattern?

\(-3, 6, -12, 24, \ldots\)

- [ ] 36
- [ ] 30
- [ ] -32
- [ ] -48
DAY 1
What is the next term in the pattern?

\(-1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\)

A) \(-\frac{1}{10}\)
B) \(-\frac{1}{16}\)
C) \(\frac{1}{16}\)
D) \(\frac{1}{10}\)

DAY 2
Which expression is equivalent to

\(2(3x - 4) - 8x + 3?\)

F) \(2x + 11\)
G) \(-2x - 5\)
H) \(2x - 5\)
J) \(-2x - 11\)

DAY 3
Based on the table, which inequality correctly shows the relationship between \(x\) and \(y\)?

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-3</td>
<td>-8</td>
<td>-10</td>
<td>-9</td>
</tr>
</tbody>
</table>

A) \(x > -y\)
B) \(2x < -y\)
C) \(x < -y\)
D) \(2x > -y\)

DAY 4
The sum of three consecutive even numbers is 42. The sum can be represented by the equation \(n + (n + 2) + (n + 4) = 42\). What does \(n\) represent?

F) The greatest number
G) The average of the numbers
H) The middle number
J) The least number

DAY 5
A job advertisement states that the position pays $12 an hour. Which equation represents the relationship between the salary \(s\) and the number of hours \(h\) worked?

A) \(s = 12 + h\)
B) \(s = 12h\)
C) \(h = 12s\)
D) \(h = 12 + s\)
**DAY 1**

Which expression is equivalent to $2^3$?

- A. $2 \cdot 3$
- B. $3 + 3$
- C. $2 + 2 + 2$
- D. $2 \cdot 2 \cdot 2$

**DAY 2**

Which number is not a solution of $-7y + 19 < 75$?

- F. 13
- G. 0
- H. −2
- J. −8

**DAY 3**

Lydia received a gift card for $25.00 worth of smoothies from the Smoothie Spot. If the cost of each smoothie is $3.25, which table best describes $b$, the balance remaining on the gift card after she buys $n$ smoothies?

- A. $\begin{array}{c|c}
    n & b \\
    \hline
    1 & $21.75 \\
    3 & $15.25 \\
    4 & $12.00 \\
    7 & $2.25 \\
  \end{array}$
- B. $\begin{array}{c|c}
    n & b \\
    \hline
    2 & $18.50 \\
    4 & $12.00 \\
    6 & $6.50 \\
    8 & $0 \\
  \end{array}$
- C. $\begin{array}{c|c}
    n & b \\
    \hline
    1 & $21.75 \\
    2 & $18.50 \\
    5 & $15.25 \\
    7 & $12.00 \\
  \end{array}$
- D. $\begin{array}{c|c}
    n & b \\
    \hline
    2 & $18.50 \\
    3 & $15.25 \\
    5 & $7.75 \\
    6 & $4.50 \\
  \end{array}$

**DAY 4**

The band is trying to raise money to take a field trip to the Rock and Roll Hall of Fame. They decide to sell sweatshirts. The equation for the amount of money $a$ that they will make for selling $t$ sweatshirts is $a = 22t - 350$. In order to make at least $2100, how many sweatshirts do the band members need to sell?

- F. 112
- G. 111
- H. 80
- J. 79

**DAY 5**

What is the solution to the equation $8x - 10 = 54$?

- A. 5.5
- B. 6.75
- C. 8
- D. 12
**DAY 1**

Which expression is equivalent to $2x(3x + 5) - x^2 + 8$?

- **A** $-x^2 + 6x + 18$
- **B** $5x^2 + 10x + 8$
- **C** $10x + 14$
- **D** $-x^2 + 16x + 8$

**DAY 2**

What is the value of $3x^2 - 5x + 2$ when $x = -4$?

- **F** $-66$
- **G** $-26$
- **H** $30$
- **I** $70$

**DAY 3**

Which graph matches the values from the table?

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>2</td>
<td>-4</td>
<td>-8</td>
</tr>
</tbody>
</table>

- **A**
- **B**
- **C**
- **D**

**DAY 4**

A rectangle has a length of 8 m and a width of 3 m. If a similar rectangle has a length of 22 m, what is its width?

- **F** 58.67 m
- **G** 17 m
- **H** 8.25 m
- **I** 9 m

**DAY 5**

Simplify the algebraic expression $4(x + 5) - 2(x + 5)$.

- **A** $2x + 10$
- **B** $2x + 30$
- **C** $6x + 30$
- **D** $-8x^2 - 200$
**DAY 1**

Which description of the relationship between \( x \) and \( y \) best represents the table below?

<table>
<thead>
<tr>
<th>( x )</th>
<th>9</th>
<th>0</th>
<th>-6</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3</td>
<td>0</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

(A) The value of \( y \) is three times the value of \( x \).
(B) The value of \( x \) is three more than the value of \( y \).
(C) The value of \( x \) is three less than the value of \( y \).
(D) The value of \( x \) is three times the value of \( y \).

**DAY 2**

Claire’s father is 6 years more than 3 times her age. If her father is 39 years old, how old is Claire?

(F) 15 years old
(G) 11 years old
(H) 7 years old
(J) 6 years old

**DAY 3**

Which expression represents the phrase “four times the sum of a number and 2”?

(A) \( 4n + 2 \)
(B) \( 4(2n) \)
(C) \( 4(n + 2) \)
(D) \( 4 \cdot n + 2 \)

**DAY 4**

What is the solution to \(-8x - 3 = -4x + 5\)?

(F) -4
(G) -2
(H) 2
(J) 4

**DAY 5**

The cheerleaders are selling tickets to a pasta dinner to raise money for new competition outfits. They plan to charge $6 per person, and their total expenses for dinner are $135. Which value for the number of tickets sold would result in the cheerleaders not making a profit?

(A) 46
(B) 45
(C) 23
(D) 22
DAY 1
Which equation matches the data in the table?

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>1</th>
<th>-2</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>-1</td>
</tr>
</tbody>
</table>

A) \( y = -x + 5 \)
B) \( y = 2x - 1 \)
C) \( y = x + 3 \)
D) \( y = -3x + 11 \)

DAY 2
The drama club charges $3 admission to the one-act play festival. Their expenses are $115. In order for the club to make exactly $110 after expenses, how many people must attend the festival?

F) 39
G) 75
H) 114
I) 152

DAY 3
Melissa’s brother is 6 years less than twice her age. The sum of her age and her brother’s age is 27. Which equation best shows this information?

A) \( x + (6 - 2x) = 27 \)
B) \( x + (2x - 6) = 27 \)
C) \( 2x - 6 = 27 \)
D) \( x = 2x - 6 \)

DAY 4
A rectangle has a length of 5 inches and a width of 3 inches. If a similar rectangle has a width of 15 inches, what is its length?

F) 5 inches
G) 9 inches
H) 15 inches
I) 25 inches

DAY 5
A bus company sells an annual bus pass for $8.50, and then charges a rider $0.25 per ride. Use the equation \( c = 0.25b + 8.5 \) to answer the following question: How much will it cost for someone to ride the bus 38 times?

A) $118.00
B) $20.00
C) $18.00
D) $9.50
DAY 1

Daniel’s recipe for 24 cookies calls for 2 \(\frac{1}{2}\) cups of flour. How much flour will Daniel need to make 60 cookies?

- **A** 1 cup
- **B** 6 \(\frac{1}{4}\) cups
- **C** 6 \(\frac{1}{2}\) cups
- **D** 7 \(\frac{1}{2}\) cups

DAY 2

What is the value of \(2x^2 + 3x - 5\) when \(x = -2\)?

- **F** -7
- **G** -3
- **H** 5
- **I** 9

DAY 3

Which graph shows a line where each value of \(y\) is three more than half of \(x\)?

- **A**
- **B**
- **C**
- **D**

DAY 4

What is the solution to \(2x + 8 < 3x - 4\)?

- **F** \(x < 12\)
- **G** \(x > \frac{12}{5}\)
- **H** \(x > 12\)
- **I** \(x < \frac{12}{5}\)

DAY 5

Simplify the expression \(3(5x - 8) + 2(4x - 1)\).

- **A** 23\(x + 26\)
- **B** 23\(x + 22\)
- **C** 23\(x - 22\)
- **D** 23\(x - 26\)
DAY 1

A swimming pool charges an annual $75 membership fee, and it costs $1.50 each time a member brings a guest. Which equation shows the yearly cost $y$ in terms of the number of guests $g$?

- A $y = 75g + 1.5$
- B $y = -1.5g + 75$
- C $y = 1.5g + 75$
- D $y = 1.5g + 75g$

DAY 2

On a certain standardized test, the equation $s = 9q + 218$ is used to determine a student's score. In this equation, $s$ is the score and $q$ is the number of questions answered correctly. If the maximum score on the test is 650, how many questions are on the test?

- F 96 questions
- G 72 questions
- H 48 questions
- I 24 questions

DAY 3

If you belong to Lowell’s Gym and plan to work out 82 times in a year, what is your total cost for the year?

<table>
<thead>
<tr>
<th>Lowell's Gym</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membership fee</td>
</tr>
<tr>
<td>Workout fee (per visit)</td>
</tr>
</tbody>
</table>

- A $89
- B $157
- C $164
- D $239

DAY 4

A publishing company must ship boxes of a particular book to bookstores around the country. The boxes used for shipping can hold a maximum of 35 pounds. If the company wants to ship at least 10 books per box, what is the maximum weight of a book that can be put in the box?

- F 3 pounds
- G 3 1/2 pounds
- H 4 pounds
- I 25 pounds

DAY 5

Which verbal description does not match the function $f(x) = -\frac{1}{3}x - 5$?

- A The function value is the difference between $x$ times $-\frac{1}{3}$ and 5.
- B The function value is 5 less than the product of $x$ and $-\frac{1}{3}$.
- C The function value is $-\frac{1}{3}$ of $x$ subtracted from 5.
- D The function value is $-\frac{1}{3}$ of $x$ decreased by 5.
DAY 1

Brian calculates the charge for each lawn he mows by using the function \( f(t) = 10t + 5.5 \), where \( t \) is the number of hours spent mowing the lawn. He always works for at least one full hour. Which statement cannot be inferred from this information?

A) Brian's hourly rate is $10.
B) Brian includes a charge of $5.50 for all lawns.
C) Brian uses $5.50 worth of gas for each job.
D) The minimum that Brian will make for each lawn is $15.50.

DAY 2

Which of the following relationships has a negative correlation?

F) A person's height and weight
G) The number of minutes spent studying and a test grade
H) The outside temperature and the number of layers of clothes a person wears
J) The number of years a person spent in school and the person's salary

DAY 3

The graph shows the value of a car over a period of years. Which of the following statements cannot be concluded from the graph?

A) The car started at a value of approximately $8400.
B) The car's value depreciates more quickly at the beginning, and then less quickly as time goes on.
C) The car is worth approximately $3000 when it is 5 years old.
D) The car should be sold before its worth drops below $2000.

DAY 4

Which function best matches the table?

<table>
<thead>
<tr>
<th>( x )</th>
<th>-3</th>
<th>0</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

F) \( y = -\frac{2}{3}x \)
G) \( y = 2x + 4 \)
H) \( y = \frac{2}{3}x + 4 \)
J) \( y = \frac{1}{3}x + 3 \)

DAY 5

To change from degrees Celsius to degrees Fahrenheit, you can use the equation \( f = \frac{9}{5}c + 32 \). If the temperature is at least 55 degrees Fahrenheit, what inequality can you use to find the temperature in Celsius?

A) \( 55 \geq \frac{9}{5}c + 32 \)
B) \( f \geq \frac{9}{5} \cdot 55 + 32 \)
C) \( 55 \leq \frac{9}{5}c + 32 \)
D) \( f \leq \frac{9}{5} \cdot 55 + 32 \)
DAY 1

A function relating two quantities is 

\[ f(x) = \frac{1}{5}x + 6. \]

What will always be true based on this function?

A. \( f(x) \) will be less than \( x \).
B. If \( x \) is positive, then \( f(x) \) will be positive.
C. If \( x \) is negative, then \( f(x) \) will be negative.
D. \( f(x) \) will be greater than \( x \).

DAY 2

In the equation \( p = -40q + 163 \), which relationship between \( p \) and \( q \) is true?

F. You cannot determine the relationship based on this information.
G. \( q \) is dependent on \( p \).
H. \( p \) and \( q \) are independent of each other.
I. \( p \) is dependent on \( q \).

DAY 3

What is the equation of the line shown?

A. \( y = 2x \)
B. \( y = -2x \)
C. \( y = \frac{1}{2}x \)
D. \( y = -\frac{1}{2}x \)

DAY 4

What is the solution to \(-8x - 3 \geq -4x + 5\)?

F. \( x \leq -2 \)
G. \( x \geq 2 \)
H. \( x \leq 2 \)
I. \( x \geq -2 \)

DAY 5

The sum of three consecutive integers is 54. What is the greatest of the three integers?

A. 17
B. 18
C. 19
D. 20
DAY 1
Darian has a picture that is 4 inches wide and 6 inches long. He wants to enlarge the picture so that it has a width of 18 inches. What is the length of the enlarged picture?

A. 27 inches
B. 24 inches
C. 20 inches
D. 12 inches

DAY 2
At a computer repair shop, the function \( f(t) = 40t + 35 \) is used to calculate fees, where \( t \) is the number of hours technicians spend working on the computer. If you pay $315 to get your computer repaired, how long did the technician work on repairing your computer?

F. 8.75 hours
G. 7.875 hours
H. 7.5 hours
I. 7 hours

DAY 3
The scatter plot shows the relationship between the size of a diamond in Carats and its retail price. What is the approximate retail price of a 0.30 Carat diamond?

A. $400
B. $600
C. $800
D. $1000

DAY 4
What is the next term in the pattern?
50, 10, 2, \( \frac{2}{5} \), __, ...

F. \( \frac{1}{25} \)
G. \( \frac{2}{25} \)
H. \( \frac{2}{10} \)
J. \( \frac{1}{5} \)

DAY 5
Which expression is equivalent to \( 5x - 35 \)?

A. \( 5(x - 35) \)
B. \( 5(x - 7) \)
C. \( 5(x + 35) \)
D. \( 5(x + 7) \)
DAY 1
Based on the information in the table, what is the relationship between the weight of the object and its volume?

<table>
<thead>
<tr>
<th>Weight (lb)</th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (in³)</td>
<td>0.5</td>
<td>1.25</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

- **A** The weight is 1.5 more than the volume.
- **B** The volume is 4 times the weight.
- **C** The weight is 4 times the volume.
- **D** The volume is 6 less than the weight.

DAY 2
What is the range of the function shown in the table?

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>2</td>
<td>-2</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

- **F** {1, 3, 5, 8}
- **G** {4}
- **H** {-2, 0, 2}
- **I** {7}

DAY 3
Which graph shows a linear function?

- **A**
- **B**
- **C**
- **D**

DAY 4
What is the solution to the equation $6(x - 9) = -12x + 36$?

- **F** $x = 5$
- **G** $x = 2.5$
- **H** $x = -1$
- **I** $x = -5$

DAY 5
Simplify the algebraic expression $5x - 2(4x - 3) + 7$.

- **A** $-3x + 1$
- **B** $13x + 13$
- **C** $-3x + 13$
- **D** $3x + 1$
**DAY 1**

The slope of a line is $-\frac{1}{4}$, and the y-intercept is $-3$. What is the equation of the line?

- A. $x + y = -3$
- B. $y = -\frac{1}{4}x + 1$
- C. $y = \frac{1}{4}x - 3$
- D. $x - 4y = -3$

**DAY 2**

A taxi company charges a $2.50 fee per ride plus an additional $2.10 per mile traveled. If the taxi fee increases to $2.75, which characteristic of this function would change?

- F. The slope
- G. The x-intercept
- H. The y-intercept
- I. There would be no changes

**DAY 3**

Which description of the relationship between $x$ and $y$ best matches the graph below?

- A. The value of $y$ is 3 times the value of $x$.
- B. The value of $x$ is 2 more than the value of $y$.
- C. The value of $x$ is 2 less than the value of $y$.
- D. The value of $x$ is 3 times the value of $y$.

**DAY 4**

In which relationship are the two quantities independent of one another?

- F. The amount of tax paid for an item and the price of the item
- G. The number of snacks bought from a snack machine and the amount of money in the machine
- H. The number of hours worked at $6.50 per hour and the amount of money earned
- I. The age of a person and the number of televisions in his or her home

**DAY 5**

The linear function $f(x) = 3x + 15$ models the cost of renting $x$ movies over the course of a year from a certain store. What range restrictions will there be if you graph this function?

- A. $f(x) \geq 0$
- B. $f(x) \geq 15$
- C. $f(x) > 0$
- D. $0 \leq f(x) \leq 15$
**DAY 1**

Which function matches the data in the table?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

- **A** $f(x) = -3x + 6$
- **B** $f(x) = -5x$
- **C** $f(x) = x^2 - 2x$
- **D** $f(x) = 3x^2 - 12$

**DAY 2**

A consultant charges her clients for her services based on an equation relating the total bill $f(h)$ to the number of hours worked $h$. The best interpretation of this function $f(h) = 125h + 150$ is:

- **F** She charges $150 per hour plus a $125 flat fee.
- **G** She charges $125 per hour plus a $150 flat fee.
- **H** She charges $275 per hour.
- **J** Her hourly charges vary from $125 to $150 depending on the job.

**DAY 3**

A rectangle has the dimensions shown. Which value for the area has the correct number of significant digits?

- **A** 55.728 square centimeters
- **B** 55.73 square centimeters
- **C** 55.7 square centimeters
- **D** 56 square centimeters

**DAY 4**

What is the range of the function $f(x) = 2x + 3$?

- **F** All real numbers
- **G** $f(x) \geq 3$
- **H** $f(x) \geq 0$
- **J** $f(x) \geq -3$

**DAY 5**

At Terry’s TVs, profit is calculated using the equation $p = 0.4c - 125$, where $p$ is the profit and $c$ is the wholesale cost. If the company makes a profit of $375 on one TV, what is the wholesale cost of the TV?

- **A** $25$
- **B** $937.50$
- **C** $1250$
- **D** $1062.50
**WEEK 15**

**DAY 1**

Which graph shows a linear function?

- **A**  
- **B**  
- **C**  
- **D**

**DAY 2**

The relationship between which two quantities can be represented by a linear function?

- The volume of a cube and its side length
- The perimeter of a rectangle and its area
- The perimeter of an equilateral triangle and its side length
- The area of a square and its side length

**DAY 3**

Which equation represents a linear function with a slope of \( \frac{2}{3} \)?

- \( y = \frac{3}{2} x \)  
- \( y = \frac{2}{3} x + 4 \)  
- \( y = x + \frac{2}{3} \)  
- \( y = \frac{2}{3} \)

**DAY 4**

What are the \( x \)- and \( y \)-intercepts of \( y = \frac{2}{5} x - 2 \)?

- \( x \)-intercept: \(-2\); \( y \)-intercept: \(5\)
- \( x \)-intercept: \(\frac{2}{5}\); \( y \)-intercept: \(-2\)
- \( x \)-intercept: \(0\); \( y \)-intercept: \(-2\)
- \( x \)-intercept: \(5\); \( y \)-intercept: \(-2\)

**DAY 5**

A system of equations is set up to determine how many pounds of hazelnut coffee and how many pounds of Colombian coffee were mixed together to make a blend. The total mixture was 20 pounds of coffee. Which of the following is not a possible solution to the system?

- \( (7, 13) \)
- \( (11, 9) \)
- \( (32, -12) \)
- \( (1, 19) \)
DAY 1

If a graph shows the relationship between the amount of sand in the top of an hourglass $y$ over time $x$, what quantity does the $x$-intercept represent?

- **A** The amount of sand that is in the top of the hourglass originally
- **B** The speed at which the sand passes to the bottom of the hourglass
- **C** The height of the hourglass
- **D** The amount of time it takes for all of the sand to pass to the bottom of the hourglass

DAY 2

Which equation represents a line parallel to the one shown?

- $y = \frac{3}{2}x + 1$
- $y = \frac{2}{3}x + 1$
- $y = \frac{3}{2}x - 1$
- $y = -\frac{2}{3}x$

DAY 3

The Whartons are planting a rectangular vegetable garden. Suppose the width of the garden is $x$ feet and the length of the garden is $5 - x$ feet. Which type of function can you use to model the area $y$ of the garden in terms of its width?

- **A** linear
- **B** quadratic
- **C** exponential decay
- **D** exponential growth

DAY 4

Your school is selling candles to raise money. If profit $y$ is related to the number of items sold $x$, what are reasonable restrictions on the domain?

- $y \geq 0$
- No restrictions
- $x \geq 0$
- $0 \leq y \leq 2000$

DAY 5

What is the solution to the system of equations graphed below?

- $(-1, 3)$
- $(-3, -1)$
- $(-1, -3)$
- $(3, -1)$
DAY 1
A store manager increases the wholesale cost of an item by 35%. Which statement best represents the functional relationship between the wholesale cost of the item and the markup on the item?

A. The markup is dependent on the wholesale cost.
B. The wholesale cost is dependent on the markup.
C. The markup and the wholesale cost are independent of each other.
D. The relationship cannot be determined.

DAY 2
Which situation can best be represented by a linear function?

F. The distance that an airplane is away from the airport as it comes in for a landing and time
G. The distance traveled by a car moving at a constant speed and time
H. The number of apples on a tree and the number of trees in the orchard
I. The height from the ground of a person riding a roller coaster and time

DAY 3
The scatter plot shows the percent of households that own a car versus the household income. What is a reasonable estimate for the percent of households that own a car if the household income is $50,000?

A. 50%
B. 65%
C. 77%
D. 85%

DAY 4
What is the slope of the line with equation $3x - 5y = 12$?

F. $-3$
G. $\frac{3}{5}$
H. $\frac{3}{5}$
I. 3

DAY 5
Joy and a friend start a business. What linear function relates the total income and each partner’s share of the profit based on the data in the table?

<table>
<thead>
<tr>
<th>Total Income (t)</th>
<th>Profit Share (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$350.00$</td>
<td>$50.00$</td>
</tr>
<tr>
<td>$425.00$</td>
<td>$87.50$</td>
</tr>
<tr>
<td>$500.00$</td>
<td>$125.00$</td>
</tr>
</tbody>
</table>

A. $p = \frac{1}{2}t - 125$
B. $p = \frac{1}{4}t - 37.5$
C. $p = t - 300$
D. $p = \frac{1}{7}t$
DAY 1
What is the range of the function $f(x) = 2x^2 + 1$ if the domain is $\{-2, 0, 3\}$?
A. $\{1, 9, 19\}$  
B. $\{0, 8, 18\}$  
C. $\{-9, 1, 19\}$  
D. $\{-2, 0, 3\}$

DAY 2
Which function has a $y$-intercept of $-2$ and a graph whose slope is $-\frac{5}{4}$?
F. $5x + 4y = 2$  
G. $5x + 2$  
H. $y = -\frac{5}{4}x + 2$  
J. $y = \frac{5}{4}x - 2$

DAY 3
The graph represents a residual plot for a data set and a linear model. Based on the residual plot, which statement best describes the goodness of fit of the linear model?
A. The line is a good fit for the data.
B. The line is in the wrong place.
C. The data are not linear.
D. The data may have relatively no correlation.

DAY 4
Which of the following cannot be represented by a linear function?
F. The amount of water $w$ coming out of a dripping faucet over $m$ minutes
G. The height $h$ of a person over $t$ years
H. The gross sales $s$ made from the sale of $c$ CDs priced at $14$ each
J. The amount of sales tax on a purchase of $d$ dollars if the rate is $6\%$

DAY 5
The graph shows the distance that a person is from home while driving at a constant speed. What quantity is represented by the $y$-intercept?
A. The speed at which the person is driving
B. The distance from home at the start
C. The amount of time it takes to get home
D. The distance from home at the end
DAY 1

Assuming that the graph below has the same x- and y-scale, which is the best estimate for the solution to the system?

\[ \begin{align*}
\text{A} & \quad (-5, 10) \\
\text{B} & \quad (-4, 2) \\
\text{C} & \quad (3, -6) \\
\text{D} & \quad (8, -4)
\end{align*} \]

DAY 2

What is the y-intercept of the function whose graph has a slope of \(-\frac{1}{3}\) and passes through the point \((-6, 4)\)?

\[ \begin{align*}
\text{A} & \quad 6 \\
\text{B} & \quad 2 \\
\text{C} & \quad -2 \\
\text{D} & \quad -6
\end{align*} \]

DAY 3

Which graph shows a function from the family \(f(x) = x^2\)?

\[ \begin{align*}
\text{A} & \quad \text{Graph 1} \\
\text{B} & \quad \text{Graph 2} \\
\text{C} & \quad \text{Graph 3} \\
\text{D} & \quad \text{Graph 4}
\end{align*} \]

DAY 4

Miguel earns a 15% commission on his sales in addition to a salary of $500 a week. His earnings can be modeled by the equation \(p = 0.15s + 500\). What restrictions on the values of \(p\) and \(s\) best fit this situation?

\[ \begin{align*}
\text{A} & \quad p \geq 0, s \text{ can be any value.} \\
\text{B} & \quad s \geq 0, p \text{ can be any value.} \\
\text{C} & \quad s \geq 500, p \geq 0 \\
\text{D} & \quad s \geq 0, p \geq 500
\end{align*} \]

DAY 5

If \(f(x) = \frac{1}{2}x\), what best describes the relationship between \(x\) and \(f(x)\)?

\[ \begin{align*}
\text{A} & \quad \text{As the value of } x \text{ increases by 1, the value of } f(x) \text{ will increase by 2.} \\
\text{B} & \quad \text{As the value of } x \text{ increases by 1, the value of } f(x) \text{ will remain the same.} \\
\text{C} & \quad \text{As the value of } x \text{ increases by 1, the value of } f(x) \text{ will increase by } \frac{1}{2}. \\
\text{D} & \quad \text{The value of } x \text{ will not affect the value of } f(x).
\end{align*} \]
DAY 1
On Saturday, Suzie’s Pretzel Stand sells a total of 12 items. Suzie charges $3 for a pretzel and $2 for a milkshake. If she took in $32 on Saturday, which system of equations can be used to determine how many pretzels and how many milkshakes were sold?

A) \[ \begin{align*} 3x + 2y &= 12 \\ x + y &= 32 \end{align*} \]
B) \[ \begin{align*} 3x - 2y &= 32 \\ x + y &= 12 \end{align*} \]
C) \[ \begin{align*} 3x - 2y &= 12 \\ x + y &= 32 \end{align*} \]
D) \[ \begin{align*} x + y &= 12 \\ 3x + 2y &= 32 \end{align*} \]

DAY 2
What is the slope of the line shown?

F) \( \frac{-4}{3} \)
G) \( \frac{-3}{4} \)
H) \( \frac{3}{4} \)
J) \( \frac{4}{3} \)

DAY 3
If the x-intercept of the function graphed below were to stay the same and the slope of the line decreased, what impact would that have on the y-intercept?

A) The y-intercept will decrease.
B) The y-intercept will remain the same.
C) The y-intercept will increase.
D) The y-intercept will double.

DAY 4
A system of equations is set up to determine the number of nickels and the number of quarters that Elissa has. Which ordered pair does not represent a valid solution to this system?

F) (0, 15)
G) (12, –5)
H) (31, 4)
J) (7, 0)

DAY 5
A gym membership costs $25 a month plus $3 per visit. This is modeled by the function \( c = 3v + 25 \), where \( c \) is the cost per month and \( v \) is the number of visits. If the slope of this function’s graph were to increase, what would that mean about the prices that the gym charges?

A) The gym raised its monthly fee.
B) The gym lowered the cost per visit.
C) The gym raised the cost per visit.
D) The gym lowered its monthly fee.
DAY 1
Which of the functions, when graphed, will result in the most narrow parabola?

A. \( y = 5x^2 \)
B. \( y = \frac{1}{5}x^2 \)
C. \( y = x^2 \)
D. \( y = \frac{1}{2}x^2 \)

DAY 2
How do the graphs of the functions \( f(x) = x^2 - 5 \) and \( g(x) = x^2 + 4 \) relate to each other?

E. The graph of \( f(x) \) is 9 units to the left of the graph of \( g(x) \).
F. The graph of \( f(x) \) is 1 unit below the graph of \( g(x) \).
G. The graph of \( f(x) \) is 9 units below the graph of \( g(x) \).
H. The graph of \( f(x) \) is 1 unit to the left of the graph of \( g(x) \).

DAY 3
The graph shows the proposed balance in Jake’s bank account if Jake saves an average of $15 a week. Which statement would not be true if the slope of the line were to increase?

A. Jake is saving more money per week.
B. Jake started with more money in his bank account.
C. It will take less time for Jake’s bank balance to reach $120.
D. After 6 weeks Jake will have more than $90 in his bank account.

DAY 4
What are the roots of the function \( f(x) = (2x - 1)(x + 5) \)?

A. \( \frac{1}{2} \) and -5
B. -1 and 5
C. 1 and -5
D. \( -\frac{1}{2} \) and 5

DAY 5
For a typical shot in basketball, the ball’s height in feet will be a function of time in seconds, modeled by an equation such as \( h = -16t^2 + 25t + 6 \). Which inequality shows the restrictions that should be put on the domain of this function?

A. \( t \leq 0 \)
B. \( h \leq 0 \)
C. \( h \geq 0 \)
D. \( t \geq 0 \)
DAY 1
What are the roots of the graphed function?

- A) 2 and 4
- B) -2 and -4
- C) -2 and 4
- D) 2 and -4

DAY 2
What is the range of the function shown?

- F) -2 < y < 3
- G) -3 < y ≤ 2
- H) -3 ≤ y ≤ 2
- I) 2 ≤ y ≤ 3

DAY 3
The graph shows the height of a baseball from the time it is thrown until the time it hits the ground. What value is not shown on the graph?

- A) The amount of time that the ball was in the air
- B) The height at which the ball started
- C) The maximum height that the ball reached
- D) The speed at which the ball was thrown

DAY 4
What are the solution(s) of the equation $x^2 - 10x - 24 = 0$?

- F) $x = -4, 6$
- G) $x = 2, -12$
- H) $x = -2, 12$
- J) $x = 4, -6$

DAY 5
What is the solution to the system of equations?

\[
\begin{align*}
3x - 4y &= 19 \\
y &= 2x - 11
\end{align*}
\]

- A) (5, -1)
- B) (-5, -21)
- C) (5, 1)
- D) (-5, -8.5)
**DAY 1**

Which graph shows a function $y = ax^2$ when $a > 1$?

- **A**
- **B**
- **C**
- **D**

**DAY 2**

Which best describes the difference between the graphs of $f(x) = x^2$ and $g(x) = x^2 - 3$?

- **F** The graph of $g(x)$ is translated 3 units up from the graph of $f(x)$.
- **G** The graph of $g(x)$ is translated 3 units left from the graph of $f(x)$.
- **H** The graph of $g(x)$ is translated 3 units right from the graph of $f(x)$.
- **I** The graph of $g(x)$ is translated 3 units down from the graph of $f(x)$.

**DAY 3**

The graph shows the height of water coming out of a fountain over time. How many seconds pass before the water reaches the ground?

- **A** 0
- **B** 2
- **C** 4
- **D** 18

**DAY 4**

What is one solution to the equation $2x^2 + 7x + 3 = 0$?

- **F** $x = 3$
- **G** $x = \frac{1}{2}$
- **H** $x = -\frac{1}{2}$
- **I** $x = -1$

**DAY 5**

What is the solution to the system of equations $4x + 2y = -8$ and $x - 2y = 13$?

- **A** $(25, 6)$
- **B** $(1, 6)$
- **C** $(-1, -6)$
- **D** $(1, -6)$
DAY 1

Which is the parent function of the graph shown?

- A. \( y = x \)
- B. \( y = |x| \)
- C. \( y = x^2 \)
- D. \( y = x^3 \)

DAY 2

What is the \( x \)-intercept of the function shown?

- F. \( x = -3 \)
- G. \( x = -2 \)
- H. \( x = 2 \)
- J. \( x = 3 \)

DAY 3

Which situation cannot be described by a linear function?

- A. The amount of commission earned on a sale if the commission rate is 12%
- B. The area of a square given its side length
- C. The cost of renting a car if the charge is $25 plus $0.15 per mile
- D. The amount paid for babysitting \( h \) hours if you charge $8.25 per hour

DAY 4

What is the equation of the line that passes through the points \((-4, 2)\) and \((-8, 8)\)?

- F. \( y = -\frac{3}{2}x - 4 \)
- G. \( y = -\frac{3}{2}x - 20 \)
- H. \( y = -\frac{3}{2}x + 8 \)
- J. \( y = -\frac{3}{2}x + 4 \)

DAY 5

What is the slope of the line given the equation \(3x - 4y = -8\)?

- A. \( m = 3 \)
- B. \( m = 2 \)
- C. \( m = \frac{3}{4} \)
- D. \( m = -3 \)
Choose a method to solve

Describe how to use the discriminant to find the number of real solutions to

Practice for greater success on your exams.
The Problem Solving Plan

Mathematical problems are a part of daily life. You need to use a good problem-solving plan to be a good problem solver. The plan used in this textbook is outlined below.

**UNDERSTAND the Problem**

First make sure you understand the problem you are asked to solve.

- **What are you asked to find?**
  Restate the question in your own words.

- **What information is given?**
  Identify the key facts given in the problem.

- **What information do you need?**
  Determine what information you need to solve the problem.

- **Do you have all the information needed?**
  Determine if you need more information.

- **Do you have too much information?**
  Determine if there is unnecessary information and eliminate it from your list of important facts.

**Make a PLAN**

Plan how to use the information you are given.

- **Have you solved similar problems?**
  Think about similar problems you have solved successfully.

- **What problem solving strategy or strategies could you use to solve this problem?**
  Choose an appropriate problem solving strategy and decide how you will use it.

**SOLVE**

Use your plan to solve the problem. Show the steps in the solution, and write a final statement that gives the solution to the problem.

**LOOK BACK**

Check your answer against the original problem.

- **Have you answered the question?**
  Make sure you have answered the original question.

- **Is the answer reasonable?**
  The answer must make sense in relation to the question.

- **Are your calculations correct?**
  Check to make sure your calculations are accurate.

- **Can you use another strategy or solve the problem in another way?**
  Using another strategy is a good way to check your answer.

- **Did you learn anything that could help you solve similar problems in the future?**
  Try to remember the types of problems you have solved and the strategies you applied.
You're the Designer

Research studies are designed to gather and analyze data in order to answer questions. The results are displayed in tables and graphs.
Study Strategy: Prepare for Your Final Exam

Math is a cumulative subject, so your final exam will probably cover all of the material you have learned since the beginning of the course. Preparation is essential for you to be successful on your final exam. It may help you to make a study timeline like the one below.

2 weeks before the final:
- Look at previous exams and homework to determine areas I need to focus on; rework problems that were incorrect or incomplete.
- Make a list of all formulas, definitions, and properties I need to know for the final.
- Create a practice exam using problems from the book that are similar to problems from each exam.

1 week before the final:
- Take the practice exam and check it. For each problem I miss, find 2 or 3 similar ones and work those.
- Work with a friend in the class to quiz each other on formulas, definitions, and properties from my list.

1 day before the final:
- Make sure I have pencils, calculator (check batteries!), and ruler.

Data Analysis and Probability

Try This

1. Create a timeline that you will use to study for your final exam.
Bar and Circle Graphs

Data displayed in bar graphs and circle graphs can be used to solve equations. In these problems, parts of the graphs are missing.

Example 1

The top part of this graph was torn off. If Warren received 15% of the votes, how many votes did Adams receive?

Step 1 Find the total number of votes. Let \( t \) represent the total.

\[
15\% \text{ of the total votes is 42 votes.} \quad 0.15 \cdot t = 42
\]

\[
0.15t = 42 \\
\quad t = 280 \text{ votes}
\]

Step 2 Find the number of votes Adams received.

Let \( a \) represent the number of votes received by Adams. Let \( h, w, s, \) and \( m \) represent the number of votes received by Hansen, Warren, Sweeney, and Marino.

\[
t = a + h + w + s + m \\
280 = a + 52 + 42 + 65 + 28 \\
280 = a + 187 \\
\quad - 187 \quad - 187 \\
93 = a
\]

Adams received 93 votes.

Try This

1. The missing bar is twice as tall as the bar for week 2. How many total miles did Kim bike in these five weeks?

2. People aged 20–29 years walked 275 more miles than the oldest age group. Find the total miles walked by all age groups.
Remember that a circle graph represents all the data in a data set. The percent represented by each section is a part of the whole data set, so the sum of all the percents must be 100%.

Example 2

A survey asked people in a neighborhood to agree or disagree with the following statement:

“We need a traffic light at Jefferson Avenue and Third Street.”

If 35% of the people disagreed with the statement, how many people had no opinion?

The number of people who answered “no opinion” is missing from the graph.

Step 1 Find the total number of people who answered the survey. Let \( t \) represent the total number of people.

\[
35\% \text{ of } \frac{t}{100} \text{ is } 63 \text{ people.}
\]

\[
0.35t = 63
\]

\[
t = 180 \text{ people}
\]

Step 2 Find the number of people who answered “no opinion.”

Let \( n \) represent the number of “no opinion” answers. Let \( d, s, g, \) and \( a \) represent the number of “disagree,” “strongly disagree,” “strongly agree,” and “agree” answers.

\[
t = n + d + s + g + a
\]

\[
180 = n + 63 + 45 + 36 + 27
\]

\[
180 = n + 171
\]

\[
-171 \quad -171
\]

\[
9 = n
\]

There were 9 people who had no opinion.

Try This

3. The students in a junior high school voted on their choice for a field trip. Sixteen students voted for the natural history museum. How many students voted for the winning choice?

4. At the fall dance recital, 40% of the tickets were sold to adults. What percent of the sales were to seniors?
Organizing and Displaying Data

Who uses this?
Nutritionists can display health information about food in bar graphs.

Objectives
Organize data in tables and graphs.
Choose a table or graph to display data.

Vocabulary
bar graph
line graph
circle graph

Bar graphs, line graphs, and circle graphs can be used to present data in a visual way.

A bar graph displays data with vertical or horizontal bars. Bar graphs are a good way to display data that can be organized into categories. Using a bar graph, you can quickly compare the categories.

EXAMPLE 1
Reading and Interpreting Bar Graphs

Use the graph to answer each question.

Fat Content of a Sub Sandwich

<table>
<thead>
<tr>
<th>Ingredient</th>
<th>Fat (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread</td>
<td>1</td>
</tr>
<tr>
<td>Ham</td>
<td>6</td>
</tr>
<tr>
<td>Turkey</td>
<td>3</td>
</tr>
<tr>
<td>Cheese</td>
<td>9</td>
</tr>
<tr>
<td>Mayonnaise</td>
<td>11</td>
</tr>
</tbody>
</table>

A Which ingredient contains the most fat?
mayonnaise
The bar for mayonnaise is the longest.

B How many more grams of fat are in ham than in turkey?
6 – 3 = 3
There are 6 grams of fat in ham and 3 grams of fat in turkey.

C How many total fat grams are in this sandwich?
1 + 6 + 3 + 9 + 11 = 30
Add the number of fat grams for each ingredient.

D What percent of the total fat grams in this sandwich are from turkey?
\[
\frac{3}{30} = \frac{1}{10} = 10\%
\]
Out of 30 total fat grams, 3 fat grams are from turkey.

Use the graph to answer each question.
1a. Which ingredient contains the least amount of fat?
1b. Which ingredients contain at least 8 grams of fat?
A double-bar graph can be used to compare two data sets. A double-bar graph has a key to distinguish between the two sets of data.

**Example 2**

**Reading and Interpreting Double Bar Graphs**

Use the graph to answer each question.

**A**

In which year did State College have the greatest average attendance for basketball?

2003

*Find the tallest orange bar.*

**B**

On average, how many more people attended a football game than a basketball game in 2001?

20,000 - 13,000 = 7000

*Find the height of each bar for 2001 and subtract.*

2. Use the graph to determine which years had the same average basketball attendance. What was the average attendance for those years?

A **line graph** displays data using line segments. Line graphs are a good way to display data that changes over a period of time.

**Example 3**

**Reading and Interpreting Line Graphs**

Use the graph to answer each question.

**A**

At what time was the temperature the warmest?

4:00 P.M. *Identify the highest point.*

**B**

During which 4-hour time period did the temperature increase the most?

From 8:00 A.M. to noon *Look for the segment with the greatest positive slope.*

3. Use the graph to estimate the difference in temperature between 4:00 A.M. and noon.
A double-line graph can be used to compare how two related data sets change over time. A double-line graph has a key to distinguish between the two sets of data.

**Example 4**

**Reading and Interpreting Double-Line Graphs**

Use the graph to answer each question.

![Airfare Between Two Cities Graph]

A. In which month(s) did airline B charge more than airline A?
   April and September  
   **Identify the points when the purple line is higher than the blue line.**

B. During which month(s) did the airlines charge the same airfare?
   May  
   **Look for the point where the data points overlap.**

**Check It Out!**

4. Use the graph to describe the general trend of the data.

A circle graph shows parts of a whole. The entire circle represents 100% of the data and each sector represents a percent of the total. Circle graphs are good for comparing each category of data to the whole set.

**Example 5**

**Reading and Interpreting Circle Graphs**

Use the graph to answer each question.

![Fibi’s Fruit Salad Recipe]

A. Which two fruits together make up half of the fruit salad?
   bananas and strawberries  
   **Look for two fruits that together make up half of the circle.**

B. Which fruit is used more than any other?
   cantaloupe  
   **Look for the largest sector of the graph.**

**Check It Out!**

5. Use the graph to determine what percent of the fruit salad is cantaloupe.
Example 6
Choosing and Creating an Appropriate Display

Use the given data to make a graph. Explain why you chose that type of graph.

A
Livestock Show Entries

<table>
<thead>
<tr>
<th>Animal</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chicken</td>
<td>38</td>
</tr>
<tr>
<td>Goat</td>
<td>10</td>
</tr>
<tr>
<td>Horse</td>
<td>32</td>
</tr>
<tr>
<td>Pig</td>
<td>12</td>
</tr>
<tr>
<td>Sheep</td>
<td>25</td>
</tr>
</tbody>
</table>

A bar graph is appropriate for this data because it will be a good way to compare categories.

Step 1 Determine an appropriate scale and interval. The scale must include all of the data values. The scale is separated into equal parts, called intervals.

Step 2 Use the data to determine the lengths of the bars. Draw bars of equal width. The bars should not touch.

Step 3 Title the graph and label the horizontal and vertical scales.

B
Division of Crops

<table>
<thead>
<tr>
<th>Crop</th>
<th>Area (acres)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>70</td>
</tr>
<tr>
<td>Fallow</td>
<td>50</td>
</tr>
<tr>
<td>Mixed vegetables</td>
<td>10</td>
</tr>
<tr>
<td>Soybeans</td>
<td>40</td>
</tr>
<tr>
<td>Wheat</td>
<td>30</td>
</tr>
</tbody>
</table>

A circle graph is appropriate for this data because it shows categories as parts of a whole.

Step 1 Calculate the percent of the total represented by each category.

Corn: \( \frac{70}{200} = 0.35 = 35\% \)

Soybeans: \( \frac{40}{200} = 0.2 = 20\% \)

Fallow: \( \frac{50}{200} = 0.25 = 25\% \)

Wheat: \( \frac{30}{200} = 0.15 = 15\% \)

Mixed vegetables: \( \frac{10}{200} = 0.05 = 5\% \)

Step 2 Find the angle measure for each sector of the graph. Since there are 360° in a circle, multiply each percent by 360°.

Corn: \( 0.35 \times 360° = 126° \)

Fallow: \( 0.25 \times 360° = 90° \)

Mixed vegetables: \( 0.05 \times 360° = 18° \)

Soybeans: \( 0.2 \times 360° = 72° \)

Wheat: \( 0.15 \times 360° = 54° \)

Step 3 Use a compass to draw a circle. Mark the center and use a straightedge to draw one radius. Then use a protractor to draw each central angle.

Step 4 Title the graph and label each sector.
Use the given data to make a graph. Explain why you chose that type of graph.

<table>
<thead>
<tr>
<th>Chinnick College Enrollment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>1930</td>
</tr>
<tr>
<td>1955</td>
</tr>
<tr>
<td>1980</td>
</tr>
<tr>
<td>2005</td>
</tr>
</tbody>
</table>

A line graph is appropriate for this data because it will show the change in enrollment over a period of time.

**Step 1** Determine the scale and interval for each set of data. Time should be plotted on the horizontal axis because it is independent.

**Step 2** Plot a point for each pair of values. Connect the points using line segments.

**Step 3** Title the graph and label the horizontal and vertical scales.

![Chinnick College Enrollment Graph](image)

6. Use the given data to make a graph. Explain why you chose that type of graph.

The data below shows how Vera spends her time during a typical 5-day week during the school year.

<table>
<thead>
<tr>
<th>Vera's Schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity</td>
</tr>
<tr>
<td>Time (h)</td>
</tr>
</tbody>
</table>

**THINK AND DISCUSS**

1. What are some comparisons you can make by looking at a bar graph?
2. Name some key components of a good line graph.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, tell which kind of graph is described.
GUIDED PRACTICE

Vocabulary Use the vocabulary from this lesson to answer the following questions.

1. In a circle graph, what does each sector represent?
2. In a line graph, how does the slope of a line segment relate to the rate of change?

Use the bar graph for Exercises 3 and 4.

3. Estimate the total number of animals at the shelter.
4. There are 3 times as many _____ as _____ at the animal shelter.

Use the double-bar graph for Exercises 5–7.

5. About how much more is a club level seat at stadium A than at stadium B?
6. Which type of seat is the closest in price at the two stadiums?
7. Describe one relationship between the ticket prices at stadium A and stadium B.

Use the line graph for Exercises 8 and 9.

8. Estimate the number of tickets sold during the week of the greatest sales.
9. Which one-week period of time saw the greatest change in sales?

Use the double-line graph for Exercises 10–12.

10. When was the support for the two candidates closest?
11. Estimate the difference in voter support for the two candidates five weeks before the election.
12. Describe the general trend(s) of voter support for the two candidates.
13. Which color is least represented in the ball playpen?

14. There are 500 balls in the playpen. How many are yellow?

15. Which two colors are approximately equally represented in the ball playpen?

16. The table shows the breakdown of Karim’s monthly budget of $100. Use the given data to make a graph. Explain why you chose that type of graph.

17. Estimate the difference in population between the tribes with the largest and the smallest population.

18. Approximately what percent of the total population shown in the table is Cherokee?

19. On what day did Ray do the most overall business?

20. On what day did Ray have the busiest lunch?

21. On Sunday, about how many times as great was the number of dinner customers as the number of lunch customers?

22. Between which two games did Marlon’s score increase the most?

23. Between which three games did Marlon’s score increase by about the same amount?
24. What was the average value per share of Juan’s two stocks in July 2004?
25. Which stock’s value changed the most over any time period?
26. Describe the trend of the values of both stocks.

27. About what percent of the total number of cars are hopper cars?
28. About what percent of the total number of cars are gondola or tank cars?

29. The table shows the weight of twin babies at various times from birth to four weeks old. Use the given data to make a graph. Explain why you chose that type of graph.

Write bar, double-bar, line, double-line, or circle to indicate the type of graph that would best display the data described.

30. attendance at a carnival each year over a ten-year period
31. attendance at two different carnivals each year over a ten-year period
32. attendance at five different carnivals during the same year
33. attendance at a carnival by age group as it relates to total attendance
34. Critical Thinking Give an example of real-world data that would best be displayed by each type of graph: line graph, circle graph, double-bar graph.

35. The first modern Olympic Games took place in 1896 in Athens, Greece. The circle graph shows the total number of medals won by several countries at the Olympic Games of 1896.

a. Which country won the most medals? Estimate the percent of the medals won by this country.

b. Which country won the second most medals? Estimate the percent of the medals won by this country.
36. **Write About It** Explain how you could use a line graph to make predictions.

37. **Test Prep**
   Which type of graph would best display the contribution of each high school basketball player to the team, in terms of points scored?
   
   A. Bar graph  
   B. Line graph  
   C. Double-line graph  
   D. Circle graph

38. At what age did Marianna have 75% more magazine subscriptions than she did at age 40?

   F. 25  
   G. 30  
   H. 35  
   I. 45

39. **Short Response** The table shows the number of students in each algebra class. Make a graph to display the data. Explain why you chose that type of graph.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr. Abrams</td>
<td>34</td>
</tr>
<tr>
<td>Ms. Belle</td>
<td>29</td>
</tr>
<tr>
<td>Mr. Marvin</td>
<td>25</td>
</tr>
<tr>
<td>Ms. Swanson</td>
<td>27</td>
</tr>
</tbody>
</table>

**CHALLENGE AND EXTEND**

Students and teachers at Lauren’s school went on one of three field trips.

40. On which trip were there more boys than girls?

41. A total of 60 people went to the museum. Estimate the number of girls who went to the museum.

42. Explain why it is not possible to determine whether fewer teachers went to the museum than to the zoo or the opera.
Objectives
Create stem-and-leaf plots.
Create frequency tables and histograms.

Vocabulary
stem-and-leaf plot
frequency
frequency table
histogram
cumulative frequency

Why learn this?
Stem-and-leaf plots can be used to organize data, like the number of students in elective classes. (See Example 1.)

A stem-and-leaf plot arranges data by dividing each data value into two parts. This allows you to see each data value.

The digits other than the last digit of each value are called a stem.

The key tells you how to read each value.

Stems are always consecutive numbers. In Example 1B, neither player has scores that start with 15, so there are no leaves in that row.

Writing Math

Example 1
Making a Stem-and-Leaf Plot

A
The numbers of students in each of the elective classes at a school are given below. Use the data to make a stem-and-leaf plot.

24, 14, 12, 25, 32, 18, 23, 24, 9, 18, 34, 28, 24, 27

Number of Students in Elective Classes

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>2 4 8 8</td>
</tr>
<tr>
<td>2</td>
<td>3 4 4 5 7 8</td>
</tr>
<tr>
<td>3</td>
<td>2 4</td>
</tr>
</tbody>
</table>

Key: 2|3 means 23

The tens digits are the stems.
The ones digits are the leaves. List the leaves from least to greatest within each row.

Title the graph and add a key.

B
Marty's and Bill's scores for ten games of bowling are given below. Use the data to make a back-to-back stem-and-leaf plot.

Marty: 137, 149, 167, 134, 121, 127, 143, 123, 168, 162
Bill: 129, 138, 141, 124, 139, 160, 149, 145, 128, 130

Bowling Scores

<table>
<thead>
<tr>
<th>Marty</th>
<th>Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 3 1</td>
<td>12 4 8 9</td>
</tr>
<tr>
<td>7 4 9</td>
<td>13 0 8 9</td>
</tr>
<tr>
<td>9 3 1</td>
<td>14 1 5 9</td>
</tr>
<tr>
<td></td>
<td>15 0</td>
</tr>
<tr>
<td>8 7 2</td>
<td>16 0</td>
</tr>
</tbody>
</table>

Key: [14][1 means 141
3][14] means 143

The first two digits are the stems.
The ones digits are the leaves.

Put Marty's scores on the left side and Bill's scores on the right.

Title the graph and add a key.
The graph shows that three of Marty's scores were higher than Bill's highest score.

Check It Out!
1. The temperatures in degrees Celsius for two weeks are given below. Use the data to make a stem-and-leaf plot.

7, 32, 34, 31, 26, 27, 23, 19, 22, 29, 30, 36, 35, 31
The frequency of a data value is the number of times it occurs. A frequency table shows the frequency of each data value. If the data is divided into intervals, the table shows the frequency of each interval.

**Example 2**

**Making a Frequency Table**

The final scores for each golfer in a tournament are given below. Use the data to make a frequency table with intervals.

77, 71, 70, 82, 75, 76, 72, 70, 77, 74, 71, 75, 68, 72, 75, 74

**Step 1** Identify the least and greatest values.

The least value is 68. The greatest value is 82.

**Step 2** Divide the data into equal intervals.

For this data set, use an interval of 3.

**Step 3** List the intervals in the first column of the table. Count the number of data values in each interval and list the count in the last column. Give the table a title.

<table>
<thead>
<tr>
<th>Golf Tournament Scores</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Scores</td>
<td>Frequency</td>
<td></td>
</tr>
<tr>
<td>68–70</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>71–73</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>74–76</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>77–79</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>80–82</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Check It Out!**

2. The numbers of days of Maria's last 15 vacations are listed below. Use the data to make a frequency table with intervals.

4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12

A histogram is a bar graph used to display the frequency of data divided into equal intervals. The bars must be of equal width and should touch, but not overlap.

**Example 3**

**Making a Histogram**

Use the frequency table in Example 2 to make a histogram.

**Step 1** Use the scale and interval from the frequency table.

**Step 2** Draw a bar for the number of scores in each interval.

All bars should be the same width. The bars should touch, but not overlap.

**Step 3** Title the graph and label the horizontal and vertical scales.

**Check It Out!**

3. Make a histogram for the number of days of Maria's last 15 vacations.

4, 8, 6, 7, 5, 4, 10, 6, 7, 14, 12, 8, 10, 15, 12
Cumulative frequency shows the frequency of all data values less than or equal to a given value. You could just count the number of values, but if the data set has many values, you might lose track. Recording the data in a cumulative frequency table can help you keep track of the data values as you count.

**Example 4** Making a Cumulative Frequency Table

The heights in inches of the players on a school basketball team are given below.

72, 68, 71, 70, 73, 69, 79, 76, 72, 75, 72, 74, 68, 70, 69, 75, 72, 71, 73, 76

a. Use the data to make a cumulative frequency table.

**Step 1** Choose intervals for the first column of the table.

**Step 2** Record the frequency of values in each interval for the second column.

**Step 3** Add the frequency of each interval to the frequencies of all the intervals before it. Put that number in the third column of the table.

**Step 4** Title the table.

<table>
<thead>
<tr>
<th>Basketball Players’ Heights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.)</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>68–70</td>
</tr>
<tr>
<td>71–73</td>
</tr>
<tr>
<td>74–76</td>
</tr>
<tr>
<td>77–79</td>
</tr>
</tbody>
</table>

b. How many players have heights under 74 in?

All heights under 74 in. are displayed in the first two rows of the table, so look at the cumulative frequency shown in the second row. There are 14 players with heights under 74 in.

**Check It Out!**

4. The numbers of vowels in each sentence of a short essay are listed below.

33, 36, 39, 37, 34, 35, 43, 35, 28, 32, 36, 35, 29, 40, 33, 41, 37

a. Use the data to make a cumulative frequency table.

b. How many sentences contain 35 vowels or fewer?

**Think and Discuss**

1. In a stem-and-leaf plot, the number of _____?_____ is always the same as the number of data values. (stems or leaves)

2. Explain how to make a histogram from a stem-and-leaf plot.

3. GET ORGANIZED Copy and complete the graphic organizer.

**Bar Graphs vs Histograms**

How are they alike? How are they different?
GUIDED PRACTICE

1. **Vocabulary** A(n) _____?_____ is a data display that shows individual data values. (stem-and-leaf plot or histogram)

2. **Sports** The ages of professional basketball players at the time the players were recruited are given. Use the data to make a stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Ages When Recruited</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
</tr>
</tbody>
</table>

3. **Weather** The average monthly rainfall for two cities (in inches) is given below. Use the data to make a back-to-back stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Average Monthly Rainfall (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austin, TX</td>
</tr>
<tr>
<td>New York, NY</td>
</tr>
</tbody>
</table>

4. **Sports** The finishing times of runners in a 5K race, to the nearest minute, are given. Use the data to make a frequency table with intervals.

<table>
<thead>
<tr>
<th>Finishing Times in 5K Race (to the nearest minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
</tr>
</tbody>
</table>

5. **Biology** The breathing intervals of gray whales are given. Use the frequency table to make a histogram for the data.

<table>
<thead>
<tr>
<th>Breathing Intervals (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>5–7</td>
</tr>
<tr>
<td>8–10</td>
</tr>
<tr>
<td>11–13</td>
</tr>
<tr>
<td>14–16</td>
</tr>
</tbody>
</table>

6. The scores made by a group of eleventh-grade students on the mathematics portion of the SAT are given.

   a. Use the data to make a cumulative frequency table.

<table>
<thead>
<tr>
<th>Scores on Mathematics Portion of SAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>520</td>
</tr>
<tr>
<td>600</td>
</tr>
</tbody>
</table>

   b. How many students scored 650 or higher on the mathematics portion of the SAT?

PRACTICE AND PROBLEM SOLVING

7. The numbers of people who visited a park each day over two weeks during different seasons are given below. Use the data to make a back-to-back stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Visitors to a Park</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer</td>
</tr>
<tr>
<td>Winter</td>
</tr>
</tbody>
</table>
8. **Weather**  The daily high temperatures in degrees Fahrenheit in a town during one month are given. Use the data to make a stem-and-leaf plot.

9. The overall GPAs of several high school seniors are given. Use the data to make a frequency table with intervals.

10. **Chemistry**  The atomic masses of the nonmetal elements are given in the table. Use the frequency table to make a histogram for the data.

   ![Atomic Masses of Nonmetal Elements](image)

11. The numbers of pretzels found in several samples of snack mix are given in the table.
   a. Use the data to make a cumulative frequency table.
   b. How many samples of snack mix had fewer than 42 pretzels?

12. **Automobiles**  The table shows gas mileage for the most economical cars in July 2004, including three hybrids.

   ![Gas Mileage of Economical Cars](image)

Make a back-to-back stem-and-leaf plot for the data.

13. Damien's math test scores are given in the table:
   a. Make a stem-and-leaf plot of Damien's test scores.
   b. Make a histogram of the test scores using intervals of 5.
   c. Make a histogram of the test scores using intervals of 10.
   d. Make a histogram of the test scores using intervals of 20.
   e. How does the size of the interval affect the appearance of the histogram?
   f. **Write About It**  Which histogram makes Damien's grades look highest? Explain.

14. **ERROR ANALYSIS**  Two students made stem-and-leaf plots for the following data: 530, 545, 550, 555, 570. Which is incorrect? Explain the error.

   ![Stem-Leaf Plots](image)

<table>
<thead>
<tr>
<th>Ticket Sales (million $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.9</td>
</tr>
<tr>
<td>19.7</td>
</tr>
<tr>
<td>18.8</td>
</tr>
<tr>
<td>13.5</td>
</tr>
<tr>
<td>13.1</td>
</tr>
<tr>
<td>11.2</td>
</tr>
<tr>
<td>10.2</td>
</tr>
<tr>
<td>7.5</td>
</tr>
<tr>
<td>6.1</td>
</tr>
<tr>
<td>5.1</td>
</tr>
</tbody>
</table>

17. **Critical Thinking** Margo's homework assignment is to make a data display of some data she finds in a newspaper. She found a frequency table with the given intervals.

<table>
<thead>
<tr>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Under 18</td>
</tr>
<tr>
<td>18–30</td>
</tr>
<tr>
<td>31–54</td>
</tr>
<tr>
<td>55 and older</td>
</tr>
</tbody>
</table>

Explain why Margo must be careful when drawing the bars of the histogram.

18. What data value occurs most often in the stem-and-leaf plot?

A) 7
B) 4.7
C) 47
D) 777

Key: 3|2 means 3.2

19. The table shows the results of a survey about time spent on the Internet each month. Which statement is NOT supported by the data in the table?

<table>
<thead>
<tr>
<th>Time Spent on the Internet per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>0–4</td>
</tr>
<tr>
<td>5–9</td>
</tr>
<tr>
<td>10–14</td>
</tr>
<tr>
<td>15–19</td>
</tr>
<tr>
<td>20–24</td>
</tr>
<tr>
<td>25–29</td>
</tr>
<tr>
<td>30–34</td>
</tr>
</tbody>
</table>

- F) The interval of 30 to 34 h/mo has the lowest frequency.
- G) More than half of those who responded spend more than 20 h/mo on the Internet.
- H) Only four people responded that they spend less than 5 h/mo on the Internet.
- J) Sixteen people responded that they spend less than 20 h/mo on the Internet.
20. The frequencies of starting salary ranges for college graduates are noted in the table. Which histogram best reflects the data?

<table>
<thead>
<tr>
<th>Salary Range ($)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>20,000–29,000</td>
<td></td>
</tr>
<tr>
<td>30,000–39,000</td>
<td></td>
</tr>
<tr>
<td>40,000–49,000</td>
<td></td>
</tr>
<tr>
<td>50,000–59,000</td>
<td></td>
</tr>
</tbody>
</table>

**Challenge and Extend**

21. The cumulative frequencies of each interval have been given. Use this information to complete the frequency column.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>13–16</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>17–20</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>21–24</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>25–28</td>
<td></td>
<td>123</td>
</tr>
</tbody>
</table>
**Objectives**
Describe the central tendency of a data set.
Create and interpret box-and-whisker plots.

**Vocabulary**
- mean
- median
- mode
- range
- outlier
- first quartile
- third quartile
- interquartile range (IQR)
- box-and-whisker plot

**Who uses this?**
Sports analysts examine data distributions to make predictions. (See Example 4.)

A measure of central tendency describes the center of a set of data. Measures of central tendency include the mean, median, and mode.

- The **mean** is the average of the data values, or the sum of the values in the set divided by the number of values in the set.
- The **median** is the middle value when the values are in numerical order, or the mean of the two middle numbers if there are an even number of values.
- The **mode** is the value or values that occur most often. A data set may have one mode or more than one mode. If no value occurs more often than another, the data set has no mode.

The **range** of a set of data is the difference between the greatest and least values in the set. The range is one measure of the spread of a data set.

### Example 1
Finding Mean, Median, Mode, and Range of a Data Set

The numbers of hours Isaac did homework on six days are 3, 8, 4, 6, 5, and 4. Find the mean, median, mode, and range of the data set.

\[
\text{mean: } \frac{3 + 4 + 4 + 5 + 6 + 8}{6} = \frac{30}{6} = 5
\]

Write the data in numerical order.

\[
\text{median: } 3, 4, 4, 5, 6, 8
\]

There is an even number of values. Find the mean of the two middle values.

\[
\text{mode: } 4
\]

4 occurs more than any other value.

\[
\text{range: } 8 - 3 = 5
\]

Subtract the least value from the greatest value.

### Check It Out!
1. The weights in pounds of five cats are 12, 14, 12, 16, and 16. Find the mean, median, mode, and range of the data set.

A value that is very different from the other values in a data set is called an **outlier**. In the data set below, one value is much greater than the other values.
**Example 2**  Determining the Effects of Outliers

Identify the outlier in the data set \{7, 10, 54, 9, 12, 8, 5\}, and determine how the outlier affects the mean, median, mode, and range of the data.

5, 7, 8, 9, 10, 12, 54  Write the data in numerical order.

The outlier is 54.  Look for a value much greater or less than the rest.

**With the Outlier:**
- **mean:** \(\frac{5 + 7 + 8 + 9 + 10 + 12 + 54}{7} = 15\)
- **median:** 5, 7, 8, 9, 10, 12, 54  The median is 9.
- **mode:** Each value occurs once.
  There is no mode.
- **range:** 54 – 5 = 49

**Without the Outlier:**
- **mean:** \(\frac{5 + 7 + 8 + 9 + 10 + 12}{6} = 8.5\)
- **median:** 5, 7, 8, 9, 10, 12  The median is 8.5.
- **mode:** Each value occurs once.
  There is no mode.
- **range:** 12 – 5 = 7

The outlier increases the mean by 6.5, the median by 0.5, and the range by 42. It has no effect on the mode.

**Check It Out!**  Identify the outlier in the data set \{21, 24, 3, 27, 30, 24\}, and determine how the outlier affects the mean, median, mode, and range of the data.

As you can see in Example 2, an outlier can strongly affect the mean of a data set, while having little or no impact on the median and mode. Therefore, the mean may not be the best measure to describe a data set that contains an outlier. In such cases, the median or mode may better describe the center of the data set.

**Example 3**  Choosing a Measure of Central Tendency

Niles scored 70, 74, 72, 71, 73, and 96 on his six geography tests. For each question, choose the mean, median, or mode, and give its value.

**A**  Which measure gives Niles’s test average?
- The average of Niles’s scores is the mean.
  \[\text{mean: } \frac{70 + 74 + 72 + 71 + 73 + 96}{6} = 76\]

**B**  Which measure best describes Niles’s typical score? Explain.
- The outlier of 96 causes the mean to be greater than all but one of the test scores, so it is not the best measure in this situation.
- The data set has no mode.
- The median best describes the typical score.
  \[\text{median: } 70, 71, 72, 73, 74, 96 \quad \text{Find the mean of the two middle values.}\]
  The median is 72.5.

**Check It Out!**  Josh scored 75, 75, 81, 84, and 85 on five tests. For each question, choose the mean, median, or mode, and give its value.

**3a.** Which measure describes the score Josh received most often?

**3b.** Which measure should Josh use to convince his parents that he is doing well in school? Explain.
Measures of central tendency describe how data cluster around one value. Another way to describe a data set is by its spread—how the data values are spread out from the center.

Quartiles divide a data set into four equal parts. Each quartile contains one-fourth of the values in the set. The **first quartile** is the median of the lower half of the data set. The second quartile is the median of the data set, and the **third quartile** is the median of the upper half of the data set.

The **interquartile range (IQR)** of a data set is the difference between the third and first quartiles. It represents the range of the middle half of the data.

A **box-and-whisker plot** can be used to show how the values in a data set are distributed. You need five values to make a box-and-whisker plot: the minimum (or least value), first quartile, median, third quartile, and maximum (or greatest value).

**Example 4**

**Sports Application**

The numbers of runs scored by a softball team in 20 games are given. Use the data to make a box-and-whisker plot.

3, 4, 8, 12, 7, 5, 4, 12, 3, 9, 11, 4, 14, 8, 2, 10, 3, 10, 9, 7

**Step 1** Order the data from least to greatest.

2, 3, 3, 3, 4, 4, 4, 5, 7, 7, 8, 8, 9, 9, 10, 10, 11, 12, 12, 14

**Step 2** Identify the five needed values.

<table>
<thead>
<tr>
<th>Minimum</th>
<th>Q1: 4</th>
<th>Q2: 7.5</th>
<th>Q3: 10</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>7.5</td>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

**Step 3** Draw a number line and plot a point above each of the five needed values. Draw a box through the first and third quartiles and a vertical line through the median. Draw lines from the box to the minimum and maximum.

**Check It Out!**

4. Use the data to make a box-and-whisker plot.

13, 14, 18, 13, 12, 17, 15, 12, 13, 19, 11, 14, 14, 18, 22, 23
Reading and Interpreting Box-and-Whisker Plots

The box-and-whisker plots show the ticket sales, in millions of dollars, of the top 25 movies in 2000 and 2007 (for the United States only).

A Which data set has a greater median? Explain.

The vertical line in the box for 2007 is farther to the right than the vertical line in the box for 2000.
The data set for 2007 has a greater median.

B Which data set has a greater interquartile range? Explain.

The length of the box for 2007 is greater than the length of the box for 2000.
The data set for 2007 has a greater interquartile range.

C About how much more were the ticket sales for the top movie in 2007 than for the top movie in 2000?

2007 maximum: about $335 million
2000 maximum: about $260 million

335 – 260 = 75

The ticket sales for the top movie in 2007 were about $75 million more than for the top movie in 2000.

Use the box-and-whisker plots above to answer each question.

5a. Which data set has a smaller range? Explain.

5b. About how much more was the median ticket sales for the top 25 movies in 2007 than in 2000?

THINK AND DISCUSS

1. Explain when the median is a value in the data set.

2. Give an example of a data set for which the mean is twice the median. Explain how you determined your answer.

3. Suppose the minimum in a data set is the same as the first quartile. How would this affect a box-and-whisker plot of the data?

4. GET ORGANIZED Copy and complete the graphic organizer. Tell which measure of central tendency answers each question.

<table>
<thead>
<tr>
<th>Measures of Central Tendency</th>
<th>Used to Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>What is the average?</td>
<td></td>
</tr>
<tr>
<td>What is the halfway point of the data?</td>
<td></td>
</tr>
<tr>
<td>What is the most common value?</td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary** What is the difference between the range and the interquartile range of a data set?

**SEE EXAMPLE 1**

Find the mean, median, mode, and range of each data set.

2. 85, 83, 85, 82
3. 12, 22, 33, 44, 44
4. 10, 26, 25, 10, 20, 22, 25, 20
5. 71, 73, 75, 78, 78, 80, 85, 86

**SEE EXAMPLE 1**

Identify the outlier in each data set, and determine how the outlier affects the mean, median, mode, and range of the data.

6. 10, 96, 12, 17, 15
7. 64, 75, 72, 13, 64

**SEE EXAMPLE 3**

Adrienne scored 82, 54, 85, 91, and 83 on her last five science tests. For each question, choose the mean, median, or mode, and give its value.

9. Which measure should Adrienne use to convince her soccer coach she is doing well in science? Explain.

**SEE EXAMPLE 4**

Use the data to make a box-and-whisker plot.

10. 21, 31, 26, 24, 28, 26
11. 12, 13, 42, 62, 62, 82

**SEE EXAMPLE 5**

The box-and-whisker plots show the scores, in thousands of points, of two players on a video game. Use the box-and-whisker plots to answer each question.

12. Which player has a higher median score? Explain.
13. Which player had the lowest score? Estimate this score.

PRACTICE AND PROBLEM SOLVING

Find the mean, median, mode, and range of each data set.

14. 75, 63, 89, 91
15. 1, 2, 2, 2, 3, 3, 3, 3, 4
16. 19, 25, 31, 19, 34, 22, 31, 34
17. 58, 58, 60, 60, 60, 61, 63

Identify the outlier in each data set, and determine how the outlier affects the mean, median, mode, and range of the data.

18. 42, 8, 54, 37, 29
19. 3, 8, 3, 3, 23, 8

Lamont bowled 153, 145, 148, and 166 in four games. For each question, choose the mean, median, or mode, and give its value.

20. Which measure gives Lamont’s average score?
21. Which measure should Lamont use to convince his parents to let him join a bowling league? Explain.

Use the data to make a box-and-whisker plot.

22. 62, 63, 62, 64, 68, 62, 62
23. 85, 90, 81, 100, 92, 85
The box-and-whisker plots show the prices, in dollars, of athletic shoes at two sports apparel stores. Use the box-and-whisker plots to answer each question.

24. Which store has the greater median price? About how much greater?

25. Which store has the smaller interquartile range? What does this tell you about the data sets?

26. Estimate the difference in price between the most expensive shoe type at Jump N Run and the most expensive shoe at Sneaks R Us.

Find the mean, median, mode, and range of each data set.

27. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
28. 5, 6, 6, 5, 5
29. 2.1, 4.3, 6.5, 1.2, 3.4
30. 0, \(\frac{1}{4}\), \(\frac{1}{2}\), \(\frac{3}{4}\), 1
31. 23, 25, 26, 25, 23
32. –3, –3, –3, –2, –2, –1
33. 1, 4, 9, 16, 25, 36
34. 1, 0, 0, 1, 1, 4
35. Estimation Estimate the mean of \(16 \frac{7}{8}\), \(12 \frac{1}{4}\), \(22 \frac{1}{10}\), \(18 \frac{5}{7}\), \(19 \frac{1}{5}\), \(13 \frac{8}{11}\), and \(13 \frac{8}{11}\).

Tell whether each statement is sometimes, always, or never true.

36. The mean is a value in the data set.
37. The median is a value in the data set.
38. If a data set has one mode, the mode is a value in the data set.
39. The mean is affected by including an outlier.
40. The mode is affected by including an outlier.

41. Sports The table shows the attendance at six football games at Jefferson High School. Which measure of central tendency best indicates the typical attendance at a football game? Why?

<table>
<thead>
<tr>
<th>Attendance at Football Games</th>
<th>Eagles vs. Bulldogs</th>
<th>743</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eagles vs. Panthers</td>
<td>768</td>
<td></td>
</tr>
<tr>
<td>Eagles vs. Coyotes</td>
<td>835</td>
<td></td>
</tr>
<tr>
<td>Eagles vs. Bears*</td>
<td>1218</td>
<td></td>
</tr>
<tr>
<td>Eagles vs. Colts</td>
<td>797</td>
<td></td>
</tr>
<tr>
<td>Eagles vs. Mustangs</td>
<td>854</td>
<td></td>
</tr>
</tbody>
</table>

*Homecoming Game

42. Weather The high temperatures in degrees Fahrenheit on 11 consecutive days were 68, 71, 75, 74, 75, 71, 73, 71, 72, 74, and 79. Find the mean, median, mode, and range of the temperatures. Describe the effect on the mean, median, mode, and range if the next day's temperature was 70 °F.

43. Advertising A home-decorating store sells five types of candles, which are priced at $3, $2, $2, $2, and $15. If the store puts an ad in the paper titled “Best Local Candle Prices,” which measure of central tendency should it advertise? Justify your answer.

Use the data to make a box-and-whisker plot.

44. 25, 28, 26, 16, 18, 15, 25, 28, 26, 16
45. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31
46. 1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4
47. 50, 52, 45, 62, 36, 55, 40, 50, 65, 33
48. The results in Olympic pole-vaulting are given as heights in meters. In the 2008 Olympic Games in Beijing, the following results occurred for the men’s pole-vault finals: 5.96, 5.85, 5.70, 5.70, 5.70, 5.70, 5.60, 5.60, 5.60, 5.45, 5.45.
   a. Find the mean, median, mode, and range of this data set. Round to the nearest hundredth if needed.
   b. The gold medal was won by Steve Hooker of Australia. What was his height in the pole-vault finals?

49. **Business** The salaries for the eight employees at a small company are shown in the stem-and-leaf plot. Find the mean and median of the salaries. Which measure better describes the typical salary of an employee at this company? Explain.

   **Salaries ($1000)**
<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0 0 3 5 5</td>
</tr>
<tr>
<td>3</td>
<td>0 5</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

   **Key:** 20 means $20,000

50. **Critical Thinking** Use the data set {1, 2, 3, 5, 8, 13, 21, 34} to complete the following.
   a. Find the mean of the data set.
   b. What happens to the mean of the data set if every number is increased by 2?
   c. What happens to the mean of the data set if every number is multiplied by 2?

51. Allison has taken 5 tests worth 100 points each. Her scores are shown in the grade book below. What score does she need on her next test to have a mean of 90%?

   **Grade Book**
<table>
<thead>
<tr>
<th>Student</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
<th>Test 5</th>
<th>Test 6</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allison</td>
<td>88</td>
<td>85</td>
<td>89</td>
<td>92</td>
<td>90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brad</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>90</td>
<td>89.2</td>
</tr>
</tbody>
</table>

52. **Astronomy** The table shows the number of moons of the planets in our solar system. What is the mean number of moons per planet? Is Earth’s value of one moon typical for the solar system? Explain.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Jupiter</th>
<th>Saturn</th>
<th>Uranus</th>
<th>Neptune</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moons</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>63</td>
<td>60</td>
<td>27</td>
<td>13</td>
</tr>
</tbody>
</table>

53. **Write About It** Explain how an outlier with a large value will affect the mean. Explain how an outlier with a small value will affect the mean.

54. Which value is always represented on a box-and-whisker plot?
   A. Mean    B. Median    C. Mode    D. Range

55. The lengths in feet of the alligators at a zoo are 9, 7, 12, 6, and 10. The lengths in feet of the crocodiles at the zoo are 13, 10, 8, 19, 18, and 16. What is the difference between the mean length of the crocodiles and the mean length of the alligators?
   F. 0.5 foot    G. 5.2 feet    H. 8 feet    I. 11.4 feet
56. The mean score on a test is 50. Which CANNOT be true?
   ⊙ Half the scores are 0, and half the scores are 100.
   ⊙ The range is 50.
   ⊙ Half the scores are 25, and half the scores are 50.
   □ Every score is 50.

57. **Short Response** The table shows the weights in pounds of six dogs. How does the mean weight of the dogs change if Rex’s weight is not included in the data set?

<table>
<thead>
<tr>
<th>Weights of Dogs (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duffy 23</td>
</tr>
<tr>
<td>Rex 62</td>
</tr>
<tr>
<td>Rocky 15</td>
</tr>
<tr>
<td>Skipper 34</td>
</tr>
<tr>
<td>Pepper 21</td>
</tr>
<tr>
<td>Sunny 19</td>
</tr>
</tbody>
</table>

58. List a set of data values with the following measures of central tendency:
   mean: 8  
   median: 7  
   mode: 6

59. Collect a set of data about your classmates or your school. For example, you might collect data about the number of points per game scored by your school’s basketball team. Use the data you collect to make a box-and-whisker plot.

60. A *weighted average* is an average in which each data value has an importance, or weight, assigned to it. A teacher uses the following weights when determining course grades: homework 25%, tests 30%, and final exam 45%. The table shows Nathalie’s scores in the class.
   a. Find the mean of Nathalie’s homework scores and the mean of her test scores.
   b. Find Nathalie’s weighted average for the class. To do so, multiply the homework mean, the test mean, and the final exam score by their corresponding weights. Then add the products.
   c. **What if...?** What would Nathalie’s mean score for the class be if her teacher did not use a weighted average?

<table>
<thead>
<tr>
<th>Homework</th>
<th>Tests</th>
<th>Final exam</th>
</tr>
</thead>
<tbody>
<tr>
<td>78, 83, 95, 82, 79, 93</td>
<td>88, 92, 81</td>
<td>90</td>
</tr>
</tbody>
</table>
A **dot plot** is a data representation that uses a number line and x's, dots, or other symbols to show frequency. Dot plots are sometimes called line plots.

### Example 1

**Making a Dot Plot**

Mrs. Montoya asked her junior and senior students how many minutes each of them spent studying math in one day, rounded to the nearest five minutes. The results are shown below. Make a dot plot showing the data for juniors and a dot plot showing the data for seniors.

<table>
<thead>
<tr>
<th>Time Spent Studying Math (min)</th>
<th>Frequency (Juniors)</th>
<th>Frequency (Seniors)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>25</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>30</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>35</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>40</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>45</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Find the least and greatest values in each data set. Then use these values to draw a number line for each graph. For each student, place a dot above the number line for the number of minutes he or she spent studying.

### Check It Out!

1. The cafeteria offers items at six different prices. John counted how many items were sold at each price for one week. Make a dot plot of the data.

<table>
<thead>
<tr>
<th>Price ($)</th>
<th>1.50</th>
<th>2.00</th>
<th>2.50</th>
<th>3.00</th>
<th>3.50</th>
<th>4.00</th>
<th>4.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Items</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>6</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>
A dot plot gives a visual representation of the distribution, or “shape”, of the data. The dot plots in Example 1 have different shapes because the data sets are distributed differently.

### Types of Distributions

<table>
<thead>
<tr>
<th>UNIFORM DISTRIBUTION</th>
<th>SYMMETRIC DISTRIBUTION</th>
<th>SKEWED DISTRIBUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Uniform Distribution" /></td>
<td><img src="image" alt="Symmetric Distribution" /></td>
<td><img src="image" alt="Skewed Distribution" /></td>
</tr>
</tbody>
</table>

In a **uniform distribution**, all data points have an approximately equal frequency.

In a **symmetric distribution**, a vertical line can be drawn and the result is a graph divided in two parts that are approximate mirror images of each other.

In a **skewed distribution**, the data is not uniform or symmetric. The data may be skewed to the right or skewed to the left.

### Example 2

#### Shapes of Data Distributions

The data table shows the number of miles run by members of two track teams during one day. Make a dot plot and determine the type of distribution for each team. Explain what the distribution means for each.

<table>
<thead>
<tr>
<th>Miles</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
<th>5.5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team A</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>Team B</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Make dot plots of the data.

The data for team A show a symmetric distribution. The distances run are evenly distributed about the mean.

The data for team B show a skewed right distribution. Most team members ran a distance greater than the mean.

### Check It Out!

2. Data for team C members are shown below. Make a dot plot and determine the type of distribution. Explain what the distribution means.

<table>
<thead>
<tr>
<th>Miles</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
<th>5.5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team C</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
1. **Biology**  Michael is collecting data for the growth of plants after one week. He planted nine seeds for each of three different types of plants and recorded his data in the table below.

<table>
<thead>
<tr>
<th>Type A (in.)</th>
<th>Type B (in.)</th>
<th>Type C (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>2.1</td>
<td>1.9</td>
</tr>
<tr>
<td>0.9</td>
<td>2.2</td>
<td>2.0</td>
</tr>
<tr>
<td>1.0</td>
<td>2.2</td>
<td>2.0</td>
</tr>
<tr>
<td>1.0</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>1.1</td>
<td>2.3</td>
<td>2.1</td>
</tr>
<tr>
<td>1.2</td>
<td>2.3</td>
<td>2.1</td>
</tr>
<tr>
<td>1.2</td>
<td>2.4</td>
<td>2.2</td>
</tr>
<tr>
<td>1.3</td>
<td>2.5</td>
<td>2.2</td>
</tr>
<tr>
<td>1.4</td>
<td>2.6</td>
<td>2.3</td>
</tr>
</tbody>
</table>

   a. Create a dot plot for each type of plant.
   b. Describe the distributions.
   c. Which data value(s) occur(s) the most often in each dot plot? the least often?
   d. For each dot plot, list the heights in order from least frequent to most frequent.

2. **Nutrition**  Julia researched grape juice brands to determine how many grams of sugar each brand contained per serving (8 fluid ounces = 1 serving). The data she collected is shown in the table.

<table>
<thead>
<tr>
<th>Grams of Sugar in Grape Juice (per serving)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15  0  36  18  30  10</td>
</tr>
<tr>
<td>30  15  35  30  36  30</td>
</tr>
<tr>
<td>36  30  38  16  35  16</td>
</tr>
</tbody>
</table>

   a. Identify any outlier(s) in the data set.
   b. Make a dot plot for the data with the outlier(s) and a dot plot for the data without the outlier(s).
   c. Describe the distribution of the data with and without the outlier(s).
   d. How does excluding the outlier(s) affect the mean, median, and mode of the data set?

3. The frequency table shows the number of siblings of each student in a class. Use the table to make a dot plot of the data, and describe the distribution.

<table>
<thead>
<tr>
<th>Number of Siblings</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
4. **School**  The list below shows which grade each member of a high school marching band belongs to.

   9, 12, 9, 10, 9, 12, 9, 9, 11, 12, 10, 9, 9, 11, 9, 10,
   10, 12, 9, 12, 11, 9, 12, 11, 10, 9, 12, 12, 9, 11, 12

   a. Make a dot plot of the data.
   
   b. Explain how you can use the dot plot to find the mean, median, and mode of the data set. Then find each of these values.

   **Use the dot plot for Exercises 5 and 6.**

5. **Write About It**  Compare stem-and-leaf plots and dot plots.
   
   a. How are they similar and how are they different?
   
   b. What information can you get from each graph?
   
   
   d. Can you make a stem-and-leaf plot given a dot plot? If so, make a stem-and-leaf plot of the data in the dot plot at right. If not, explain why not.

6. **Write About It**  Compare histograms and dot plots.
   
   a. How are they similar and how are they different?
   
   b. What information can you get from each graph?
   
   c. Can you make a dot plot given a histogram? Explain.
   
   d. Can you make a histogram given a dot plot? If so, make a histogram of the data in the dot plot at right. If not, explain why not.

7. **Multi-Step**  Gather data on the heights of people in your classroom. Separate the data for males from the data for females. Make two dot plots representing the data collected for each group. Compare the dot plots and the distributions of the data.

8. The dot plot at right shows an example of a **bimodal distribution**. Why is this an appropriate name for this type of distribution?

9. **Critical Thinking**  Magdalene and Peter conducted the same experiment. Both of their data sets had the same mean. Both made dot plots of their data that showed symmetric distributions, but Peter’s dot plot shows a greater range than Magdalene’s dot plot. Identify which plot below belongs to Peter and which belongs to Magdalene.
Use Technology to Make Graphs

You can use a spreadsheet program to create bar graphs, line graphs, and circle graphs. You can also use a graphing calculator to make a box-and-whisker plot.

Use with Data Distributions

Activity 1

Many colors are used on the flags of the 50 United States. The table shows the number of flags that use each color. Use a spreadsheet program to make a bar graph to display the data.

<table>
<thead>
<tr>
<th>Color</th>
<th>Black</th>
<th>Blue</th>
<th>Brown</th>
<th>Gold</th>
<th>Green</th>
<th>Purple</th>
<th>Red</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>27</td>
<td>46</td>
<td>20</td>
<td>36</td>
<td>24</td>
<td>4</td>
<td>34</td>
<td>42</td>
</tr>
</tbody>
</table>

1. Enter the data from the table in the first two columns of the spreadsheet.

2. Select the cells containing the titles and the data.
   Then click the Chart Wizard icon, Click Column from the list on the left, and then choose the small picture of a vertical bar graph. Click Next.

3. The next screen shows the range of cells used to make the graph. Click Next.

4. Give the chart a title and enter titles for the x-axis and y-axis. Click the Legend tab, and then click the box next to Show Legend to turn off the key. (A key is needed when making a double-bar graph.) Click Next.

5. Click Finish to place the chart in the spreadsheet.
Try This

1. The table shows the average number of hours of sleep people at different ages get each night. Use a spreadsheet program to make a bar graph to display the data.

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>3–9</th>
<th>10–13</th>
<th>14–18</th>
<th>19–30</th>
<th>31–45</th>
<th>46–50</th>
<th>51+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleep (h)</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7.5</td>
<td>6</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Activity 2

Adrianne is a waitress at a restaurant. The amounts Adrianne made in tips during her last 15 shifts are listed below. Use a graphing calculator to make a box-and-whisker plot to display the data. Give the minimum, first quartile, median, third quartile, and maximum values.

$58, $63, $40, $44, $57, $59, $61, $53, $54, $58, $57, $57, $58, $56

1. To make a list of the data, press \( \text{STAT} \), select \( \text{Edit} \), and enter the values in List 1 (L1). Press \( \text{ENTER} \) after each value.

2. To use the \( \text{STAT PLOT} \) editor to set up the box-and-whisker plot, press \( \text{2nd} \) \( \text{Y=} \), and then \( \text{ENTER} \).

   Press \( \text{ENTER} \) to select \( \text{Plot 1} \).

3. Select \( \text{On} \). Then use the arrow keys to choose the fifth type of graph, a box-and-whisker plot. \( \text{Xlist} \) should be \( \text{L1} \) and \( \text{Freq:} \) should be \( 1 \).

4. Press \( \text{ZOOM} \) and select \( 9: \text{ZoomStat} \) to see the graph in the statistics window.

5. Use \( \text{TRACE} \) and the arrow keys to move the cursor along the graph to the five important values: minimum (MinX), first quartile (Q1), median (MED), third quartile (Q3), and maximum (MaxX).

   - minimum: 40
   - first quartile: 54
   - median: 57
   - third quartile: 58
   - maximum: 63

Try This

2. The average length in inches of the ten longest bones in the human body are listed. Use a graphing calculator to make a box-and-whisker plot to display the data. What are the minimum, first quartile, median, third quartile, and maximum values of the data set?

1-4 Misleading Graphs and Statistics

**Objectives**
- Recognize misleading graphs.
- Recognize misleading statistics.

**Vocabulary**
- random sample

**Why learn this?**
A misleading graph can be used to distort the results of a student council election. (See Example 3.)

Graphs can be used to influence what people believe. The way a data set is displayed can influence how the data are interpreted.

**Example 1** Misleading Bar Graphs
The graph shows the size of tomatoes on plants that were treated with different fertilizers.

A Explain why the graph is misleading.
The scale on the vertical axis begins at 80. This exaggerates the differences between the sizes of the bars.

B What might someone believe because of the graph?
Someone might believe that the tomato treated with fertilizer D is much larger than the other tomatoes. It is only 3.3 grams larger than the tomato treated with fertilizer B.

**Check It Out!**
1. Who might want to use the graph above? Explain.

**Example 2** Misleading Line Graphs
The graph shows the average price of gasoline in the U.S. in September.

A Explain why the graph is misleading.
The intervals on the vertical axis are not equal.

B What might people be influenced to believe by the graph?
Someone might believe that the price of gasoline increased the most between 1995 and 1997. However, the change between 1995 and 1997 was only $0.14/gal while the change between 1999 and 2001 was $0.17/gal.

**Check It Out!**
2. Who might want to use the graph above? Explain.
A circle graph compares each category of a data set to the whole. When any category is not represented in the graph, it may appear that another category represents a greater percentage of the total than it should.

**Example 3** Misleading Circle Graphs

The graph shows what percent of the total votes were received by three candidates for student council president.

A Explain why the graph is misleading.
   The sections of the graph do not add to 100%, so the votes for at least one of the candidates is not represented.

B What might people be influenced to believe by the graph?
   Someone might believe that Smith won the election.

3. Who might want to use the graph above? Explain.

Statistics can be misleading because of the way the data is collected or the way the results are reported. A sufficiently large random sample is a good way to collect unbiased data. In a random sample, all members of the group being surveyed have an equal chance of being selected.

**Example 4** Misleading Statistics

A researcher surveys people leaving a basketball game about what they like to watch on TV. Explain why the following statement is misleading: “80% of people like to watch sports on TV.”

The sample is biased because people who attend sporting events are more likely to watch sports on TV than people who watch TV but do not attend sporting events.

4. A researcher asks 4 people if they have seasonal allergies. Three people respond yes. Explain why the following statement is misleading: “75% of people have seasonal allergies.”

**Think and Discuss**

1. Give an example of a situation in which someone might intentionally try to make a graph misleading.

2. GET ORGANIZED Copy and complete the graphic organizer. Add more boxes if needed.
GUIDED PRACTICE

1. **Vocabulary** Explain in your own words what the term *random sample* means.

2. The graph shows the average salaries of employees at three companies.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

3. The graph shows hotel occupancy in San Francisco over four years.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

4. The graph shows the nutritional information for a granola bar.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

5. Three students were surveyed about their favorite teacher. Two students answer Mr. Gregory, and one answers Mr. Blaine. Explain why the following statement is misleading: “Mr. Gregory is the favorite teacher of a majority of the students.”

6. A researcher surveys people at a shopping mall about whether they favor enlarging the size of the mall parking lot. Explain why the following statement is misleading: “85% of the community is in favor of enlarging the parking lot.”
7. The graph shows the median rent for men and women in a metropolitan area.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

8. The graph shows the export prices of Colombian arabica coffee over nine years.
   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

   a. Explain why the graph is misleading.
   b. What might someone believe because of the graph?
   c. Who might want to use this graph? Explain.

10. A college math course has one section with 240 students and 8 sections with 30 students. Explain why the following statement is misleading: “The average class size for the course is 53 students.”

11. The table shows scores from the women’s gymnastics finals in the floor exercise at the 2004 Summer Olympic Games.
    a. Find the average score for the women in the finals.
    b. Would it be misleading to describe the data using the average score? Explain.
    c. Make a graph for this data that could convince someone that the difference between the first place score and the eighth place score was very small.
12. ERROR ANALYSIS The graph shows the population of a city over time. Which conclusion is incorrect? Explain why the conclusion is incorrect and how the graph was misleading.

A. The population has grown rapidly over 20 years.
B. The population has grown slowly over 20 years.

13. The table shows the average connection speeds of some broadband Internet service providers.

<table>
<thead>
<tr>
<th>Provider</th>
<th>Connection Speed (Kbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speedy Online</td>
<td>954</td>
</tr>
<tr>
<td>TelQuick</td>
<td>914</td>
</tr>
<tr>
<td>Alacrity</td>
<td>858</td>
</tr>
</tbody>
</table>

a. Construct a display that suggests that Speedy Online is much faster than the other services.
b. Construct a display that suggests that all of the services offer about the same connection speeds.
c. Write About It Where might you expect to see your graph from part b? Explain.

14. Critical Thinking Explain how a graph can show truthful data but still be misleading.

15. What might someone be influenced to believe because of the graph?

A. The measles vaccine was introduced when the mortality rate was at its highest.
B. The measles vaccine was unnecessary.
C. The measles vaccine dramatically decreased the mortality rate.
D. The measles vaccine increased the mortality rate.

16. The table shows the number of votes cast in the 2000 U.S. presidential election and in the 2002 French presidential election. What additional information is needed to determine whether the following statement is misleading?

"American voters are more likely to vote than French voters."

F. The number of candidates in each election
G. The legal voting age in France
H. The number of registered voters in the United States in 2000 and France in 2002
I. The number of polling locations in the United States in 2000 and France in 2002
17. **Logic** A fingerprint analyst is studying a fingerprint that was found in the chemistry lab. He reports that the fingerprint belongs to Dr. Arenson. Below are two questions the analyst was asked and the answers he gave.

**Question 1:** What are the chances that the fingerprint belongs to someone else who has the same fingerprint as Dr. Arenson?

**Answer:** One in several billion.

**Question 2:** What are the chances that the fingerprint was wrongly identified?

**Answer:** About 1 in 100.

**a.** What is the difference between the two questions?
**b.** What does the answer to question 1 lead you to believe?
**c.** Who do you think might have asked question 1?
**d.** What does the answer to question 2 lead you to believe?
**e.** Who do you think might have asked question 2?

18. **History** Graphs like the one at right were created by Florence Nightingale. Nightingale served as a nurse during the Crimean War and was concerned with the unsanitary conditions the soldiers lived in. Each “wedge” of the circle represents a month between April 1854 and March 1855.

**a.** What do you think Florence Nightingale wanted to show with this graph?
**b.** Who do you think Nightingale showed the graph to?
Sampling and Bias

If you wanted to collect data about a very large group of people, you would most likely need to survey a smaller group. The large group that contains all the people you could survey is called a population. The smaller group is called a sample.

You have learned that a random sample is a good way of collecting data that is unbiased. There are different ways of selecting a random sample.

You have learned that a random sample is a good way of collecting data that is unbiased. There are different ways of selecting a random sample.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Random Sample</td>
<td>Every member of the population has an equal chance of being chosen for the sample.</td>
<td>The names of all students in your class are placed in a hat and three names are chosen at random.</td>
</tr>
<tr>
<td>Stratified Random Sample</td>
<td>The population is divided into similar groups. Then a simple random sample is chosen from each group.</td>
<td>Your class is divided into boys and girls and two students are chosen at random from each group.</td>
</tr>
<tr>
<td>Systematic Random Sample</td>
<td>A member of the population is chosen for the sample at a regular interval.</td>
<td>Every third student who comes into the classroom is chosen.</td>
</tr>
</tbody>
</table>

Example 1

In each situation, identify the population and the sample. Tell whether each sample is a simple, stratified, or systematic random sample.

A For one week, the manager of a pet supplies store asks every tenth customer what brand of pet food they buy.
   population: all customers at the pet supplies store during one week
   sample: every tenth customer during that same week
   The sample is systematic because one member of the population is chosen for the sample at a regular interval.

B Every person who enters a theater one evening places their ticket stub in a bowl. The theater owner chooses five ticket stubs to award prizes.
   population: all people who put their ticket stub in the bowl
   sample: five ticket stubs chosen from the bowl
   The sample is simple because every member of the population has the same chance of being chosen.

C One student from each classroom at a school is chosen at random for a committee.
   population: all students at a school
   sample: one student from each classroom
   The sample is stratified because the population is divided into similar groups and one member is chosen at random from each group.
Try This

1. Choose a topic to research. Describe the population, why you need to choose a sample, and the sample.

2. Choose whether to use a simple, stratified, or systematic random sample. Explain your choice and the process you would use to choose your sample.

A random sample is not biased because no part of the population is favored over another. In a biased sample, one or more parts of the population have an advantage for being chosen for the sample.

There are two main types of biased samples.

<table>
<thead>
<tr>
<th>Biased Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong></td>
</tr>
<tr>
<td>Convenience Sample</td>
</tr>
<tr>
<td>Voluntary Response Sample</td>
</tr>
</tbody>
</table>

Example 2

In each situation, identify the population and the sample. Tell whether each sample is a convenience or voluntary response sample. Explain why the sample is biased.

A website asks visitors to complete a survey about Internet usage.

Population: all visitors to the website
Sample: those visitors who choose to complete the survey

The sample is a voluntary response sample because visitors to the website chose whether to complete the survey. The sample is biased because those visitors who choose to complete the survey may not be representative of all visitors to the website.

A reporter asks people leaving a shopping center through one door about their shopping habits.

Population: all people at the shopping center
Sample: people who leave from one door at the shopping center

The sample is a convenience sample because the people leaving through the chosen door are easily accessed. The sample is biased because people leaving through another door do not have an opportunity to be chosen.
A **two-way table** is a useful way to organize data that can be categorized by two variables. Suppose you asked 20 children and adults whether they liked broccoli. The table shows one way to arrange the data.

The **joint relative frequencies** are the values in each category divided by the total number of values, shown by the shaded cells in the table. Each value is divided by 20, the total number of individuals.

The **marginal relative frequencies** are found by adding the joint relative frequencies in each row and column.

### Example 1
**Finding Joint and Marginal Relative Frequencies**

The table shows the results of a poll of 80 randomly selected high school students who were asked if they prefer math or English. Make a table of the joint and marginal relative frequencies.

<table>
<thead>
<tr>
<th></th>
<th>9th grade</th>
<th>10th grade</th>
<th>11th grade</th>
<th>12th grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>10</td>
<td>12</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>English</td>
<td>12</td>
<td>11</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Divide each value by the total of 80 to find the joint relative frequencies, and add each row and column to find the marginal relative frequencies.

<table>
<thead>
<tr>
<th></th>
<th>9th grade</th>
<th>10th grade</th>
<th>11th grade</th>
<th>12th grade</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>0.125</td>
<td>0.15</td>
<td>0.1375</td>
<td>0.1</td>
<td>0.5125</td>
</tr>
<tr>
<td>English</td>
<td>0.15</td>
<td>0.1375</td>
<td>0.1</td>
<td>0.1</td>
<td>0.4875</td>
</tr>
<tr>
<td>Total</td>
<td>0.275</td>
<td>0.2875</td>
<td>0.2375</td>
<td>0.2</td>
<td>1</td>
</tr>
</tbody>
</table>

### Check It Out!
1. The table shows the number of books sold at a library sale. Make a table of the joint and marginal relative frequencies.
To find a **conditional relative frequency**, divide the joint relative frequency by the marginal relative frequency. Conditional relative frequencies can be used to find conditional probabilities.

### Example 2

**Using Conditional Relative Frequency to Find Probability**

A sociologist collected data on the types of pets in 100 randomly selected households, and summarized the results in a table.

A. Make a table of the joint and marginal relative frequencies.

<table>
<thead>
<tr>
<th>Owns a dog</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Owns a cat</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>0.15</td>
<td>0.24</td>
<td>0.39</td>
</tr>
<tr>
<td>No</td>
<td>0.18</td>
<td>0.43</td>
<td>0.61</td>
</tr>
<tr>
<td>Total</td>
<td>0.33</td>
<td>0.67</td>
<td>1</td>
</tr>
</tbody>
</table>

B. If you are given that a household has a dog, what is the probability that the household also has a cat? Use the conditional relative frequency for the row with the condition “Owns a dog.” The total for households with dogs is 0.39, or 39%. Out of these, 0.15, or 15%, also have cats. The conditional relative frequency is \[ \frac{0.15}{0.39} \approx 0.38. \] Given that a household has a dog, there is a probability of about 0.38 that the household also has a cat.

The classes at a dance academy include ballet and tap dancing. Enrollment in these classes is shown in the table.

<table>
<thead>
<tr>
<th>Ballet</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>38</td>
<td>52</td>
</tr>
<tr>
<td>No</td>
<td>86</td>
<td>24</td>
</tr>
</tbody>
</table>

2a. Copy and complete the table of the joint relative frequencies and marginal relative frequencies.

<table>
<thead>
<tr>
<th>Ballet</th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

2b. If you are given that a student is taking ballet, what is the probability that the student is not taking tap?

Notice that in Example 2, the conditional relative frequency could have been found from the original data:

\[ \frac{0.15}{0.39} = \frac{15}{39} \approx 0.38 \]
Comparing Conditional Probabilities

Tomas is trying to decide on the best possible route to drive to work. He has a choice of three possible routes. On each day, he randomly selects a route and keeps track of whether he is late. After a 40-day trial, his notes look like this.

Use conditional probabilities to determine the best route for Tomas to take to work.

Create a table of joint and marginal relative frequencies. There are 40 data values, so divide each frequency by 40.

To find the conditional probabilities, divide the joint relative frequency of being late by the marginal relative frequency in each row.

\[
P(\text{being late if driving Route A}) = \frac{0.1}{0.35} \approx 0.29
\]

\[
P(\text{being late if driving Route B}) = \frac{0.075}{0.25} = 0.3
\]

\[
P(\text{being late if driving Route C}) = \frac{0.1}{0.4} = 0.25
\]

The probability of being late is least for Route C. Based on the sample, Tomas is least likely to be late if he takes Route C.

3. Francine is evaluating three driving schools. She asked 50 people who attended the schools whether they passed their driving tests on the first try.

Use conditional probabilities to determine which is the best school.

Think and Discuss

1. Describe the relationship between joint relative frequencies and marginal relative frequencies.

2. Explain how to find the conditional relative frequencies from a two-way table showing joint and marginal relative frequencies.

3. Get Organized Copy and complete the graphic organizer at right. In each column, explain how to find the relative frequency from a two-way table.
Vocabulary

1. The ___?___ relative frequencies are the sums of each row and column in a two-way table. (joint, marginal, or conditional)

2. You can compare ___?___ probabilities to evaluate the best one out of a number of options. (joint, marginal, or conditional)

3. The table shows the results of a poll of randomly selected high school students who were asked if they prefer to hear all-school announcements in the morning or afternoon.

<table>
<thead>
<tr>
<th></th>
<th>Underclassmen</th>
<th>Upperclassmen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td>8</td>
<td>14</td>
</tr>
<tr>
<td>Afternoon</td>
<td>18</td>
<td>10</td>
</tr>
</tbody>
</table>

Make a table of the joint and marginal relative frequencies.

4. Customer Service The table shows the results of a customer satisfaction survey for a cellular service provider, by location of the customer. In the survey, customers were asked whether they would recommend a plan with the provider to a friend.

<table>
<thead>
<tr>
<th></th>
<th>Arlington</th>
<th>Towson</th>
<th>Parkville</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>40</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>No</td>
<td>18</td>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

Make a table of the joint and marginal relative frequencies. Round to the nearest hundredth where appropriate.

5. School Pamela has collected data on the number of students in the sophomore class who play a sport or play a musical instrument.

<table>
<thead>
<tr>
<th>Plays a sport</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plays an instrument</td>
<td>47</td>
<td>38</td>
</tr>
<tr>
<td>No</td>
<td>51</td>
<td>67</td>
</tr>
</tbody>
</table>

a. Copy and complete the table of the joint and marginal relative frequencies. Round to the nearest hundredth where appropriate.

b. If you are given that a student plays an instrument, what is the probability that the student also plays a sport? Round your answer to the nearest hundredth.

c. If you are given that a student plays a sport, what is the probability that the student also plays an instrument? Round your answer to the nearest hundredth.
6. **Business** Roberto is the owner of a car dealership. He is assessing the success rates of his top three salespeople in order to offer one of them a promotion. Over two months, for each attempted sale, he records whether the salesperson made a successful sale or not. The results are shown in the chart below.

<table>
<thead>
<tr>
<th></th>
<th>Successful</th>
<th>Unsuccessful</th>
</tr>
</thead>
<tbody>
<tr>
<td>Becky</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Raul</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Darrell</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

a. Make a table of the joint relative frequencies and marginal relative frequencies. Round to the nearest hundredth where appropriate.
b. Find the probability that each salesperson will make a successful sale. Round to the nearest hundredth where appropriate.
c. Determine which salesperson has the highest success rate.

---

**PRACTICE AND PROBLEM SOLVING**

7. **Fundraising** The table shows the number of T-shirts and sweatshirts sold at a fundraiser during parent visitation night at Preston High School.

<table>
<thead>
<tr>
<th></th>
<th>Students</th>
<th>Adults</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Shirts</td>
<td>16</td>
<td>23</td>
</tr>
<tr>
<td>Sweatshirts</td>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

Make a table of the joint relative frequencies and marginal relative frequencies.

8. **Write About It** Describe in your own words the process you use to write marginal relative frequencies for data given in a two-way table.

9. **Customer Service** The claims handlers at a car insurance company help customers with insurance issues when there has been an accident, so their customer service skills are very important.

The claims handlers at the Trust Auto Insurance Company are divided into three teams. For one month, a customer satisfaction survey was given for each team. The results of the surveys are shown below.

<table>
<thead>
<tr>
<th></th>
<th>Satisfied</th>
<th>Dissatisfied</th>
</tr>
</thead>
<tbody>
<tr>
<td>Team 1</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>Team 2</td>
<td>34</td>
<td>12</td>
</tr>
<tr>
<td>Team 3</td>
<td>34</td>
<td>10</td>
</tr>
</tbody>
</table>

a. Make a table of the joint relative frequencies and marginal relative frequencies. Round to the nearest hundredth where appropriate.
b. Find the probability that a customer will be satisfied after working with each team. Round to the nearest hundredth where appropriate.
c. Determine which team has the highest rate of customer satisfaction.

10. **Critical Thinking** What do you notice about the value that always falls in the cell to the lower right of a two-way table when marginal relative frequencies have been written in? What does this value represent?
11. **ERROR ANALYSIS** One hundred adults and children were randomly selected and asked whether they spoke more than one language fluently. The data were recorded in a two-way table. Maria and Brennan each used the data to make the tables of joint relative frequencies shown below, but their results are slightly different. The difference is shaded. Can you tell by looking at the tables which of them made an error? Explain.

<table>
<thead>
<tr>
<th>Maria’s table</th>
<th>Brennan’s table</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Children</strong></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>0.15</td>
</tr>
<tr>
<td>No</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Adults</strong></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>0.1</td>
</tr>
<tr>
<td>No</td>
<td>0.6</td>
</tr>
</tbody>
</table>

12. **Estimation** A total of 107 brownies and muffins was sold at a school bake sale. The joint relative frequency representing muffins sold to seniors was 0.48. Use mental math to find approximately how many muffins were sold to seniors.

13. **Public Transit** A town planning committee is considering a new system for public transit. Residents of the town were randomly selected to answer two questions: “Do you work within 5 miles of your home?” and “Would you use the new system to get to work, if it were available?” The results are shown below.

<table>
<thead>
<tr>
<th>Work less than 5 miles from home?</th>
<th>Use new system?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>24</td>
</tr>
<tr>
<td>No</td>
<td>44</td>
</tr>
<tr>
<td><strong>Yes</strong></td>
<td>32</td>
</tr>
<tr>
<td><strong>No</strong></td>
<td>20</td>
</tr>
</tbody>
</table>

a. Make a table of the joint relative frequencies and marginal relative frequencies. Round to the nearest hundredth where appropriate.

b. If residents work less than 5 miles from home, what is the probability that they would use the new system? Round to the nearest hundredth.

c. If residents are willing to use the new system, what is the probability that they don't work less than 5 miles from home? Round to the nearest hundredth.

14. Students and teachers at a school were polled to see if they were in favor of extending the parking lot into part of the athletic fields. The results of the poll are shown in the two-way table.

<table>
<thead>
<tr>
<th>In Favor</th>
<th>Not in Favor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students</td>
<td>16</td>
</tr>
<tr>
<td>Teachers</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>23</td>
</tr>
</tbody>
</table>

Which of the following statements is false?

A. Thirty-nine students were polled in all.
B. Fourteen teachers were polled in all.
C. Twenty-three students are not in favor of extending the parking lot.
D. Nine teachers are in favor of extending the parking lot.
15. A group of students were polled to find out how many were planning to major in a scientific field of study in college. The results of the poll are shown in the two-way table.

<table>
<thead>
<tr>
<th>Class</th>
<th>Majoring in a science field</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>Junior</td>
<td>150</td>
</tr>
<tr>
<td>Senior</td>
<td>112</td>
</tr>
</tbody>
</table>

Which of the following statements is true?

A. Three hundred sixty students were polled in all.
B. A student in the senior class is more likely to be planning on a scientific major than a nonscientific major.
C. A student planning on a scientific major is more likely to be a junior than a senior.
D. More seniors than juniors plan to enter a scientific field of study.

16. **Gridded Response** A group of children and adults were polled about whether they watch a particular TV show. The survey results, showing the joint relative frequencies and marginal relative frequencies, are shown in the two-way table.

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Adults</td>
<td>0.25</td>
<td>x</td>
</tr>
<tr>
<td>Total</td>
<td>0.55</td>
<td>0.45</td>
</tr>
</tbody>
</table>

What is the value of x?

**CHALLENGE AND EXTEND**

The table shows the joint relative frequencies for data on how many children and teenagers attended a fair in one evening, and whether each bought a booklet of tickets for rides at the entrance gate.

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children</td>
<td>0.125</td>
<td>0.1</td>
</tr>
<tr>
<td>Teenagers</td>
<td>0.725</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Use the table to answer questions 17–20. Round answers to the nearest hundredth where appropriate.

17. Find the marginal relative frequencies for the data.

18. Based on this data, use a percentage to express how likely it is that tomorrow evening a teenager at the fair will buy a ticket booklet at the entrance. Round your answer to the nearest whole percent, if necessary.

19. If the data represent 80 teenagers and children altogether, how many children will have bought a ticket booklet at the entrance?

20. If 12 children did not buy ticket booklets at the entrance, then how many children and teenagers altogether does the data represent?
21. A poll with the options of 'yes' and 'no' was given. If the marginal relative frequency of 'yes' is 1.0, what was the marginal relative frequency of 'no'?

<table>
<thead>
<tr>
<th></th>
<th>Yes</th>
<th>No</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>0.24</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Group 2</td>
<td>0.76</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Total</td>
<td>1.0</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

22. **Short Response** What is the maximum a marginal relative frequency can be, and why?
Vocabulary

bar graph  frequency  outcome
box-and-whisker plot  frequency table  outlier
circle graph  histogram  prediction
complement  independent events  probability
cumulative frequency  interquartile range (IQR)  random sample
dependent events  line graph  range
equally likely  mean  sample space
event  median  stem-and-leaf plot
experiment  mode  theoretical probability
experimental probability  odds  third quartile
fair  outcome

Complete the sentences below with vocabulary words from the list above.

1. A(n) ____?____ is one possible result of an experiment.
2. The ____?____ is the difference between the third and first quartiles.
3. Two events are ____?____ if the occurrence of one event does not affect the probability of the other.

1-1 Organizing and Displaying Data

The circle graph shows the post-graduation plans for a high school's 500 graduating seniors.

How many seniors plan to attend a four-year college?
50% of 500
0.50 \times 500 = 250
250 students plan to attend a four-year college.

4. In which year did the same number of boys and girls participate?
5. How many more boys than girls participated during 2004?
**1-2 Frequency and Histograms**

**Example**

- The lives (in hours) of each light bulb in a test are given below. Use the data to make a frequency table and a histogram.

<table>
<thead>
<tr>
<th>Life Bulb Life (h)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>18–20</td>
<td>3</td>
</tr>
<tr>
<td>21–23</td>
<td>5</td>
</tr>
<tr>
<td>24–26</td>
<td>4</td>
</tr>
<tr>
<td>27–29</td>
<td>2</td>
</tr>
</tbody>
</table>

Identify the least and greatest values. Divide the data into equal intervals.

**Exercises**

6. The weights of packages shipped by an online retailer are given below. Use the data to make a stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Package Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14 9 22 24 7 1 19 22 28 18 12</td>
</tr>
</tbody>
</table>

7. The numbers of people who attended two different plays are shown in the table below. Make a back-to-back stem-and-leaf plot.

<table>
<thead>
<tr>
<th>Play Attendance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comedy Camp</td>
</tr>
<tr>
<td>Days and Days</td>
</tr>
<tr>
<td>104 62 83 102</td>
</tr>
<tr>
<td>104 120 81</td>
</tr>
<tr>
<td>126 122</td>
</tr>
<tr>
<td>109 128 82</td>
</tr>
<tr>
<td>132 139</td>
</tr>
</tbody>
</table>

8. The capacities of the gas tanks on several new vehicles are shown below. Use the data to make a frequency table with intervals.

<table>
<thead>
<tr>
<th>Gas Tank Capacity (gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 12 12 15 18 26 25 12 15 18 20 21</td>
</tr>
<tr>
<td>10 12 16 15 16 18 25 21 18 20 21</td>
</tr>
</tbody>
</table>

9. Use your frequency table from Exercise 9 to make a histogram.

**1-3 Data Distributions**

**Example**

- Consider the following ages of the winners of an art contest: 13, 15, 14, 18, 12, 10, 11, 13. Find the mean and mode of the data.

\[
\text{mean: } \frac{10 + 11 + 12 + 13 + 13 + 14 + 15 + 18}{8} = \frac{106}{8} = 13.25
\]

mode: 13 occurs most often.

**Exercises**

Find the mean, median, mode, and range of each data set.

10. Years of playing experience of the members of a musical ensemble:

5, 14, 25, 7, 8, 10, 12, 33, 12

Herman has five pairs of cowboy boots. The prices were $120, $137, $120, $145, and $482.

mean: $200.80 median: $137 mode: $120

11. Which value best describes the price Herman paid? Explain.

Use the data to make a box-and-whisker plot.

12. 25, 28, 2, 24, 28, 21, 18, 29, 31, 12, 6, 19, 27, 3
1-4 Misleading Graphs and Statistics

**Example**
- Explain why the graph is misleading.

The graph shows the cost of admission to an amusement park over 20 years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Adult Ticket Price $</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>40</td>
</tr>
<tr>
<td>1984</td>
<td>50</td>
</tr>
<tr>
<td>1988</td>
<td>60</td>
</tr>
<tr>
<td>1992</td>
<td>70</td>
</tr>
<tr>
<td>1996</td>
<td>80</td>
</tr>
<tr>
<td>2000</td>
<td>90</td>
</tr>
</tbody>
</table>

**Exercises**
- The graph shows the cost of admission to an amusement park over 20 years.

13. Explain why the graph is misleading.

14. What might someone believe because of the graph?

1-5 Experimental Probability

**Examples**
- The manager of a photo processing lab inspects 500 photos and finds that 4 have flaws.
  - What is the experimental probability that a photo is flawed?
    \[
    P(\text{flawed}) = \frac{\text{number of times the event occurs}}{\text{number of trials}} = \frac{4}{500} = 0.8\%
    \]
  - In one month, the lab processes 13,000 photos. Predict the number that are likely to be flawed.
    \[
    0.008 \cdot 13,000 = 104 \quad \text{Find 0.8\% of 13,000.}
    \]
  - 104 photos are likely to be flawed.

**Exercises**
- A manufacturer inspects 800 batteries and finds that 796 have no defects.
  - What is the experimental probability that a battery chosen at random has no defects?
  - There are 25,000 batteries in storage. How many batteries are likely to have no defects?
  - Another storage area holds 50,000 batteries. How many batteries are likely to have a defect?

1-6 Theoretical Probability

**Example**
- A jar contains red, green, brown, and blue marbles. The probability of choosing red is 0.30, of choosing green is 0.20, and of choosing brown is 0.25. Find the probability of choosing blue.
  \[
  P(\text{blue}) = \frac{1}{P(\text{not blue})} = \frac{1}{1 - (0.30 + 0.20 + 0.25)} = \frac{1}{0.45} = 0.2222
  \]
  \[
  P(\text{blue}) = 0.25
  \]

**Exercises**
- Find the theoretical probability of each outcome.
  18. Rolling a number less than 4 on a standard number cube
  19. Randomly selecting a month that starts with “J” from all month names
  20. Randomly selecting a vowel from the letters in EQUATION
Independent and Dependent Events

**Examples**

A hardware store shelf holds 12 cans of red paint, 4 cans of yellow paint, and 6 cans of black paint.

- Syd selects one can at random and replaces it. Then she selects another can at random. What is the probability that Syd selects a red can and then a yellow can?
  
  \[ P(\text{red, yellow}) = P(\text{red}) \cdot P(\text{yellow}) \]
  
  \[ = \frac{12}{22} \cdot \frac{4}{22} \quad \text{Independent events} \]
  
  \[ = \frac{48}{484} = \frac{12}{121} \]

- Gene selects one can at random and then selects another can at random from the remaining cans. What is the probability that Gene selects two cans of black paint?
  
  \[ P(\text{black, black}) = P(\text{black}) \cdot P(\text{black after black}) \]
  
  \[ = \frac{6}{22} \cdot \frac{5}{21} \quad \text{Dependent events} \]
  
  \[ = \frac{30}{462} = \frac{5}{77} \]

**Exercises**

Tell whether each set of events is independent or dependent. Explain your answers.

21. A computer generates a random number and then generates another random number.
22. You roll two number cubes. One is a 6 and the other is a 1.
23. Two audience members are called to the stage.
24. A yellow ball is drawn and set aside. Then a green ball is drawn.
25. A golden ball is drawn and set aside. Then another golden ball is drawn.
26. A green ball is drawn and replaced. Then another green ball is drawn.
1. The table shows the population of Oakville. Use the data to make a graph. Explain why you chose that type of graph.

2. Which ten-year period saw the greatest change in population?

3. Describe the trend in Oakville's population.

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>20,851</td>
</tr>
<tr>
<td>1980</td>
<td>14,229</td>
</tr>
<tr>
<td>1990</td>
<td>11,198</td>
</tr>
<tr>
<td>2000</td>
<td>9,579</td>
</tr>
</tbody>
</table>

The high temperatures in degrees Fahrenheit for two weeks are given: 64, 66, 63, 58, 59, 55, 51, 54, 61, 62, 68, 70, 63, 63.

4. Use the data to make a stem-and-leaf plot.

5. Use the data to make a frequency table with intervals.

6. Use your frequency table from Problem 5 to make a histogram.

The lengths of statements during a town council meeting are given.

7. Find the mean, median, mode, and range of the data.

8. Use the data to make a box-and-whisker plot.

<table>
<thead>
<tr>
<th>Length of Statement (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6  25  12  14  2  13  38</td>
</tr>
<tr>
<td>22 21 14 3 8 5 17</td>
</tr>
</tbody>
</table>

A manufacturer inspects 500 watches and finds that 498 have no defects.

9. What is the experimental probability that a watch chosen at random has no defects?

10. There are 30,000 watches in a warehouse. Predict the number of watches that are likely to have no defects.

The graph shows how the money raised by a charity is spent.

11. Explain why the graph is misleading.

12. What might someone believe because of the graph?

13. Who might want to use this graph?

14. A bag contains 12 cards, each with a different month of the year printed on it. A card is selected at random. What is the probability that the month begins with A?

15. The odds of spinning red on a spinner are 2:7. What is the probability of not spinning red?

16. A bag has 14 red marbles and 10 white marbles. Rosa randomly picks two marbles from the bag, one at a time. What is the probability that Rosa picks two white marbles?
### Chapter 2: Equations

<table>
<thead>
<tr>
<th>LAB</th>
<th>Model One-Step Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Solving Equations by Adding or Subtracting</td>
</tr>
<tr>
<td>2-2</td>
<td>Solving Equations by Multiplying or Dividing</td>
</tr>
<tr>
<td>LAB</td>
<td>Solve Equations by Graphing</td>
</tr>
<tr>
<td>2-3</td>
<td>Solving Two-Step and Multi-Step Equations</td>
</tr>
<tr>
<td>LAB</td>
<td>Model Equations with Variables on Both Sides</td>
</tr>
<tr>
<td>2-4</td>
<td>Solving Equations with Variables on Both Sides</td>
</tr>
<tr>
<td>LAB</td>
<td>Solve Equations Graphically</td>
</tr>
<tr>
<td>2-5</td>
<td>Algebraic Proof</td>
</tr>
<tr>
<td>2-6</td>
<td>Solving for a Variable</td>
</tr>
<tr>
<td>2-7</td>
<td>Solving Absolute-Value Equations</td>
</tr>
<tr>
<td>2-8</td>
<td>Rates, Ratios, and Proportions</td>
</tr>
<tr>
<td>2-9</td>
<td>Applications of Proportions</td>
</tr>
<tr>
<td>2-10</td>
<td>Precision and Accuracy</td>
</tr>
</tbody>
</table>

### Chapter Focus

- Choose procedures to solve equations efficiently.
- Differentiate between accuracy and precision.

### All in Proportion

A common use of equations and proportional relationships is the construction of scale models.

---

**Learn It Online**

Chapter Project Online

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- [Image of scale model construction](image)
## Reading Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

Pay attention to the **margin notes**. Know-It Note icons point out key information. Writing Math notes, Helpful Hints, and Caution notes help you understand concepts and avoid common mistakes.

The **Glossary** is found in the back of your textbook. Use it as a resource when you need the definition of an unfamiliar word or property.

The **Index** is located at the end of your textbook. Use it to locate the page where a particular concept is taught.

The **Problem-Solving Handbook** is found in the back of your textbook. These pages review strategies that can help you solve real-world problems.

### Try This

Use your textbook for the following problems.

1. Use the index to find the page where a term from this chapter is defined.
2. Describe how a strategy from the Problem Solving Workbook can be used in this chapter.
3. Use the glossary to find the definition of a term from this chapter.
Model One-Step Equations

You can use algebra tiles and an equation mat to model and solve equations. To find the value of the variable, place or remove tiles to get the x-tile by itself on one side of the mat. You must place or remove the same number of yellow tiles or the same number of red tiles on both sides.

**Activity**

Use algebra tiles to model and solve \( x + 6 = 2 \).

<table>
<thead>
<tr>
<th>MODEL</th>
<th>ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Model x + 6 on the left side of the mat and 2 on the right side of the mat." /></td>
<td>( x + 6 = 2 )</td>
</tr>
<tr>
<td><img src="image" alt="Place 6 red tiles on both sides of the mat. This represents adding −6 to both sides of the equation." /></td>
<td>( x + 6 + (−6) = 2 + (−6) )</td>
</tr>
<tr>
<td><img src="image" alt="Remove zero pairs from both sides of the mat." /></td>
<td>( x + 0 = 0 + (−4) )</td>
</tr>
<tr>
<td><img src="image" alt="One x-tile is equivalent to 4 red tiles." /></td>
<td>( x = −4 )</td>
</tr>
</tbody>
</table>

**Try This**

Use algebra tiles to model and solve each equation.

1. \( x + 2 = 5 \)  
2. \( x − 7 = 8 \)  
3. \( x − 5 = 9 \)  
4. \( x + 4 = 7 \)
Objective
Solve one-step equations in one variable by using addition or subtraction.

Vocabulary
- equation
- solution of an equation

Who uses this?
Athletes can use an equation to estimate their maximum heart rates. (See Example 4.)

An equation is a mathematical statement that two expressions are equal. A solution of an equation is a value of the variable that makes the equation true.

To find solutions, isolate the variable. A variable is isolated when it appears by itself on one side of an equation, and not at all on the other side. Isolate a variable by using inverse operations, which “undo” operations on the variable.

An equation is like a balanced scale. To keep the balance, perform the same operation on both sides.

EXAMPLE 1
Solving Equations by Using Addition

Solve each equation.

A  \[ x - 10 = 4 \]
\[
\begin{align*}
  x - 10 & = 4 \\
  + 10 & + 10 \\
  x & = 14
\end{align*}
\]

Check  \[
\begin{align*}
  x - 10 & = 4 \\
  14 - 10 & = 4 \\
  4 & = 4 \checkmark
\end{align*}
\]

Since 10 is subtracted from \( x \), add 10 to both sides to undo the subtraction.

To check your solution, substitute 14 for \( x \) in the original equation.

B  \[ \frac{2}{5} = m - \frac{1}{5} \]
\[
\begin{align*}
  \frac{2}{5} & = m - \frac{1}{5} \\
  + \frac{1}{5} & + \frac{1}{5} \\
  \frac{3}{5} & = m
\end{align*}
\]

Since \( \frac{1}{5} \) is subtracted from \( m \), add \( \frac{1}{5} \) to both sides to undo the subtraction.

Writing Math
Solutions are sometimes written in a solution set. For Example 1A, the solution set is \{14\}. For Example 1B, the solution set is \( \left\{ \frac{3}{5} \right\} \).

CHECK IT OUT!
Solve each equation. Check your answer.

1a.  \[ n - 3.2 = 5.6 \]
1b.  \[ -6 = k - 6 \]
1c.  \[ 16 = m - 9 \]
EXAMPLE 2  Solving Equations by Using Subtraction

Solve each equation. Check your answer.

A  \( x + 7 = 9 \)

Since 7 is added to \( x \), subtract 7 from both sides to undo the addition.

\[
\begin{align*}
  x + 7 &= 9 \\
  x + 7 &\quad \quad \text{(Original equation)} \\
  x + 7 - 7 &= 9 - 7 \\
  x &= 2 \\
  \text{Check} \\
  x + 7 &= 9 \\
  2 + 7 &= 9 \\
  9 &= 9 \\
  \checkmark
\end{align*}
\]

B  \( 0.7 = r + 0.4 \)

Since 0.4 is added to \( r \), subtract 0.4 from both sides to undo the addition.

\[
\begin{align*}
  0.7 &= r + 0.4 \\
  0.7 - 0.4 &= r + 0.4 - 0.4 \\
  0.3 &= r \\
  \text{Check} \\
  0.7 &= r + 0.4 \\
  0.7 &= 0.3 + 0.4 \\
  0.7 &= 0.7 \\
  \checkmark
\end{align*}
\]

EXAMPLE 3  Solving Equations by Adding the Opposite

Solve \(-8 + b = 2\).

Since \(-8\) is added to \( b \), add 8 to both sides.

\[
\begin{align*}
  -8 + b &= 2 \\
  -8 + b &\quad \quad \text{(Original equation)} \\
  -8 + b + 8 &= 2 + 8 \\
  b &= 10 \\
\end{align*}
\]

Solve each equation. Check your answer.

2a. \( d + \frac{1}{2} = 1 \)  
2b. \(-5 = k + 5\)  
2c. \( 6 + t = 14 \)

Remember that subtracting is the same as adding the opposite. When solving equations, you will sometimes find it easier to add an opposite to both sides instead of subtracting. For example, this method may be useful when the equation contains negative numbers.

EXAMPLE 4  Solving Equations by Adding the Opposite

Solve each equation. Check your answer.

3a. \(-2.3 + m = 7\)  
3b. \(-\frac{3}{4} + z = \frac{5}{4}\)  
3c. \(-11 + x = 33\)

Student to Student  Zero As a Solution

I used to get confused when I got a solution of 0. But my teacher reminded me that 0 is a number just like any other number, so it can be a solution of an equation. Just check your answer and see if it works.

\[
\begin{align*}
  x + 6 &= 6 \\
  x + 6 &\quad \quad \text{(Original equation)} \\
  x + 6 - 6 &= 6 - 6 \\
  x &= 0 \\
  \text{Check} \\
  x + 6 &= 6 \\
  0 + 6 &= 6 \\
  6 &= 6 \\
  \checkmark
\end{align*}
\]

Ama Walker  Carson High School

Chapter 2  Equations
**Example 4**  
*Fitness Application*

A person’s maximum heart rate is the highest rate, in beats per minute, that the person’s heart should reach. One method to estimate maximum heart rate states that your age added to your maximum heart rate is 220. Using this method, write and solve an equation to find the maximum heart rate of a 15-year-old.

<table>
<thead>
<tr>
<th>Age added to maximum heart rate</th>
<th>is 220.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a + r = 220</td>
<td></td>
</tr>
</tbody>
</table>

Write an equation to represent the relationship.

Substitute 15 for a. Since 15 is added to r, subtract 15 from both sides to undo the addition.

\[ a + r = 220 \]
\[ 15 + r = 220 \]
\[ -15 \]
\[ r = 205 \]

The maximum heart rate for a 15-year-old is 205 beats per minute. Since age added to maximum heart rate is 220, the answer should be less than 220. So 205 is a reasonable answer.

**4. What if...?** Use the method above to find a person’s age if the person’s maximum heart rate is 185 beats per minute.

The properties of equality allow you to perform inverse operations, as in the previous examples. These properties say that you can perform the same operation on both sides of an equation.

**Properties of Equality**

<table>
<thead>
<tr>
<th>WORDS</th>
<th>NUMBERS</th>
<th>ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition Property of Equality</strong></td>
<td>You can add the same number to both sides of an equation, and the statement will still be true.</td>
<td>[ 3 = 3 ] [ 3 + 2 = 3 + 2 ] [ 5 = 5 ]</td>
</tr>
<tr>
<td><strong>Subtraction Property of Equality</strong></td>
<td>You can subtract the same number from both sides of an equation, and the statement will still be true.</td>
<td>[ 7 = 7 ] [ 7 - 5 = 7 - 5 ] [ 2 = 2 ]</td>
</tr>
</tbody>
</table>

**Think and Discuss**

1. Describe how the Addition and Subtraction Properties of Equality are like a balanced scale.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.
1. **Vocabulary** Will the solution of an equation such as \( x - 3 = 9 \) be a variable or a number? Explain.

Solve each equation. Check your answer.

**SEE EXAMPLE 1**

2. \( s - 5 = 3 \)

3. \( 17 = w - 4 \)

4. \( k - 8 = -7 \)

5. \( x - 3.9 = 12.4 \)

6. \( 8.4 = y - 4.6 \)

7. \( \frac{3}{8} = t - \frac{1}{8} \)

**SEE EXAMPLE 2**

8. \( t + 5 = -25 \)

9. \( 9 = s + 9 \)

10. \( 42 = m + 36 \)

11. \( 2.8 = z + 0.5 \)

12. \( b + \frac{2}{3} = 2 \)

13. \( n + 1.8 = 3 \)

**SEE EXAMPLE 3**

14. \( -10 + d = 7 \)

15. \( 20 = -12 + v \)

16. \( -46 + q = 5 \)

17. \( 2.8 = -0.9 + y \)

18. \( -\frac{2}{3} + c = \frac{2}{3} \)

19. \( -\frac{5}{6} + p = 2 \)

20. **Geology** In 1673, the Hope diamond was reduced from its original weight by about 45 carats, resulting in a diamond weighing about 67 carats. Write and solve an equation to find how many carats the original diamond weighed. Show that your answer is reasonable.

**PRACTICE AND PROBLEM SOLVING**

Solve each equation. Check your answer.

21. \( 1 = k - 8 \)

22. \( u - 15 = -8 \)

23. \( x - 7 = 10 \)

24. \( -9 = p - 2 \)

25. \( \frac{3}{7} = p - \frac{1}{7} \)

26. \( q - 0.5 = 1.5 \)

27. \( 6 = t - 4.5 \)

28. \( 4 \frac{2}{3} = r - \frac{1}{3} \)

29. \( 6 = x - 3 \)

30. \( 1.75 = k - 0.75 \)

31. \( 19 + a = 19 \)

32. \( 4 = 3.1 + y \)

33. \( m + 20 = 3 \)

34. \( -12 = c + 3 \)

35. \( v + 2300 = -800 \)

36. \( b + 42 = 300 \)

37. \( 3.5 = n + 4 \)

38. \( b + \frac{1}{2} = \frac{1}{2} \)

39. \( x + 5.34 = 5.39 \)

40. \( 2 = d + \frac{1}{4} \)

41. \( -12 + f = 3 \)

42. \( -9 = -4 + g \)

43. \( -1200 + j = 345 \)

44. \( 90 = -22 + a \)

45. \( 26 = -4 + y \)

46. \( \frac{3}{4} = -\frac{1}{4} + w \)

47. \( -\frac{1}{6} + h = \frac{1}{6} \)

48. \( -5.2 + a = -8 \)

49. **Finance** Luis deposited $500 into his bank account. He now has $4732. Write and solve an equation to find how much was in his account before the deposit. Show that your answer is reasonable.

50. **ERROR ANALYSIS** Below are two possible solutions to \( x + 12.5 = 21.6 \). Which is incorrect? Explain the error.

   **A**
   
   \[
   \begin{align*}
   x + 12.5 &= 21.6 \\
   -12.5 &= -12.5 \\
   x &= 9.1
   \end{align*}
   \]

   **B**
   
   \[
   \begin{align*}
   x + 12.5 &= 21.6 \\
   +12.5 &= +12.5 \\
   x &= 34.1
   \end{align*}
   \]
Write an equation to represent each relationship. Then solve the equation.

51. Ten less than a number is equal to 12.
52. A number decreased by 13 is equal to 7.
53. Eight more than a number is 16.
54. A number minus 3 is –8.
55. The sum of 5 and a number is 6.
56. Two less than a number is –5.
57. The difference of a number and 4 is 9.

58. **Geology** The sum of the Atlantic Ocean’s average depth (in feet) and its greatest depth is 43,126. Use the information in the graph to write and solve an equation to find the average depth of the Atlantic Ocean. Show that your answer is reasonable.

59. **School** Helene’s marching band needs money to travel to a competition. Band members have raised $560. They need to raise a total of $1680. Write and solve an equation to find how much more they need. Show that your answer is reasonable.

60. **Economics** When you receive a loan to make a purchase, you often must make a down payment in cash. The amount of the loan is the purchase cost minus the down payment. Riva made a down payment of $1500 on a used car. She received a loan of $2600. Write and solve an equation to find the cost of the car. Show that your answer is reasonable.

**Geometry** The angles in each pair are complementary. Write and solve an equation to find each value of \( x \). (Hint: The measures of complementary angles add to 90°.)

61. \[ 63^\circ - x^\circ \]
62. \[ 42^\circ - x^\circ \]
63. \[ x^\circ - 15^\circ \]

**Rates** are often used to describe how quickly something is moving or changing.

a. A wildfire spreads at a rate of 1000 acres per day. How many acres will the fire cover in 2 days? Show that your answer is reasonable.

b. How many acres will the fire cover in 5 days? Explain how you found your answer.

c. Another wildfire spread for 7 days and covered a total of 780 square miles. How can you estimate the number of square miles the fire covered per day?
65. **Statistics**  The range of a set of scores is 28, and the lowest score is 47. Write and solve an equation to find the highest score. *(Hint: In a data set, the range is the difference between the highest and the lowest values.)* Show that your answer is reasonable.

66. **Write About It**  Describe a real-world situation that can be modeled by \( x + 5 = 25 \). Tell what the variable represents in your situation. Then solve the equation and tell what the solution means in the context of your problem.

67. **Critical Thinking**  Without solving, tell whether the solution of \(-3 + z = 10\) will be greater than 10 or less than 10. Explain.

---

68. **Test Prep**

Which situation is best represented by \( x - 32 = 8 \)?

- A. Logan withdrew $32 from her bank account. After her withdrawal, her balance was $8. How much was originally in her account?
- B. Daniel has 32 baseball cards. Joseph has 8 fewer baseball cards than Daniel. How many baseball cards does Joseph have?
- C. Room A contains 32 desks. Room B has 8 fewer desks. How many desks are in Room B?
- D. Janelle bought a bag of 32 craft sticks for a project. She used 8 craft sticks. How many craft sticks does she have left?

69. For which equation is \( a = 8 \) a solution?

- F. \( 15 - a = 10 \)
- G. \( 10 + a = 23 \)
- H. \( a - 18 = 26 \)
- I. \( a + 8 = 16 \)

70. **Short Response**  Julianna used a gift card to pay for an $18 haircut. The remaining balance on the card was $22.

a. Write an equation that can be used to determine the original value of the card.

b. Solve your equation to find the original value of the card.

---

### CHALLENGE AND EXTEND

Solve each equation. Check your answer.

- \( \frac{3}{5} + b = \frac{4}{5} \)
- \( x - \frac{7}{4} = \frac{2}{3} \)
- \( x + \frac{7}{4} = \frac{2}{3} \)
- \( x - \frac{4}{9} = \frac{4}{9} \)

- \( \) If \( p - 4 = 2 \), find the value of \( 5p - 20 \).
- \( \) If \( t + 6 = 21 \), find the value of \( -2t \).
- \( \) If \( x + 3 = 15 \), find the value of \( 18 + 6x \).
- \( \) If \( 2 + n = -11 \), find the value of \( 6n \).
Area of Composite Figures

Review the area formulas for squares, rectangles, and triangles in the table below.

<table>
<thead>
<tr>
<th>Squares</th>
<th>Rectangles</th>
<th>Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Square" /></td>
<td><img src="image" alt="Rectangle" /></td>
<td><img src="image" alt="Triangle" /></td>
</tr>
<tr>
<td>[A = s^2]</td>
<td>[A = \ell w]</td>
<td>[A = \frac{1}{2}bh]</td>
</tr>
</tbody>
</table>

A composite figure is a figure that is composed of basic shapes. You can divide composite figures into combinations of squares, rectangles, and triangles to find their areas.

**Example**

Find the area of the figure shown.

Divide the figure into a rectangle and a right triangle. Notice that you do not know the base or the height of the triangle. Use \(b\) and \(h\) to represent these lengths.

The bottom of the rectangle is 16 units long; the top of the rectangle is 8 units long plus the base of the triangle. Use this information to write and solve an equation.

\[b + 8 = 16\]
\[-8 - 8\]
\[b = 8\]

The right side of the figure is 13 units long: 7 units from the rectangle plus the height of the triangle. Use this information to write and solve an equation.

\[h + 7 = 13\]
\[-7 - 7\]
\[h = 6\]

The area of the figure is the sum of the areas of the rectangle and the triangle.

\[A = \ell w + \frac{1}{2}bh\]

\[A = 16(7) + \frac{1}{2}(8)(6)\]

\[A = 112 + 24\]

\[A = 136 \text{ square units}\]

**Try This**

Find the area of each composite figure.

1. 
   ![Diagram 1](image)

2. 
   ![Diagram 2](image)

3. 
   ![Diagram 3](image)
**Objective**
Solve one-step equations in one variable by using multiplication or division.

**Who uses this?**
Pilots can make quick calculations by solving one-step equations. (See Example 4.)

Solving an equation that contains multiplication or division is similar to solving an equation that contains addition or subtraction. Use inverse operations to undo the operations on the variable.

Remember that an equation is like a balanced scale. To keep the balance, whatever you do on one side of the equation, you must also do on the other side.

### Example 1
**Solving Equations by Using Multiplication**

Solve each equation. Check your answer.

**A** 
\[-4 = \frac{k}{-5}\]

\((-5)(-4) = (-5)\left(\frac{k}{-5}\right)\]

\[20 = k\]

Since \(k\) is divided by \(-5\), multiply both sides by \(-5\) to undo the division.

**Check**
\[-4 = \frac{k}{-5}\]

\[\begin{array}{c|c|c}
-4 & 20 & \checkmark \\
\hline
-5 & -5 & \\
\end{array}\]

To check your solution, substitute 20 for \(k\) in the original equation.

**B** 
\[\frac{m}{3} = 1.5\]

\[(3)\left(\frac{m}{3}\right) = (3)(1.5)\]

\[m = 4.5\]

Since \(m\) is divided by 3, multiply both sides by 3 to undo the division.

**Check**
\[\frac{m}{3} = 1.5\]

\[\begin{array}{c|c|c}
4.5 & 1.5 & \checkmark \\
\hline
3 &  \ \\
\end{array}\]

To check your solution, substitute 1.5 for \(m\) in the original equation.

### Check It Out!
Solve each equation. Check your answer.

1a. \(\frac{p}{5} = 10\)

1b. \(-13 = \frac{y}{3}\)

1c. \(\frac{c}{8} = 7\)
**Example 2**  
Solving Equations by Using Division  

Solve each equation. Check your answers.

**A**  
\[
7x = 56
\]
\[
\frac{7x}{7} = \frac{56}{7}
\]
\[
x = 8
\]
Since x is multiplied by 7, divide both sides by 7 to undo the multiplication.

**Check**  
\[
\frac{7x}{7} = \frac{56}{7}
\]
\[
\frac{56}{7} = \frac{56}{7}
\]
✓

**B**  
\[
13 = -2w
\]
\[
\frac{13}{-2} = \frac{-2w}{-2}
\]
\[
-6.5 = w
\]
Since w is multiplied by -2, divide both sides by -2 to undo the multiplication.

**Check**  
\[
\frac{13}{-2} = \frac{-2w}{-2}
\]
\[
-6.5 = w
\]
To check your solution, substitute -6.5 for w in the original equation.

**Check It Out!**  
Solve each equation. Check your answer.

2a. \[16 = 4c\]  
2b. \[0.5y = -10\]  
2c. \[15k = 75\]

Remember that dividing is the same as multiplying by the reciprocal. When solving equations, you will sometimes find it easier to multiply by a reciprocal instead of dividing. This is often true when an equation contains fractions.

**Example 3**  
Solving Equations That Contain Fractions  

Solve each equation.

**A**  
\[
\frac{5}{9} v = 35
\]
\[
\left(\frac{9}{5}\right) \frac{5}{9} v = \left(\frac{9}{5}\right) 35
\]
\[
v = 63
\]
The reciprocal of \(\frac{5}{9}\) is \(\frac{9}{5}\). Since v is multiplied by \(\frac{5}{9}\), multiply both sides by \(\frac{9}{5}\).

**B**  
\[
\frac{4y}{3} = \frac{2}{3}
\]
\[
\frac{5}{2} = \frac{4y}{3}
\]
\[
\frac{5}{2} = \frac{4}{3} y
\]
\[\frac{4y}{3}\] is the same as \(\frac{4}{3}y\).

\[
\left(\frac{3}{4}\right) \frac{5}{2} = \left(\frac{3}{4}\right) \frac{4}{3} y
\]
\[
\frac{15}{8} = y
\]
The reciprocal of \(\frac{4}{3}\) is \(\frac{3}{4}\). Since y is multiplied by \(\frac{4}{3}\), multiply both sides by \(\frac{3}{4}\).

**Check It Out!**  
Solve each equation. Check your answer.

3a. \[-\frac{1}{4} = \frac{1}{5} b\]  
3b. \[\frac{4j}{6} = \frac{2}{3}\]  
3c. \[\frac{1}{6} w = 102\]
**Aviation Application**

The distance in miles from the airport that a plane should begin descending, divided by 3, equals the plane’s height above the ground in thousands of feet. If a plane is 10,000 feet above the ground, write and solve an equation to find the distance at which the pilot should begin descending.

<table>
<thead>
<tr>
<th>Distance divided by 3 equals height in thousands of feet.</th>
</tr>
</thead>
</table>
| \[
\frac{d}{3} = h
\]

Write an equation to represent the relationship.

Substitute 10 for \(h\). Since \(d\) is divided by 3, multiply both sides by 3 to undo the division.

\[
\left(\frac{3}{3}\right) = \left(\frac{3}{3}\right)10 \\
\]

\[
d = 30
\]

The pilot should begin descending 30 miles from the airport.

4. **What if…?** A plane began descending 45 miles from the airport. Use the equation above to find how high the plane was flying when the descent began.

You have now used four properties of equality to solve equations. These properties are summarized in the box below.

**Properties of Equality**

<table>
<thead>
<tr>
<th>WORDS</th>
<th>NUMBERS</th>
<th>ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition Property of Equality</strong></td>
<td>You can add the same number to both sides of an equation, and the statement will still be true.</td>
<td>[3 = 3]</td>
</tr>
<tr>
<td></td>
<td>[3 + 2 = 3 + 2]</td>
<td>[a + c = b + c]</td>
</tr>
<tr>
<td><strong>Subtraction Property of Equality</strong></td>
<td>You can subtract the same number from both sides of an equation, and the statement will still be true.</td>
<td>[7 = 7]</td>
</tr>
<tr>
<td></td>
<td>[7 - 5 = 7 - 5]</td>
<td>[a - c = b - c]</td>
</tr>
<tr>
<td><strong>Multiplication Property of Equality</strong></td>
<td>You can multiply both sides of an equation by the same number, and the statement will still be true.</td>
<td>[6 = 6]</td>
</tr>
<tr>
<td></td>
<td>[6(3) = 6(3)]</td>
<td>[ac = bc]</td>
</tr>
<tr>
<td><strong>Division Property of Equality</strong></td>
<td>You can divide both sides of an equation by the same nonzero number, and the statement will still be true.</td>
<td>[8 = 8]</td>
</tr>
<tr>
<td></td>
<td>[\frac{8}{4} = \frac{8}{4}]</td>
<td>[(c \neq 0)]</td>
</tr>
<tr>
<td></td>
<td>[2 = 2]</td>
<td>[\frac{a}{c} = \frac{b}{c}]</td>
</tr>
</tbody>
</table>
THINK AND DISCUSS
1. Tell how the Multiplication and Division Properties of Equality are similar to the Addition and Subtraction Properties of Equality.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an equation that can be solved by using the given property, and solve it.

GUIDED PRACTICE
Solve each equation. Check your answer.

SEE EXAMPLE 1
1. \( \frac{k}{4} = 8 \)  
2. \( \frac{z}{3} = -9 \)  
3. \( -2 = \frac{w}{-7} \)
4. \( 6 = \frac{t}{-5} \)  
5. \( \frac{g}{1.9} = 10 \)  
6. \( 2.4 = \frac{b}{5} \)

SEE EXAMPLE 2
7. \( 4x = 28 \)  
8. \( -64 = 8c \)  
9. \( -9j = -45 \)
10. \( 84 = -12a \)  
11. \( 4m = 10 \)  
12. \( 2.8 = -2h \)

SEE EXAMPLE 3
13. \( \frac{1}{2}d = 7 \)  
14. \( 15 = \frac{5}{6}f \)  
15. \( \frac{2}{3}s = -6 \)
16. \( 9 = -\frac{3}{8}r \)  
17. \( \frac{1}{10} = \frac{4}{5}y \)  
18. \( \frac{1}{4}v = -\frac{3}{4} \)

SEE EXAMPLE 4
19. Recreation The Baseball Birthday Batter Package at a minor league ballpark costs $192. The package includes tickets, drinks, and cake for a group of 16 children. Write and solve an equation to find the cost per child.

20. Nutrition An orange contains about 80 milligrams of vitamin C, which is 10 times as much as an apple contains. Write and solve an equation to find the amount of vitamin C in an apple.

PRACTICE AND PROBLEM SOLVING
Solve each equation. Check your answer.

21. \( \frac{x}{2} = 12 \)  
22. \( -40 = \frac{b}{5} \)  
23. \( -\frac{j}{6} = 6 \)  
24. \( -\frac{n}{3} = -4 \)
25. \( -\frac{q}{5} = 30 \)  
26. \( 1.6 = \frac{d}{3} \)  
27. \( \frac{u}{10} = 5.5 \)  
28. \( \frac{h}{8.1} = -4 \)
29. \( 5t = -15 \)  
30. \( 49 = 7c \)  
31. \( -12 = -12u \)  
32. \( -7m = 63 \)
33. \( -52 = -4c \)  
34. \( 11 = -2z \)  
35. \( 5f = 1.5 \)  
36. \( -8.4 = -4n \)
Solve each equation. Check your answer.

37. \( \frac{5}{2} k = 5 \)  
38. \( -9 = \frac{3}{4} d \)  
39. \( -\frac{5}{8} b = 10 \)  
40. \( -\frac{4}{5} g = -12 \)

41. \( \frac{4}{7} t = -2 \)  
42. \( -\frac{4}{5} p = \frac{2}{3} \)  
43. \( \frac{2}{3} = -\frac{1}{3} q \)  
44. \( -\frac{5}{8} = -\frac{3}{4} a \)

45. **Finance** After taxes, Alexandra’s take-home pay is \( \frac{7}{10} \) of her salary before taxes. Write and solve an equation to find Alexandra’s salary before taxes for the pay period that resulted in $392 of take-home pay.

46. **Earth Science** Your weight on the Moon is about \( \frac{1}{6} \) of your weight on Earth. Write and solve an equation to show how much a person weighs on Earth if he weighs 16 pounds on the Moon. How could you check that your answer is reasonable?

47. **Error Analysis** For the equation \( x \div 3 = 15 \), a student found the value of \( x \) to be 5. Explain the error. What is the correct answer?

48. **Geometry** The perimeter of a square is given. Write and solve an equation to find the length of each side of the square.

49. \( P = 36 \text{ in.} \)
50. \( P = 84 \text{ in.} \)
51. \( P = 100 \text{ yd} \)
52. \( P = 16.4 \text{ cm} \)

Write an equation to represent each relationship. Then solve the equation.

52. Five times a number is 45.
53. A number multiplied by negative 3 is 12.
54. A number divided by 4 is equal to 10.
55. The quotient of a number and 3 is negative 8.

56. **Statistics** The mean height of the students in Marta’s class is 60 in. There are 18 students in her class. Write and solve an equation to find the total measure of all students’ heights. (Hint: The mean is found by dividing the sum of all data values by the number of data values.)

57. **Finance** Lisa earned $6.25 per hour at her after-school job. Each week she earned $50. Write and solve an equation to show how many hours she worked each week.

58. **Critical Thinking** Will the solution of \( \frac{x}{6} = 4 \) be greater than 4 or less than 4? Explain.

59. **Consumer Economics** Dion’s long-distance phone bill was $13.80. His long-distance calls cost $0.05 per minute. Write and solve an equation to find the number of minutes he was charged for. Show that your answer is reasonable.

60. **Nutrition** An 8 oz cup of coffee has about 184 mg of caffeine. This is 5 times as much caffeine as in a 12 oz soft drink. Write and solve an equation to find about how much caffeine is in a 12 oz caffeinated soft drink. Round your answer to the nearest whole number. Show that your answer is reasonable.

Use the equation \( 8y = 4x \) to find \( y \) for each value of \( x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 4x )</th>
<th>( 8y = 4x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>4(-4)  = -16</td>
<td>8y = -16</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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16 Chapter 2 Equations
Solve each equation. Check your answer.

66. \( \frac{m}{6} = 1 \)  
67. \( 4x = 28 \)  
68. \( 1.2h = 14.4 \)  
69. \( \frac{1}{5}x = 121 \)  
70. \( 2w = 26 \)  
71. \( 4b = \frac{3}{4} \)  
72. \( 5y = 11 \)  
73. \( \frac{n}{1.9} = 3 \)

Biology Use the table for Exercises 74 and 75.

<table>
<thead>
<tr>
<th>Animal</th>
<th>At Birth (g)</th>
<th>Adult Female (g)</th>
<th>Adult Male (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hamster</td>
<td>2</td>
<td>130</td>
<td>110</td>
</tr>
<tr>
<td>Guinea pig</td>
<td>85</td>
<td>800</td>
<td>1050</td>
</tr>
<tr>
<td>Rat</td>
<td>5</td>
<td>275</td>
<td>480</td>
</tr>
</tbody>
</table>

74. The mean weight of an adult male rat is 16 times the mean weight of an adult male mouse. Write and solve an equation to find the mean weight of an adult male mouse. Show that your answer is reasonable.

75. On average, a hamster at birth weighs \( \frac{2}{3} \) the weight of a gerbil at birth. Write and solve an equation to find the average weight of a gerbil at birth. Show that your answer is reasonable.

76. Write About It Describe a real-world situation that can be modeled by \( 3x = 42 \). Solve the equation and tell what the solution means in the context of your problem.

77. Which situation does NOT represent the equation \( \frac{d}{2} = 10 \)?
   A. Leo bought a box of pencils. He gave half of them to his brother. They each got 10 pencils. How many pencils were in the box Leo bought?
   B. Kasey evenly divided her money from babysitting into two bank accounts. She put $10 in each account. How much did Kasey earn?
   C. Gilbert cut a piece of ribbon into 2-inch strips. When he was done, he had ten 2-inch strips. How long was the ribbon to start?
   D. Mattie had 2 more CDs than her sister Leona. If Leona had 10 CDs, how many CDs did Mattie have?

78. Which equation below shows a correct first step for solving \( 3x = -12 \)?
   A. \( 3x + 3 = -12 + 3 \)  
   B. \( 3x - 3 = -12 - 3 \)  
   C. \( 3(3x) = 3(-12) \)  
   D. \( \frac{3x}{3} = \frac{-12}{3} \)

2-2 Solving Equations by Multiplying or Dividing
79. In a regular pentagon, all of the angles are equal in measure. The sum of the angle measures is 540°. Which of the following equations could be used to find the measure of each angle?

- \( \frac{x}{540} = 5 \)
- \( 5x = 540 \)
- \( \frac{x}{5} = 540 \)

80. For which equation is \( m = 10 \) a solution?

- \( 5 = 2m \)
- \( 5m = 2 \)
- \( \frac{m}{2} = 5 \)
- \( \frac{m}{10} = 2 \)

81. **Short Response** Luisa bought 6 cans of cat food that each cost the same amount. She spent a total of $4.80.

   a. Write an equation to determine the cost of one can of cat food. Tell what each part of your equation represents.

   b. Solve your equation to find the cost of one can of cat food. Show each step.

---

**CHALLENGE AND EXTEND**

Solve each equation. Check your answer.

82. \( \left( \frac{3}{5} \right) b = \frac{4}{5} \)

83. \( \left( \frac{1}{3} \right) x = 2 \frac{2}{3} \)

84. \( \left( \frac{5}{4} \right) x = -52 \frac{1}{5} \)

85. \( \left( -2 \frac{9}{10} \right) k = -26 \frac{1}{10} \)

86. \( \left( \frac{1}{2} \right) w = 15 \frac{1}{3} \)

87. \( \left( 2 \frac{1}{4} \right) d = 4 \frac{1}{2} \)

Find each indicated value.

88. If \( 2p = 4 \), find the value of \( 6p + 10 \).

89. If \( 6t = 24 \), find the value of \( -5t \).

90. If \( 3x = 15 \), find the value of \( 12 - 4x \).

91. If \( \frac{n}{2} = -11 \), find the value of \( 6n \).

92. To isolate \( x \) in \( ax = b \), what should you divide both sides by?

93. To isolate \( x \) in \( \frac{x}{a} = b \), what operation should you perform on both sides of the equation?

94. **Travel** The formula \( d = rt \) gives the distance \( d \) that is traveled at a rate \( r \) in time \( t \).

   a. If \( d = 400 \) and \( r = 25 \), what is the value of \( t \)?

   b. If \( d = 400 \) and \( r = 50 \), what is the value of \( t \)?

   c. **What if...?** How did \( t \) change when \( r \) increased from 25 to 50?

   d. **What if...?** If \( r \) is doubled while \( d \) remains the same, what is the effect on \( t \)?
Solve Equations by Graphing

You can use graphs to solve equations. As you complete this activity, you will learn some of the connections between graphs and equations.

Activity

Solve $3x - 4 = 5$.

1. Press $Y=1$. In $Y_1$, enter the left side of the equation, $3x - 4$.

   $Y_1=3x-4$  ENTER

   In $Y_2$, enter the right side of the equation, 5.

   $Y_2=5$  ENTER

2. Press GRAPH. Press TRACE. The display will show the $x$- and $y$-values of a point on the first line. Press the right arrow key several times. Notice that the $x$- and $y$-values change.

3. Continue to trace as close as possible to the intersection of the two lines. The $x$-value of this point 2.9787… is an approximation of the solution. The solution is about 3.

4. While still in trace mode, to check, press 3  ENTER. The display will show the $y$-value when $x = 3$. When $x = 3$, $y = 5$. So 3 is the solution. You can also check this solution by substituting 3 for $x$ in the equation:

   \[
   \text{Check} \quad \frac{3x - 4 = 5}{3(3) - 4} = 5 \quad \frac{9 - 4}{5} = 5 \quad \frac{5}{5} \checkmark
   \]

Try This

1. Solve $3x - 4 = 2$, $3x - 4 = 17$, and $3x - 4 = -7$ by graphing.

2. What does each line represent?

3. Describe a procedure for finding the solution of $3x - 4 = y$ for any value of $y$.

4. Solve $\frac{1}{2}x - 7 = -4$, $\frac{1}{2}x - 7 = 0$, and $\frac{1}{2}x - 7 = 2$ by graphing.
2-3

Solving Two-Step and Multi-Step Equations

Objective
Solve equations in one variable that contain more than one operation.

Why learn this?
Equations containing more than one operation can model real-world situations, such as the cost of a music club membership.

Alex belongs to a music club. In this club, students can buy a student discount card for $19.95. This card allows them to buy CDs for $3.95 each. After one year, Alex has spent $63.40.

To find the number of CDs $c$ that Alex bought, you can solve an equation.

\[
\text{Cost per CD} \rightarrow 3.95c + 19.95 = 63.40 \rightarrow \text{Total cost}
\]

Notice that this equation contains multiplication and addition. Equations that contain more than one operation require more than one step to solve. Identify the operations in the equation and the order in which they are applied to the variable. Then use inverse operations and work backward to undo them one at a time.

**EXAMPLE 1**

Solving Two-Step Equations

Solve $10 = 6 - 2x$. Check your answer.

\[
10 = 6 - 2x \\
\underline{- 6} \quad \underline{- 6} \\
4 = -2x \\
\underline{-2} \quad \underline{-2} \\
-2 = 1x \\
-2 = x
\]

Check

\[\begin{array}{c|c}
10 & 6 - 2(-2) \\
10 & 6 - (-4) \\
10 & 10 \checkmark \\
\end{array}\]

Solve each equation. Check your answer.

1a. $-4 + 7x = 3$
1b. $1.5 = 1.2y - 5.7$
1c. $\frac{n}{7} + 2 = 2$
EXAMPLE 2
Solving Two-Step Equations That Contain Fractions

Solve \( \frac{q}{15} - \frac{1}{5} = \frac{3}{5} \).

Method 1 Use fraction operations.

\[
\frac{q}{15} - \frac{1}{5} = \frac{3}{5}
\]

Since \( \frac{1}{5} \) is subtracted from \( \frac{q}{15} \), add \( \frac{1}{5} \) to both sides to undo the subtraction.

\[
+ \frac{1}{5} + \frac{1}{5}
\]

\[
\frac{q}{15} = \frac{4}{5}
\]

Since \( q \) is divided by 15, multiply both sides by 15 to undo the division.

\[
15 \left( \frac{q}{15} \right) = 15 \left( \frac{4}{5} \right)
\]

\[
q = \frac{60}{5}
\]

\[
q = 12
\]

Method 2 Multiply by the least common denominator (LCD) to clear the fractions.

\[
\frac{q}{15} - \frac{1}{5} = \frac{3}{5}
\]

Multiply both sides by 15, the LCD of the fractions.

\[
15 \left( \frac{q}{15} \right) - 15 \left( \frac{1}{5} \right) = 15 \left( \frac{3}{5} \right)
\]

Distribute 15 on the left side.

\[
q - 3 = 9
\]

Simplify.

\[
\underline{+3 \hspace{1cm} +3}
\]

\[
q = 12
\]

Solve each equation. Check your answer.

2a. \( \frac{2x}{5} - \frac{1}{2} = 5 \)  2b. \( \frac{3}{4} u + \frac{1}{2} = \frac{7}{8} \)  2c. \( \frac{1}{5} n - \frac{1}{3} = \frac{8}{3} \)

Equations that are more complicated may have to be simplified before they can be solved. You may have to use the Distributive Property or combine like terms before you begin using inverse operations.

EXAMPLE 3
Simplifying Before Solving Equations

Solve each equation.

A

\[
6x + 3 - 8x = 13
\]

Use the Commutative Property of Addition.

\[
6x - 8x + 3 = 13
\]

Combine like terms.

\[
-2x + 3 = 13
\]

Since 3 is added to \(-2x\), subtract 3 from both sides to undo the addition.

\[
-2x = 10
\]

\[
\underline{-2 \hspace{1cm} -2}
\]

\[
\frac{-2x}{-2} = \frac{10}{-2}
\]

Since \( x \) is multiplied by \(-2\), divide both sides by \(-2\) to undo the multiplication.
Solve each equation.

\[ 9 = 6 - (x + 2) \]

\[ 9 = 6 + (-1)(x + 2) \]
\[ 9 = 6 + (-1)(x) + (-1)(2) \]
\[ 9 = 6 - x - 2 \]
\[ 9 = 6 - 2 - x \]
\[ 9 = 4 - x \]

\[ \frac{-4}{-4} \]
\[ \frac{5}{-1} = \frac{-x}{-1} \]
\[ -5 = x \]

**Helpful Hint**

You can think of an opposite sign as a coefficient of \(-1\).

\(- (x + 2) = -1(x + 2)\)

and \(-x = -1x\).

**Example 4**

**Problem-Solving Application**

Alex belongs to a music club. In this club, students can buy a student discount card for $19.95. This card allows them to buy CDs for $3.95 each. After one year, Alex has spent $63.40. Write and solve an equation to find how many CDs Alex bought during the year.

1. **Understand the Problem**
   The answer will be the number of CDs that Alex bought during the year.

   List the important information:
   - Alex paid $19.95 for a student discount card.
   - Alex pays $3.95 for each CD purchased.
   - After one year, Alex has spent $63.40.

2. **Make a Plan**
   Let \( c \) represent the number of CDs that Alex purchased. That means Alex has spent $3.95\( c \). However, Alex must also add the amount spent on the card. Write an equation to represent this situation.

   \[ \text{total cost} = \text{cost of compact discs} + \text{cost of discount card} \]

   \[ 63.40 = 3.95c + 19.95 \]
Solve

\[ 63.40 = 3.95c + 19.95 \]

Since 19.95 is added to 3.95c, subtract 19.95 from both sides to undo the addition.

\[ \begin{align*}
-19.95 & \quad -19.95 \\
43.45 & = 3.95c \\
\frac{43.45}{3.95} & = \frac{3.95c}{3.95} \\
11 & = c
\end{align*} \]

Alex bought 11 CDs during the year.

Look Back

Check that the answer is reasonable. The cost per CD is about $4, so if Alex bought 11 CDs, this amount is about \( 11 \times 4 = 44 \). Add the cost of the discount card, which is about $20: \( 44 + 20 = 64 \). So the total cost was about $64, which is close to the amount given in the problem, $63.40.

\[ 4. \] Sara paid $15.95 to become a member at a gym. She then paid a monthly membership fee. Her total cost for 12 months was $735.95. How much was the monthly fee?

### Example

Solving Equations to Find an Indicated Value

If \( 3a + 12 = 30 \), find the value of \( a + 4 \).

Step 1  Find the value of \( a \).

\[ \begin{align*}
3a + 12 & = 30 \\
-12 & \quad -12 \\
3a & = 18
\end{align*} \]

Since 12 is added to \( 3a \), subtract 12 from both sides to undo the addition.

\[ \begin{align*}
\frac{3a}{3} & = \frac{18}{3} \\
a & = 6
\end{align*} \]

Since \( a \) is multiplied by 3, divide both sides by 3 to undo the multiplication.

Step 2  Find the value of \( a + 4 \).

\[ \begin{align*}
a + 4 \\
6 + 4
\end{align*} \]

To find the value of \( a + 4 \), substitute 6 for \( a \).

\[ \begin{align*}
& \text{Simplify.} \\
& 10
\end{align*} \]

\[ 5. \] If \( 2x + 4 = -24 \), find the value of \( 3x \).

Think and Discuss

1. Explain the steps you would follow to solve \( 2x + 1 = 7 \). How is this procedure different from the one you would follow to solve \( 2x - 1 = 7 \)?

2. Get Organized  Copy and complete the graphic organizer. In each box, write and solve a multi-step equation. Use addition, subtraction, multiplication, and division at least one time each.

Know it! Note

2-3 Solving Two-Step and Multi-Step Equations  23
GUIDED PRACTICE

Solve each equation. Check your answer.

1. \(4a + 3 = 11\)
2. \(8 = 3r - 1\)
3. \(42 = -2d + 6\)
4. \(x + 0.3 = 3.3\)
5. \(15y + 31 = 61\)
6. \(9 - c = -13\)
7. \(\frac{x}{6} + 4 = 15\)
8. \(\frac{1}{3}y + \frac{1}{4} = \frac{5}{12}\)
9. \(\frac{2}{7}j - \frac{1}{7} = \frac{3}{14}\)
10. \(15 = \frac{a}{3} - 2\)
11. \(4 - \frac{m}{2} = 10\)
12. \(\frac{x}{8} - \frac{1}{2} = 6\)
13. \(28 = 8x + 12 - 7x\)
14. \(2y - 7 + 5y = 0\)
15. \(2.4 = 3(m + 4)\)
16. \(3(x - 4) = 48\)
17. \(4t - 7 = t = 19\)
18. \((5 - 2w) + 8w = 15\)

SEE EXAMPLE 3

19. Transportation Paul bought a student discount card for the bus. The card cost $7 and allows him to buy daily bus passes for $1.50. After one month, Paul spent $29.50. How many daily bus passes did Paul buy?

20. If \(3x - 13 = 8\), find the value of \(x - 4\).
21. If \(3(x + 1) = 7\), find the value of \(3x\).
22. If \(-3(y - 1) = 9\), find the value of \(\frac{1}{2}y\).
23. If \(4 - 7x = 39\), find the value of \(x + 1\).

PRACTICE AND PROBLEM SOLVING

Solve each equation. Check your answer.

24. \(5 = 2g + 1\)
25. \(6h - 7 = 17\)
26. \(0.6v + 2.1 = 4.5\)
27. \(3x + 3 = 18\)
28. \(0.6g + 11 = 5\)
29. \(32 = 5 - 3t\)
30. \(2d + \frac{1}{5} = \frac{3}{5}\)
31. \(1 = 2x + \frac{1}{2}\)
32. \(\frac{x}{2} + 1 = \frac{3}{2}\)
33. \(\frac{2}{3} = \frac{4j}{6}\)
34. \(\frac{3}{4} = \frac{3}{8}x - \frac{3}{2}\)
35. \(\frac{1}{5} - \frac{x}{5} = -\frac{2}{5}\)
36. \(6 = -2(7 - c)\)
37. \(5(h - 4) = 8\)
38. \(-3x - 8 + 4x = 17\)
39. \(4x + 6x = 30\)
40. \(2(x + 3) = 10\)
41. \(17 = 3(p - 5) + 8\)

42. Consumer Economics Jennifer is saving money to buy a bike. The bike costs $245. She has $125 saved, and each week she adds $15 to her savings. How long will it take her to save enough money to buy the bike?

43. If \(2x + 13 = 17\), find the value of \(3x + 1\).
44. If \(-(x - 1) = 5\), find the value of \(-4x\).
45. If \(5(y + 10) = 40\), find the value of \(\frac{1}{4}y\).
46. If \(9 - 6x = 45\), find the value of \(x - 4\).

Geometry Write and solve an equation to find the value of \(x\) for each triangle. (Hint: The sum of the angle measures in any triangle is 180°.)

47. \(\triangle\) with angles \(30°\) and \(63°\), find the value of \(2x + 7\).
48. \(\triangle\) with angles \(115°\) and \(x\), find the value of \(x\).
49. \(\triangle\) with angles \(60°\) and \(60°\), find the value of \(4x - 80\).
Write an equation to represent each relationship. Solve each equation.

50. Seven less than twice a number equals 19.
51. Eight decreased by 3 times a number equals 2.
52. The sum of two times a number and 5 is 11.

53. **History** In 1963, Dr. Martin Luther King Jr. began his famous “I have a dream” speech with the words “Five score years ago, a great American, in whose symbolic shadow we stand, signed the Emancipation Proclamation.” The proclamation was signed by President Abraham Lincoln in 1863.
   a. Using the dates given, write and solve an equation that can be used to find the number of years in a score.
   b. How many score would represent 60?

Solve each equation. Check your answer.

54. $3t + 44 = 50$
55. $3(x - 2) = 18$
56. $15 = \frac{c}{3} - 2$
57. $2x + 6.5 = 15.5$

58. $3.9w - 17.9 = -2.3$
59. $17 = x - 3(x + 1)$
60. $5x + 9 = 39$
61. $15 + 5.5m = 70$

**Biology** Use the graph for Exercises 62 and 63.

62. The height of an ostrich is 20 inches more than 4 times the height of a kiwi. Write and solve an equation to find the height of a kiwi. Show that your answer is reasonable.

63. Five times the height of a kakapo minus 70 equals the height of an emu. Write and solve an equation to find the height of a kakapo. Show that your answer is reasonable.

64. The sum of two consecutive whole numbers is 57. What are the two numbers? *(Hint: Let $n$ represent the first number. Then $n + 1$ is the next consecutive whole number.)*

65. Stan’s, Mark’s, and Wayne’s ages are consecutive whole numbers. Stan is the youngest, and Wayne is the oldest. The sum of their ages is 111. Find their ages.

66. The sum of two consecutive even whole numbers is 206. What are the two numbers? *(Hint: Let $n$ represent the first number. What expression can you use to represent the second number?)*

67. a. The cost of fighting a certain forest fire is $225 per acre. Complete the table.
   b. Write an equation for the relationship between the cost $c$ of fighting the fire and the number of acres $n$.

<table>
<thead>
<tr>
<th>Acres</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>22,500</td>
</tr>
<tr>
<td>200</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td></td>
</tr>
</tbody>
</table>
68. **Critical Thinking** The equation \(2(m - 8) + 3 = 17\) has more than one solution method. Give at least two different “first steps” to solve this equation.

69. **Write About It** Write a series of steps that you can use to solve any multi-step equation.

---

**Test Prep**

70. Lin sold 4 more shirts than Greg. Fran sold 3 times as many shirts as Lin. In total, the three sold 51 shirts. Which represents the number of shirts Greg sold?

- (A) \(3g = 51\)
- (B) \(3 + g = 51\)
- (C) \(8 + 5g = 51\)
- (D) \(16 + 5g = 51\)

71. If \(\frac{4m - 3}{7} = 3\), what is the value of \(7m - 5\)?

- (F) 6
- (G) 10.5
- (H) 37
- (J) 68.5

72. The equation \(c = 48 + 0.06m\) represents the cost \(c\) of renting a car and driving \(m\) miles. Which statement best describes this cost?

- (A) The cost is a flat rate of $0.06 per mile.
- (B) The cost is $0.48 for the first mile and $0.06 for each additional mile.
- (C) The cost is a $48 fee plus $0.06 per mile.
- (D) The cost is a $6 fee plus $0.48 per mile.

73. **Gridded Response** A telemarketer earns $150 a week plus $2 for each call that results in a sale. Last week she earned a total of $204. How many of her calls resulted in sales?

---

**Challenge and Extend**

Solve each equation. Check your answer.

74. \(\frac{9}{2}x + 18 + 3x = \frac{11}{2}\)

75. \(\frac{15}{4}x - 15 = \frac{33}{4}\)

76. \((x + 6) - (2x + 7) - 3x = -9\)

77. \((4x + 2) - (12x + 8) + 2(5x - 3) = 6 + 11\)

78. Find a value for \(b\) so that the solution of \(4x + 3b = -1\) is \(x = 2\).

79. Find a value for \(b\) so that the solution of \(2x - 3b = 0\) is \(x = -9\).

80. **Business** The formula \(p = nc - e\) gives the profit \(p\) when a number of items \(n\) are each sold at a cost \(c\) and expenses \(e\) are subtracted.

   a. If \(p = 2500\), \(n = 2000\), and \(e = 800\), what is the value of \(c\)?

   b. If \(p = 2500\), \(n = 1000\), and \(e = 800\), what is the value of \(c\)?

   c. What if...? If \(n\) is divided in half while \(p\) and \(e\) remain the same, what is the effect on \(c\)?
Model Equations with Variables on Both Sides

Algebra tile models can help you understand how to solve equations with variables on both sides.

**Activity**

Use algebra tiles to model and solve $5x - 2 = 2x + 10$.

<table>
<thead>
<tr>
<th>MODEL</th>
<th>ALGEBRA</th>
</tr>
</thead>
</table>
| ![Model](model.png)  
Model $5x - 2$ on the left side of the mat and $2x + 10$ on the right side. Remember that $5x - 2$ is the same as $5x + (-2)$.  
$5x - 2 = 2x + 10$ |  
Remove $2x$-tiles from both sides. This represents subtracting $2x$ from both sides of the equation.  
$5x - 2 - 2x = 2x + 10$  
$3x = 10$ |  
Place $2$ yellow tiles on both sides. This represents adding $2$ to both sides of the equation. Remove zero pairs.  
$3x - 2 + 2 = 10 + 2$  
$3x = 12$ |  
Separate each side into $3$ equal groups. Each group is $\frac{1}{3}$ of the side. One $x$-tile is equivalent to $4$ yellow tiles.  
$\frac{1}{3}(3x) = \frac{1}{3}(12)$  
$x = 4$ |

**Try This**

Use algebra tiles to model and solve each equation.

1. $3x + 2 = 2x + 5$  
2. $5x + 12 = 2x + 3$  
3. $9x - 5 = 6x + 13$  
4. $x = -2x + 9$
**Objective**
Solve equations in one variable that contain variable terms on both sides.

**Vocabulary**
identity

---

**Why learn this?**
You can compare prices and find the best value.

Many phone companies offer low rates for long-distance calls without requiring customers to sign up for their services. To compare rates, solve an equation with variables on both sides.

To solve an equation like this, use inverse operations to “collect” variable terms on one side of the equation.

---

**Example 1**

Solving Equations with Variables on Both Sides

Solve each equation.

**A**

\[ 7k = 4k + 15 \]

\[ -4k \quad -4k \]

\[ 3k = 15 \]

\[ k = 5 \]

To collect the variable terms on one side, subtract 4k from both sides.

Since \( k \) is multiplied by 3, divide both sides by 3 to undo the multiplication.

**B**

\[ 5x - 2 = 3x + 4 \]

\[ -3x \quad -3x \]

\[ 2x - 2 = 4 \]

\[ +2 \quad +2 \]

\[ 2x = 6 \]

\[ x = 3 \]

To collect the variable terms on one side, subtract 3x from both sides.

Since 2 is subtracted from 2x, add 2 to both sides to undo the subtraction.

Since \( x \) is multiplied by 2, divide both sides by 2 to undo the multiplication.

**Check**

\[ 5x - 2 = 3x + 4 \]

\[ 5(3) - 2 \quad 3(3) + 4 \]

\[ 15 - 2 = 9 + 4 \]

\[ 13 = 13 \checkmark \]

---

**Check It Out!**

Solve each equation. Check your answer.

1a. \( 4b + 2 = 3b \)

1b. \( 0.5 + 0.3y = 0.7y - 0.3 \)

To solve more complicated equations, you may need to first simplify by using the Distributive Property or combining like terms.
EXAMPLE 2

Simplifying Each Side Before Solving Equations

Solve each equation.

A \[2(y + 6) = 3y\]

\[
\begin{align*}
2(y + 6) &= 3y \\
2y + 12 &= 3y \\
-2y &= -2y \\
12 &= y
\end{align*}
\]

Distribute 2 to the expression in parentheses.

To collect the variable terms on one side, subtract 2y from both sides.

Check \[2(y + 6) = 3y\]

\[
\begin{array}{c|c|c}
2(12 + 6) & 3(12) \\
2(18) & 36 \\
36 & 36 \checkmark
\end{array}
\]

B \[2k - 5 = 3(1 - 2k)\]

\[
\begin{align*}
2k - 5 &= 3(1 - 2k) \\
2k - 5 &= 3(1) - 3(2k) \\
2k - 5 &= 3 - 6k \\
+6k &= +6k \\
8k - 5 &= 3 \\
+5 &= +5 \\
8k &= 8 \\
\frac{8}{8} &= 1 \\
k &= 1
\end{align*}
\]

Distribute 3 to the expression in parentheses.

To collect the variable terms on one side, add 6k to both sides.

Since 5 is subtracted from 8k, add 5 to both sides.

Since k is multiplied by 8, divide both sides by 8.

C \[3 - 5b + 2b = -2 - 2(1 - b)\]

\[
\begin{align*}
3 - 5b + 2b &= -2 - 2(1 - b) \\
3 - 5b + 2b &= -2 - 2(1) - 2(-b) \\
3 - 5b + 2b &= -2 - 2 + 2b \\
+3b &= +3b \\
\frac{3}{7} &= -4 + 5b \\
\frac{4}{7} &= \frac{5b}{5} \\
1.4 &= b
\end{align*}
\]

Distribute \(-2\) to the expression in parentheses.

Combine like terms.

Add 3b to both sides.

Since \(-4\) is added to 5b, add 4 to both sides.

Since b is multiplied by 5, divide both sides by 5.

Check each solution. Check your answer.

2a. \[\frac{1}{2}(b + 6) = \frac{3}{2}b - 1\]

2b. \[3x + 15 - 9 = 2(x + 2)\]

An identity is an equation that is always true, no matter what value is substituted for the variable. The solutions of an identity are all real numbers. Some equations are always false. These equations have no solutions.
**EXAMPLE 3**

**Infinitely Many Solutions or No Solutions**

Solve each equation.

**A**

\[ x + 4 - 6x = 6 - 5x - 2 \]

Identify like terms.

\[ 4 - 5x = 4 - 5x \]

Combine like terms on the left and the right.

\[ \underline{+5x} \underline{+5x} \]

Add 5x to both sides.

\[ 4 = 4 \checkmark \]

True statement

The equation \( x + 4 - 6x = 6 - 5x - 2 \) is an identity. All values of \( x \) will make the equation true. All real numbers are solutions.

**B**

\[ -8x + 6 + 9x = -17 + x \]

Identify like terms.

\[ -8x + 6 + 9x = -17 + x \]

Combine like terms.

\[ \underline{-x} \underline{-x} \]

Subtract \( x \) from both sides.

\[ 6 = -17 \checkmark \]

False statement

The equation \(-8x + 6 + 9x = -17 + x\) is always false. There is no value of \( x \) that will make the equation true. There are no solutions.

**CHECK IT OUT!**

Solve each equation.

3a. \( 4y + 7 - y = 10 + 3y \)

3b. \( 2c + 7 + c = -14 + 3c + 21 \)

**EXAMPLE 4**

**Consumer Application**

The long-distance rates of two phone companies are shown in the table. How long is a call that costs the same amount no matter which company is used? What is the cost of that call?

<table>
<thead>
<tr>
<th>Phone Company</th>
<th>Charges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company A</td>
<td>36¢ plus 3¢ per minute</td>
</tr>
<tr>
<td>Company B</td>
<td>6¢ per minute</td>
</tr>
</tbody>
</table>

Let \( m \) represent minutes, and write expressions for each company’s cost.

Let is \[ 36¢ \] plus \[ 3¢ \] per minute \[ \times \] number of minutes \[ \text{the same as} \] \[ 6¢ \] per minute \[ \times \] number of minutes?

\[ 36 + 3m = 6 \]

To collect the variable terms on one side, subtract \( 3m \) from both sides.

\[ \frac{36}{3} = \frac{3m}{3} \]

Since \( m \) is multiplied by 3, divide both sides by 3 to undo the multiplication.

\[ 12 = m \]

The charges will be the same for a 12-minute call using either phone service. To find the cost of this call, evaluate either expression for \( m = 12 \):

\[ 36 + 3m = 36 + 3(12) = 36 + 36 = 72 \]

\[ 6m = 6(12) = 72 \]

The cost of a 12-minute call through either company is 72¢.

**CHECK IT OUT!**

4. Four times Greg’s age, decreased by 3 is equal to 3 times Greg’s age, increased by 7. How old is Greg?
THINK AND DISCUSS

1. Tell which of the following is an identity. Explain your answer.
   a. $4(a + 3) - 6 = 3(a + 3) - 6$  
   b. $8.3x - 9 + 0.7x = 2 + 9x - 11$

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of an equation that has the indicated number of solutions.

   An equation with variables on both sides can have...

   one solution:  
   many solutions:  
   no solution:

GUIDED PRACTICE

1. **Vocabulary** How can you recognize an identity?

   Solve each equation. Check your answer.

   2. $2c - 5 = c + 4$
   3. $8r + 4 = 10 + 2r$
   4. $2x - 1 = x + 11$
   5. $28 - 0.3y = 0.7y - 12$
   6. $-2(x + 3) = 4x - 3$
   7. $3c - 4c + 1 = 5c + 2 + 3$
   8. $5 + 3(q - 4) = 2(q + 1)$
   9. $5 - (t + 3) = -1 + 2(t - 3)$
   10. $7x - 4 = -2x + 1 + 9x - 5$
   11. $8x + 6 - 9x = 2 - x - 15$
   12. $6y = 8 - 9 + 6y$
   13. $6 - 2x - 1 = 4x + 8 - 6x - 3$
   
   14. **Consumer Economics** A house-painting company charges $376 plus $12 per hour. Another painting company charges $280 plus $15 per hour.
      a. How long is a job for which both companies will charge the same amount?
      b. What will that cost be?

   PRACTICE AND PROBLEM SOLVING

   Solve each equation. Check your answer.

   15. $7a - 17 = 4a + 1$
   16. $2b - 5 = 8b + 1$
   17. $4x - 2 = 3x + 4$
   18. $2x - 5 = 4x - 1$
   19. $8x - 2 = 3x + 12.25$
   20. $5x + 2 = 3x$
   21. $3c - 5 = 2c + 5$
   22. $-17 - 2x = 6 - x$
   23. $3(t - 1) = 9 + t$
   24. $5 - x - 2 = 3 + 4x + 5$
   25. $2(x + 4) = 3(x - 2)$
   26. $3m - 10 = 2(4m - 5)$
   27. $5 - (n - 4) = 3(n + 2)$
   28. $6(x + 7) - 20 = 6x$
   29. $8(x + 1) = 4x - 8$
   30. $x - 4 - 3x = -2x - 3 - 1$
   31. $-2(x + 2) = -2x + 1$
   32. $2(x + 4) - 5 = 2x + 3$
33. **Sports** Justin and Tyson are beginning an exercise program to train for football season. Justin weighs 150 lb and hopes to gain 2 lb per week. Tyson weighs 195 lb and hopes to lose 1 lb per week.
   a. If the plan works, in how many weeks will the boys weigh the same amount? What will that weight be?

Write an equation to represent each relationship. Then solve the equation.

34. Three times the sum of a number and 4 is the same as 18 more than the number.

35. A number decreased by 30 is the same as 14 minus 3 times the number.

36. Two less than 2 times a number is the same as the number plus 64.

Solve each equation. Check your answer.

37. \(2x - 2 = 4x + 6\)

38. \(3x + 5 = 2x + 2\)

39. \(4x + 3 = 5x - 4\)

40. \(-\frac{2}{5}p + 2 = \frac{1}{5}p + 11\)

41. \(5x + 24 = 2x + 15\)

42. \(5x - 10 = 14 - 3x\)

43. \(12 - 6x = 10 - 5x\)

44. \(5x - 7 = -6x - 29\)

45. \(1.8x + 2.8 = 2.5x + 2.1\)

46. \(2.6x + 18 = 2.4x + 22\)

47. \(1 - 3x = 2x + 8\)

48. \(\frac{1}{2}(8 - 6h) = h\)

49. \(3(x + 1) = 2x + 7\)

50. \(9x - 8 + 4x = 7x + 16\)

51. \(3(2x - 1) + 5 = 6(x + 1)\)

52. **Travel** Rapid Rental Car company charges a $40 rental fee, $15 for gas, and $0.25 per mile driven. For the same car, Capital Cars charges $45 for rental and gas and $0.35 per mile.
   a. Find the number of miles for which the companies’ charges will be the same. Then find that charge. Show that your answers are reasonable.
   b. The Barre family estimates that they will drive about 95 miles during their vacation to Hershey, Pennsylvania. Which company should they rent their car from? Explain.
   c. **What if…?** The Barres have extended their vacation and now estimate that they will drive about 120 miles. Should they still rent from the same company as in part b? Why or why not?
   d. Give a general rule for deciding which company to rent from.

53. **Geometry** The triangles shown have the same perimeter. What is the value of \(x\)?

54. a. A fire currently covers 420 acres and continues to spread at a rate of 60 acres per day. How many total acres will be covered in the next 2 days? Show that your answer is reasonable.
   b. Write an expression for the total area covered by the fire in \(d\) days.
   c. The firefighters estimate that they can put out the fire at a rate of 80 acres per day. Write an expression for the total area that the firefighters can put out in \(d\) days.
   d. Set the expressions in parts b and c equal. Solve for \(d\). What does \(d\) represent?
55. **Critical Thinking**  Write an equation with variables on both sides that has no solution.

56. **Biology**  The graph shows the maximum recorded speeds of the four fastest mammals.

<table>
<thead>
<tr>
<th>Mammal</th>
<th>Maximum speed (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cheetah</td>
<td>65</td>
</tr>
<tr>
<td>Pronghorn antelope</td>
<td>55</td>
</tr>
<tr>
<td>Springbok</td>
<td>50</td>
</tr>
<tr>
<td>Thompson’s gazelle</td>
<td>47</td>
</tr>
</tbody>
</table>

*Source: The Top 10 of Everything*

a. Write an expression for the distance in miles that a Thompson’s gazelle can run at top speed in \( x \) hours.

b. Write an expression for the distance in miles that a cheetah can run at top speed in \( x \) hours.

c. A cheetah and a Thompson’s gazelle are running at their top speeds. The cheetah is one mile behind the gazelle. Write an expression for the distance the cheetah must run to catch up with the gazelle.

d. Write and solve an equation that represents how long the cheetah will have to run at top speed to catch up with the gazelle.

e. A cheetah can maintain its top speed for only 300 yards. Will the cheetah be able to catch the gazelle? Explain.

57. **Write About It**  Write a series of steps that you can use to solve any equation with variables on both sides.

58. Lindsey’s monthly magazine subscription costs $1.25 per issue. Kenzie’s monthly subscription costs $1.50 per issue, but she received her first 2 issues free. Which equation can be used to find the number of months after which the girls will have paid the same amount?

- \[ 1.25m = 1.50m - 2 \]
- \[ 1.25m = 1.50(m - 2) \]
- \[ 1.25m = 1.50m - 2m \]
- \[ 1.25m = 3m - 1.50 \]

59. What is the numerical solution of the equation \( 7 \) times a number equals \( 3 \) less than \( 5 \) times that number?

- \(-1.5\)
- \(0.25\)
- \(\frac{2}{3}\)
- \(4\)

60. Three packs of markers cost $9.00 less than 5 packs of markers. Which equation best represents this situation?

- \[ 5x + 9 = 3x \]
- \[ 3x + 9 = 5x \]
- \[ 3x - 9 = 5x \]
- \[ 9 - 3x = 5x \]

61. Nicole has $120. If she saves $20 per week, in how many days will she have $500?

- \(19\)
- \(25\)
- \(133\)
- \(175\)

62. **Gridded Response**  Solve \(-2(x - 1) + 5x = 2(2x - 1)\).
Challenge and Extend

Solve each equation.

63. \[4x + 2[4 - 2(x + 2)] = 2x - 4\]

64. \[\frac{x + 5}{2} + \frac{x - 1}{2} = \frac{x - 1}{3}\]

65. \[\frac{2}{3} w - \frac{1}{4} = \frac{2}{3} \left( w - \frac{1}{4} \right)\]

66. \[-5 - 7 - 3f = -f - 2(f + 6)\]

67. \[\frac{2}{3} x + \frac{1}{2} = \frac{3}{5} x - \frac{5}{6}\]

68. \[x - \frac{1}{4} = \frac{x}{3} + 7 \frac{3}{4}\]

69. Find three consecutive integers such that twice the greatest integer is 2 less than 3 times the least integer.

70. Find three consecutive integers such that twice the least integer is 12 more than the greatest integer.

71. Rob had twice as much money as Sam. Then Sam gave Rob 1 quarter, 2 nickels, and 3 pennies. Rob then gave Sam 8 dimes. If they now have the same amount of money, how much money did Rob originally have? Check your answer.

Career Path

Beth Simmons
Biology major

Q: What math classes did you take in high school?
A: Algebra 1 and 2, Geometry, and Precalculus

Q: What math classes have you taken in college?
A: Two calculus classes and a calculus-based physics class

Q: How do you use math?
A: I use math a lot in physics. Sometimes I would think a calculus topic was totally useless, and then we would use it in physics class! In biology, I use math to understand populations.

Q: What career options are you considering?
A: When I graduate, I could teach, or I could go to graduate school and do more research. I have a lot of options.
Solve Equations Graphically

You can use graphs to solve equations. As you complete this activity, you will explore connections between graphs and equations.

Activity 1

Use a graphing calculator to solve \( x + 1 = 3x - 5 \).

1. Write the equation \( x + 1 = 3x - 5 \) as two separate equations:
   
   \[
   y = x + 1 \\
   y = 3x - 5
   \]

2. Press \( \text{Y} = \). Enter the first equation, \( y = x + 1 \), in \( Y_1 \) and the second equation, \( y = 3x - 5 \), in \( Y_2 \).

3. Press \( \text{2nd} \text{ TABLE} \) to use the \( \text{TABLE} \) function.

4. Scroll through the values using \( \text{\leftarrow} \) and \( \text{\rightarrow} \). Look for values where \( Y_1 \) and \( Y_2 \) are equal, and then find the corresponding \( x \)-value. This \( x \)-value is the solution of the equation.

5. Press \( \text{GRAPH} \) and verify where the lines intersect.

6. You can check your answer by solving the equation algebraically.

\[
\begin{align*}
  x + 1 &= 3x - 5 \\
  x - x + 1 &= 3x - x - 5 \\
  1 &= 2x - 5 \\
  1 + 5 &= 2x - 5 + 5 \\
  6 &= 2x \\
  6 &= 2x \\
  2 &= \frac{2}{2} \\
  3 &= x
\end{align*}
\]
Try This

Solve each equation by graphing.
1. \(2x + 1 = x + 8\)  
2. \(4x - 3 = 2x + 9\)  
3. \(2x - 5 = x - 9\)  
4. \(-x + 3 = 5 - 2x\)  
5. \(x - 10 = -3x + 2\)  
6. \(-6x + 5 = -x\)

Activity 2

Use a graphing calculator to solve \(1 - 4x + 3x = -1 - 3(2 - x)\).

1. Press \(\text{Y} =\) and enter each side of the equation into \(Y_1\) and \(Y_2\). You do not need to simplify first.

2. Press \(\text{2nd} \ \text{TABLE}\) to use the \text{TABLE} function. Scroll until you find the value of \(x\) for which the \(y\)-values are the same.

3. Press \(\text{GRAPH}\) and verify where the lines intersect.

4. You can check your answer by solving the equation algebraically.

\[
1 - 4x + 3x = -1 - 3(2 - x) \\
1 - x = -1 - 6 + 3x \\
1 - x - 3x = -7 + 3x - 3x \\
1 - 4x = -7 \\
1 - 1 - 4x = -7 - 1 \\
-4x = -8 \\
-4x = -8 \\
\frac{-4}{-4} = \frac{-8}{-4} \\
x = 2 \checkmark
\]

Try This

Solve each equation by graphing.
7. \(-5x + 2(x - 2) = 5x + 4\)  
8. \(3(x + 2) = -x + 18\)  
9. \(2x - 5x + 4 = 2(x - 1) + 16\)
Algebraic Proof

Who uses this?
Game designers and animators solve equations to simulate motion. (See Example 2.)

A proof is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.

If you’ve ever solved an equation in Algebra, then you’ve already done a proof! An algebraic proof uses algebraic properties such as the properties of equality and the Distributive Property.

Properties of Equality

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td>If ( a = b ), then ( a + c = b + c ).</td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td>If ( a = b ), then ( a - c = b - c ).</td>
</tr>
<tr>
<td>Multiplication Property of Equality</td>
<td>If ( a = b ), then ( ac = bc ).</td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td>If ( a = b ) and ( c \neq 0 ), then ( \frac{a}{c} = \frac{b}{c} ).</td>
</tr>
<tr>
<td>Reflexive Property of Equality</td>
<td>( a = a )</td>
</tr>
<tr>
<td>Symmetric Property of Equality</td>
<td>If ( a = b ), then ( b = a ).</td>
</tr>
<tr>
<td>Transitive Property of Equality</td>
<td>If ( a = b ) and ( b = c ), then ( a = c ).</td>
</tr>
<tr>
<td>Substitution Property of Equality</td>
<td>If ( a = b ), then ( b ) can be substituted for ( a ) in any expression.</td>
</tr>
</tbody>
</table>

As you have learned, if you start with a true statement and each logical step is valid, then your conclusion is valid.

An important part of writing a proof is giving justifications to show that every step is valid. For each justification, you can use a definition, postulate, property, or a piece of information that is given.

Example 1
Solving an Equation in Algebra

Solve the equation \(-5 = 3n + 1\). Write a justification for each step.

\[-5 = 3n + 1 \quad \text{Given equation}\]
\[-5 - 1 = 3n \quad \text{Subtraction Property of Equality}\]
\[-6 = 3n \quad \text{Simplify}\]
\[-6 \div 3 = 3n \quad \text{Division Property of Equality}\]
\[-2 = n \quad \text{Simplify}\]
\[n = -2 \quad \text{Symmetric Property of Equality}\]

Check It Out! 1. Solve the equation \(\frac{1}{2}t = -7\). Write a justification for each step.
Problem-Solving Application

To simulate the motion of an object in a computer game, the designer uses the formula \( sr = 3.6p \) to find the number of pixels the object must travel during each second of animation. In the formula, \( s \) is the desired speed of the object in kilometers per hour, \( r \) is the scale of pixels per meter, and \( p \) is the number of pixels traveled per second.

The graphics in a game are based on a scale of 6 pixels per meter. The designer wants to simulate a vehicle moving at 75 km/h. How many pixels must the vehicle travel each second? Solve the equation for \( p \) and justify each step.

1. **Understand the Problem**

   The answer will be the number of pixels traveled per second.
   
   List the important information:
   
   - \( sr = 3.6p \)
   - \( s = 75 \) km/h
   - \( p \): pixels traveled per second
   - \( r = 6 \) pixels per meter

2. **Make a Plan**

   Substitute the given information into the formula and solve.

3. **Solve**

   \[ sr = 3.6p \]
   \[ (75)(6) = 3.6p \]
   \[ 450 = 3.6p \]
   \[ \frac{450}{3.6} = \frac{3.6p}{3.6} \]
   \[ 125 = p \]
   \[ p = 125 \text{ pixels} \]

4. **Look Back**

   Check your answer by substituting it back into the original formula.
   
   \[ sr = 3.6p \]
   \[ (75)(6) = 3.6(125) \]
   \[ 450 = 450 \] ✓

2. **What is the temperature in degrees Celsius \( C \) when it is 86°F?**

   Solve the equation \( C = \frac{5}{9}(F - 32) \) for \( C \) and justify each step.

Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.
EXAMPLE 3

Solving an Equation in Geometry

Write a justification for each step.

\[ KM = KL + LM \]
\[ 5x - 4 = (x + 3) + (2x - 1) \]  
Segment Addition Postulate
\[ 5x - 4 = 3x + 2 \]  
Substitution Property of Equality
\[ 2x - 4 = 2 \]  
Simplify.
\[ 2x = 6 \]  
Subtraction Property of Equality
\[ x = 3 \]  
Addition Property of Equality
\[ x = 3 \]  
Division Property of Equality

CHECK IT OUT!

3. Write a justification for each step.

\[ m\angle ABC = m\angle ABD + m\angle DBC \]
\[ 8x^\circ = (3x + 5)^\circ + (6x - 16)^\circ \]  
Segment Addition Postulate
\[ 8x = 9x - 11 \]  
Substitution Property of Equality
\[ -x = -11 \]  
Simplify.
\[ x = 11 \]  
Subtraction Property of Equality

You have learned that segments with equal lengths are congruent and angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.

Properties of Congruence

**SYMBOLS**

**EXAMPLE**

| Reflexive Property of Congruence | EF ≅ EF |
| Symmetric Property of Congruence | If \( \angle 1 \equiv \angle 2 \), then \( \angle 2 \equiv \angle 1 \). |
| Transitive Property of Congruence | If \( \overline{PQ} \equiv \overline{RS} \) and \( \overline{RS} \equiv \overline{TU} \), then \( \overline{PQ} \equiv \overline{TU} \). |

**EXAMPLE 4**

Identifying Properties of Equality and Congruence

Identify the property that justifies each statement.

A. \( m\angle 1 = m\angle 1 \)  
Reflex. Prop. of =

B. \( \overline{XY} \equiv \overline{VW} \), so \( \overline{VW} \equiv \overline{XY} \).  
Sym. Prop. of ≅

C. \( \angle ABC \equiv \angle ABC \)  
Reflex. Prop. of ≅

D. \( \angle 1 \equiv \angle 2 \), and \( \angle 2 \equiv \angle 3 \). So \( \angle 1 \equiv \angle 3 \).  
Trans. Prop. of ≅

Identify the property that justifies each statement.

4a. \( DE = GH \), so \( GH = DE \).  
4b. \( 94^\circ = 94^\circ \)

4c. \( 0 = a \), and \( a = x \). So \( 0 = x \).  
4d. \( \angle A \equiv \angle Y \), so \( \angle Y \equiv \angle A \).
THINK AND DISCUSS
1. Tell what property you would use to solve the equation $k/6 = 3.5$.
2. Explain when to use a congruence symbol instead of an equal sign.
3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of the property, using the correct symbol.

<table>
<thead>
<tr>
<th>Property</th>
<th>Equality</th>
<th>Congruence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflexive</td>
<td>x = x</td>
<td>x ≅ x</td>
</tr>
<tr>
<td>Symmetric</td>
<td>x = y</td>
<td>x ≅ y</td>
</tr>
<tr>
<td>Transitive</td>
<td>x = y, y = z</td>
<td>x ≅ y, y ≅ z</td>
</tr>
</tbody>
</table>

GUIDED PRACTICE

1. Vocabulary Write the definition of proof in your own words.

Multi-Step Solve each equation. Write a justification for each step.

2. $y + 1 = 5$
3. $t - 3.2 = -8.3$
4. $2p - 30 = -4p + 6$
5. $x + 3\over 2 = 8$
6. $\frac{1}{2}n = \frac{3}{4}$

8. Nutrition Amy’s favorite breakfast cereal has 102 Calories per serving. The equation $C = 9f + 90$ relates the grams of fat $f$ in one serving to the Calories $C$ in one serving. How many grams of fat are in one serving of the cereal? Solve the equation for $f$ and justify each step.

9. Movie Rentals The equation $C = 5.75 + 0.89m$ relates the number of movie rentals $m$ to the monthly cost $C$ of a movie club membership. How many movies did Elias rent this month if his membership cost $11.98? Solve the equation for $m$ and justify each step.

Write a justification for each step.

10. $5y + 6 = 2y + 21$

11. $9n + 5$

12. $\overline{AB} \cong \overline{AB}$

13. $\angle 1 = \angle 2$, and $\angle 2 = \angle 4$. So $\angle 1 = \angle 4$.

14. $x = y$, so $y = x$.

15. $\overline{ST} \cong \overline{YZ}$, and $\overline{YZ} \cong \overline{PR}$. So $\overline{ST} \cong \overline{PR}$.
Independent Practice

For Exercises See Example
16–21 1
22 2
23–24 3
25–28 4

Extra Practice
See Extra Practice for more Skills Practice and Applications Practice exercises.

PRACTICE AND PROBLEM SOLVING

Multi-Step Solve each equation. Write a justification for each step.

16. \(5x - 3 = 4(x + 2)\)  17. \(1.6 = 3.2n\)  18. \(\frac{z}{3} - 2 = -10\)

19. \(-(h + 3) = 72\)  20. \(9y + 17 = -19\)  21. \(\frac{1}{2}(p - 16) = 13\)

22. **Ecology** The equation \(T = 0.03c + 0.05b\) relates the numbers of cans \(c\) and bottles \(b\) collected in a recycling rally to the total dollars \(T\) raised. How many cans were collected if $147 was raised and 150 bottles were collected? Solve the equation for \(c\) and justify each step.

Write a justification for each step.

23. \(m\angle XYZ = m\angle 2 + m\angle 3\)  24. \(m\angle WYV = m\angle 1 + m\angle 2\)

\(4n - 6 = 58 + (2n - 12)\)  \(5n = 3(n - 2) + 58\)

\(4n - 6 = 2n + 46\)  \(5n = 3n - 6 + 58\)

\(2n - 6 = 46\)  \(5n = 3n + 52\)

\(2n = 52\)  \(2n = 52\)

\(n = 26\)  \(n = 26\)

Identify the property that justifies each statement.

25. \(KL \cong PR\), so \(PR \cong KL\).

26. \(412 = 412\)

27. If \(a = b\) and \(b = 0\), then \(a = 0\).

28. figure \(A \cong \text{figure} A\)

29. **Estimation** Round the numbers in the equation \(2(3.1x - 0.87) = 94.36\) to the nearest whole number and estimate the solution. Then solve the equation, justifying each step. Compare your estimate to the exact solution.

Use the indicated property to complete each statement.

30. Reflexive Property of Equality: \(3x - 1 = \) ?

31. Transitive Property of Congruence: If \(\angle A \cong \angle X\) and \(\angle X \cong \angle T\), then \(\) ? .

32. Symmetric Property of Congruence: If \(BC \cong NP\), then \(\) ? .

33. **Recreation** The north campground is midway between the Northpoint Overlook and the waterfall. Use the midpoint formula to find the values of \(x\) and \(y\), and justify each step.

34. **Business** A computer repair technician charges $35 for each job plus $21 per hour of labor and 110% of the cost of parts. The total charge for a 3-hour job was $169.50. What was the cost of parts for this job? Write and solve an equation and justify each step in the solution.

35. **Finance** Morgan spent a total of $1,733.65 on her car last year. She spent $92.50 on registration, $79.96 on maintenance, and $983 on insurance. She spent the remaining money on gas. She drove a total of 10,820 miles.

a. How much on average did the gas cost per mile? Write and solve an equation and justify each step in the solution.

b. **What if...?** Suppose Morgan’s car averages 32 miles per gallon of gas. How much on average did Morgan pay for a gallon of gas?

36. **Critical Thinking** Use the definition of segment congruence and the properties of equality to show that all three properties of congruence are true for segments.
37. Recall from Algebra 1 that the Multiplication and Division Properties of Inequality tell you to reverse the inequality sign when multiplying or dividing by a negative number.
   a. Solve the inequality $x + 15 \leq 63$ and write a justification for each step.
   b. Solve the inequality $-2x > 36$ and write a justification for each step.

38. **Write About It** Compare the conclusion of a deductive proof and a conjecture based on inductive reasoning.

39. Which could NOT be used to justify the statement $\overline{AB} \cong \overline{CD}$?
   - (A) Definition of congruence
   - (B) Reflexive Property of Congruence
   - (C) Symmetric Property of Congruence
   - (D) Transitive Property of Congruence

40. A club membership costs $35 plus $3 each time $t$ the member uses the pool. Which equation represents the total cost $C$ of the membership?
   - (F) $35 = C + 3t$
   - (G) $C + 35 = 3t$
   - (H) $C = 35 + 3t$
   - (J) $C = 35t + 3$

41. Which statement is true by the Reflexive Property of Equality?
   - (A) $x = 35$
   - (B) $\overline{CD} = \overline{CD}$
   - (C) $\overline{RT} \cong \overline{TR}$
   - (D) $CD = CD$

42. **Gridded Response** In the triangle, $m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$. If $m\angle 3 = 2m\angle 1$ and $m\angle 1 = m\angle 2$, find $m\angle 3$ in degrees.

### CHALLENGE AND EXTEND

43. In the gate, $PA = QB$, $QB = RA$, and $PA = 18$ in. Find $PR$, and justify each step.

44. **Critical Thinking** Explain why there is no Addition Property of Congruence.

45. **Algebra** Justify each step in the solution of the inequality $7 - 3x > 19$. 

---

*Images from Brand X Pictures/ Getty Images, or Paul A Souders/CORBIS*
Objectives
Solve a formula for a given variable.
Solve an equation in two or more variables for one of the variables.

Vocabulary
formula
literal equation

Who uses this?
Athletes can “rearrange” the distance formula to calculate their average speed.

Many wheelchair athletes compete in marathons, which cover about 26.2 miles. Using the time \( t \) it took to complete the race, the distance \( d \), and the formula \( d = rt \), racers can find their average speed \( r \).

A **formula** is an equation that states a rule for a relationship among quantities.

In the formula \( d = rt \), \( d \) is isolated. You can “rearrange” a formula to isolate any variable by using inverse operations. This is called **solving for a variable**.

### Solving for a Variable

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Locate the variable you are asked to solve for in the equation.</td>
</tr>
<tr>
<td>2</td>
<td>Identify the operations on this variable and the order in which they are applied.</td>
</tr>
<tr>
<td>3</td>
<td>Use inverse operations to undo operations and isolate the variable.</td>
</tr>
</tbody>
</table>

### Example 1

**Sports Application**

In 2004, Ernst Van Dyk won the wheelchair race of the Boston Marathon with a time of about 1.3 hours. The race was about 26.2 miles. What was his average speed? Use the formula \( d = rt \) and round your answer to the nearest tenth.

The question asks for speed, so first solve the formula \( d = rt \) for \( r \).

\[
\frac{d}{t} = r \quad \text{Locate } r \text{ in the equation.}
\]

\[
\frac{d}{t} = \frac{rt}{t} \quad \text{Since } r \text{ is multiplied by } t, \text{ divide both sides by } t \text{ to undo the multiplication.}
\]

\[
\frac{d}{t} = r, \text{ or } r = \frac{d}{t}
\]

Now use this formula and the information given in the problem.

\[
r = \frac{d}{t} = \frac{26.2}{1.3} \approx 20.2
\]

Van Dyk's average speed was about 20.2 miles per hour.

1. Solve the formula \( d = rt \) for \( t \). Find the time in hours that it would take Van Dyk to travel 26.2 miles if his average speed was 18 miles per hour. Round to the nearest hundredth.
### Example 2: Solving Formulas for a Variable

#### A

The formula for a Fahrenheit temperature in terms of degrees Celsius is \( F = \frac{9}{5}C + 32 \). Solve for \( C \).

\[
F = \frac{9}{5}C + 32 \\
F - 32 = \frac{9}{5}C \\
\left(\frac{5}{9}\right)(F - 32) = \left(\frac{5}{9}\right)\frac{9}{5}C \\
\frac{5}{9}(F - 32) = C
\]

Since \( C \) is multiplied by \( \frac{9}{5} \), divide both sides by \( \frac{9}{5} \) (multiply by \( \frac{5}{9} \)) to undo the multiplication.

#### B

The formula for a person's typing speed is \( s = \frac{w - 10e}{m} \), where \( s \) is speed in words per minute, \( w \) is number of words typed, \( e \) is number of errors, and \( m \) is number of minutes typing. Solve for \( w \).

\[
s = \frac{w - 10e}{m} \\
ms = m\left(\frac{w - 10e}{m}\right) \\
ms = w - 10e \\
ms + 10e = w
\]

Since \( 10e \) is subtracted from \( w \), add \( 10e \) to both sides to undo the subtraction.

### Example 3: Solving Literal Equations for a Variable

#### A

Solve \( m - n = 5 \) for \( m \).

\[
m - n = 5 \\
+m + n = 5 + n \\
\]

Since \( n \) is subtracted from \( m \), add \( n \) to both sides to undo the subtraction.

#### B

Solve \( \frac{m}{k} = x \) for \( k \).

\[
\frac{m}{k} = x \\
k\left(\frac{m}{k}\right) = kx \\
m = kx \\
\frac{m}{x} = \frac{kx}{x} \\
\frac{m}{x} = k
\]

Since \( k \) is multiplied by \( x \), divide both sides by \( x \) to undo the multiplication.

### Check It Out!

2. The formula for an object's final velocity \( f \) is \( f = i - gt \), where \( i \) is the object's initial velocity, \( g \) is acceleration due to gravity, and \( t \) is time. Solve for \( i \).

A formula is a type of **literal equation**. A **literal equation** is an equation with two or more variables. To solve for one of the variables, use inverse operations.

### Example 3: Solving Literal Equations for a Variable

#### A

Solve \( m - n = 5 \) for \( m \).

\[
m - n = 5 \\
+m + n = 5 + n \\
\]

Since \( n \) is subtracted from \( m \), add \( n \) to both sides to undo the subtraction.

#### B

Solve \( \frac{m}{k} = x \) for \( k \).

\[
\frac{m}{k} = x \\
k\left(\frac{m}{k}\right) = kx \\
m = kx \\
\frac{m}{x} = \frac{kx}{x} \\
\frac{m}{x} = k
\]

Since \( k \) is multiplied by \( x \), divide both sides by \( x \) to undo the multiplication.
THINK AND DISCUSS

1. Describe a situation in which a formula could be used more easily if it were “rearranged.” Include the formula in your description.

2. Explain how to solve \( P = 2\ell + 2w \) for \( w \).

3. GET ORGANIZED Copy and complete the graphic organizer. Write a formula that is used in each subject. Then solve the formula for each of its variables.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry</td>
<td></td>
</tr>
<tr>
<td>Physical science</td>
<td></td>
</tr>
<tr>
<td>Earth science</td>
<td></td>
</tr>
</tbody>
</table>

GUIDED PRACTICE

1. **Vocabulary** Explain why a \textit{formula} is a type of \textit{literal equation}.

2. **Construction** The formula \( a = 46c \) gives the floor area \( a \) in square meters that can be wired using \( c \) circuits.
   a. Solve \( a = 46c \) for \( c \).
   b. If a room is 322 square meters, how many circuits are required to wire this room?

3. The formula for the volume of a rectangular prism with length \( \ell \), width \( w \), and height \( h \) is \( V = \ell wh \). Solve this formula for \( w \).

4. Solve \( st + 3t = 6 \) for \( s \).

5. Solve \( m - 4n = 8 \) for \( m \).

6. Solve \( \frac{f + 4}{g} = 6 \) for \( f \).

7. Solve \( b + c = \frac{10}{a} \) for \( a \).

PRACTICE AND PROBLEM SOLVING

8. **Geometry** The formula \( C = 2\pi r \) relates the circumference \( C \) of a circle to its radius \( r \).
   (Recall that \( \pi \) is the constant ratio of circumference to diameter.)
   a. Solve \( C = 2\pi r \) for \( r \).
   b. If a circle’s circumference is 15 inches, what is its radius? Leave the symbol \( \pi \) in your answer.

9. **Finance** The formula \( A = P + I \) shows that the total amount of money \( A \) received from an investment equals the principal \( P \) (the original amount of money invested) plus the interest \( I \). Solve this formula for \( I \).

10. Solve \( -2 = 4r + s \) for \( s \).

11. Solve \( xy - 5 = k \) for \( x \).

12. Solve \( \frac{m}{n} = p - 6 \) for \( n \).

13. Solve \( \frac{x - 2}{y} = z \) for \( y \).
Solve for the indicated variable.

14. \( S = 180n - 360 \) for \( n \)

15. \( x = \frac{1}{5} - g \) for \( x \)

16. \( A = \frac{1}{2} bh \) for \( b \)

17. \( y = mx + b \) for \( x \)

18. \( a = 3n + 1 \) for \( n \)

19. \( PV = nRT \) for \( T \)

20. \( T + M = R \) for \( T \)

21. \( M = T - R \) for \( T \)

22. \( PV = nRT \) for \( R \)

23. \( 2a + 2b = c \) for \( b \)

24. \( 5p + 9c = p \) for \( c \)

25. \( ax + r = 7 \) for \( r \)

26. \( 3x + 7y = 2 \) for \( y \)

27. \( 4y + 3x = 5 \) for \( x \)

28. \( y = 3x + 3b \) for \( b \)

29. **Estimation** The table shows the flying time and distance traveled for five flights on a certain airplane.
   
a. Use the data in the table to write a rule that estimates the relationship between flying time \( t \) and distance traveled \( d \).
   
b. Use your rule from part a to estimate the time that it takes the airplane to fly 1300 miles.
   
c. Solve your rule for \( d \).
   
d. Use your rule from part c to estimate the distance the airplane can fly in 8 hours.

30. **Sports** To find a baseball pitcher’s earned run average (ERA), you can use the formula \( Ei = 9r \), where \( E \) represents ERA, \( i \) represents number of innings pitched, and \( r \) represents number of earned runs allowed. Solve the equation for \( E \). What is a pitcher’s ERA if he allows 5 earned runs in 18 innings pitched?

31. **Meteorology** For altitudes up to 36,000 feet, the relationship between temperature and altitude can be described by the formula \( t = -0.0035a + g \), where \( t \) is the temperature in degrees Fahrenheit, \( a \) is the altitude in feet, and \( g \) is the ground temperature in degrees Fahrenheit. Solve this formula for \( a \).

32. **Write About It** In your own words, explain how to solve a literal equation for one of the variables.

33. **Critical Thinking** How is solving \( a – ab = c \) for \( a \) different from the problems in this lesson? How might you solve this equation for \( a \)?

34. a. Suppose firefighters can extinguish a wildfire at a rate of 60 acres per day. Use this information to complete the table.
   
b. Use the last row in the table to write an equation for acres \( A \) extinguished in terms of the number of days \( d \).
   
c. Graph the points in the table with \( Days \) on the horizontal axis and \( Acres \) on the vertical axis. Describe the graph.

<table>
<thead>
<tr>
<th>Days</th>
<th>Acres</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>( d )</td>
<td></td>
</tr>
</tbody>
</table>
35. Which equation is the result of solving $9 + 3x = 2y$ for $x$?

A. $\frac{9 + 3y}{2} = x$  
B. $\frac{2}{3}y - 9 = x$  
C. $x = \frac{2}{3}y - 3$  
D. $x = 2y - 3$

36. Which of the following is a correct method for solving $2a - 5b = 10$ for $b$?

C. Subtract $5b$ from both sides, then divide both sides by 2.

D. Subtract $2a$ from both sides, then divide both sides by $-5$.

37. The formula for the volume of a rectangular prism is $V = \ell w h$. Anna wants to make a cardboard box with a length of 7 inches, a width of 5 inches, and a volume of 210 cubic inches. Which variable does Anna need to solve for in order to build her box?

A. $V$  
B. $\ell$  
C. $w$  
D. $h$

### CHALLENGE AND EXTEND

Solve for the indicated variable.

38. $3.3x + r = 23.1$ for $x$

39. $\frac{2}{5}a - \frac{3}{4}b = c$ for $a$

40. $\frac{3}{5}x + 1.4y = \frac{2}{5}$ for $y$

41. $t = \frac{d}{500} + \frac{1}{2}$ for $d$

42. $s = \frac{1}{2}gr^2$ for $g$

43. $v^2 = u^2 + 2as$ for $s$

44. Solve $y = mx + 6$ for $m$. What can you say about $y$ if $m = 0$?

45. **Entertainment** The formula

$$S = \frac{h \cdot w \cdot f \cdot t}{35,000}$$

gives the approximate size in kilobytes (Kb) of a compressed video. The variables $h$ and $w$ represent the height and width of the frame measured in pixels, $f$ is the number of frames per second (fps) the video plays, and $t$ is the time the video plays in seconds. Estimate the time a movie trailer will play if it has a frame height of 320 pixels, has a frame width of 144 pixels, plays at 15 fps, and has a size of 2370 Kb.
Objective
Solve equations in one variable that contain absolute-value expressions.

Why learn this?
Engineers can solve absolute-value equations to calculate the length of the deck of a bridge. (See Example 3.)

Recall that the absolute value of a number is that number’s distance from zero on a number line. For example, \(|-5| = 5\) and \(|5| = 5\).

For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and \(-5\) have an absolute value of 5.

To write this statement using algebra, you would write \(|x| = 5\). This equation asks, “What values of \(x\) have an absolute value of 5?” The solutions are 5 and \(-5\). Notice that this equation has two solutions.

To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation. Then you must consider two cases.

**Example 1**

Solving Absolute-Value Equations

Solve each equation.

**A** \(|x| = 4\)

Think: What numbers are 4 units from 0?

Case 1 \(x = -4\)

Case 2 \(x = 4\)

Rewrite the equation as two cases.

The solutions are \(-4\) and 4.
Solve each equation.

**B** \[ 4|x + 2| = 24 \]

\[ \frac{4|x + 2|}{4} = \frac{24}{4} \]

\[ |x + 2| = 6 \]

**Case 1**

\[ x + 2 = -6 \]

\[ x = -8 \]

**Case 2**

\[ x + 2 = 6 \]

\[ x = 4 \]

The solutions are -8 and 4.

Solve each equation. Check your answer.

1a. \[ |x| - 3 = 4 \]

1b. \[ 8 = |x - 2.5| \]

The table summarizes the steps for solving absolute-value equations.

<table>
<thead>
<tr>
<th><strong>Solving an Absolute-Value Equation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Use inverse operations to isolate the absolute-value expression.</td>
</tr>
<tr>
<td>2. Rewrite the resulting equation as two cases that do not involve absolute values.</td>
</tr>
<tr>
<td>3. Solve the equation in each of the two cases.</td>
</tr>
</tbody>
</table>

Not all absolute-value equations have two solutions. If the absolute-value expression equals 0, there is one solution. If an equation states that an absolute value is negative, there are no solutions.

**Example 2** Special Cases of Absolute-Value Equations

Solve each equation.

**A** \[ |x + 3| + 4 = 4 \]

\[ |x + 3| + 4 = 4 \]

\[ -4 \quad -4 \]

\[ |x + 3| = 0 \]

\[ x + 3 = 0 \]

\[ x = -3 \]

**B** \[ 5 = |x + 2| + 8 \]

\[ 5 = |x + 2| + 8 \]

\[ -8 \quad -8 \]

\[ -3 = |x + 2| \]

\[ x \]

Absolute value cannot be negative.

This equation has no solution.

Solve each equation.

2a. \[ 2 - |2x - 5| = 7 \]

2b. \[ -6 + |x - 4| = -6 \]
**Engineering Application**

Sydney Harbour Bridge in Australia is 1149 meters long. Because of changes in temperature, the bridge can expand or contract by as much as 420 millimeters. Write and solve an absolute-value equation to find the minimum and maximum lengths of the bridge.

First convert millimeters to meters.

\[ 420 \text{ mm} = 0.420 \text{ m} \quad \text{Move the decimal point three places to the left.} \]

The length of the bridge can vary by 0.42 m, so find two numbers that are 0.42 units away from 1149 on a number line.

You can find these numbers by using the absolute-value equation \( |x - 1149| = 0.42 \). Solve the equation by rewriting it as two cases.

**Case 1**

\[
\begin{align*}
  x &- 1149 = -0.42 \\
  +1149 &+1149 \\
  x & = 1148.58
\end{align*}
\]

**Case 2**

\[
\begin{align*}
  x &- 1149 = 0.42 \\
  +1149 &+1149 \\
  x & = 1149.42
\end{align*}
\]

The minimum length of the bridge is 1148.58 m, and the maximum length is 1149.42 m.

3. Sydney Harbour Bridge is 134 meters tall. The height of the bridge can rise or fall by 180 millimeters because of changes in temperature. Write and solve an absolute-value equation to find the minimum and maximum heights of the bridge.

**THINK AND DISCUSS**

1. Explain the steps you would use to solve the equation \( \frac{1}{3}|x - 3| = 2 \).

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of an absolute-value equation that has the indicated number of solutions, and then solve.
Solve each equation.

1. \(|x| = 6\)  
2. \(9 = |x + 5|\)  
3. \(|3x| + 2 = 8\)
4. \(2|x| = 18\)  
5. \(\left| x + \frac{1}{2} \right| = 1\)  
6. \(|x - 3| - 6 = 2\)
7. \(-8 = |x|\)  
8. \(|x| = 0\)  
9. \(|x + 4| = -7\)
10. \(7 = |3x + 9| + 7\)  
11. \(|2.8 - x| + 1.5 = 1.5\)  
12. \(5|x| + 7 + 14 = 8\)

13. **Communication** Barry’s walkie-talkie has a range of 2 mi. Barry is traveling on a straight highway and is at mile marker 207. Write and solve an absolute-value equation to find the minimum and maximum mile marker from 207 that Barry’s walkie-talkie will reach.

Solve each equation.

14. \(|x| = \frac{1}{5}\)  
15. \(2x - 4 = 22\)  
16. \(18 = 3|x - 1|\)
17. \(-2|x| = -4\)  
18. \(3|x| - 12 = 18\)  
19. \(|x - 42.04| = 23.24\)
20. \(\left| \frac{2}{3}x - \frac{2}{3} \right| = \frac{2}{3}\)  
21. \(3x + 1 = 13\)  
22. \(|-2x + 3| = 5.8\)
23. \(|4x| + 9 = 9\)  
24. \(8 = 7 - |x|\)  
25. \(|x| + 6 = 12 - 6\)
26. \(|x - 3| + 14 = 5\)  
27. \(0 = \left| \frac{2}{3} - x \right|\)  
28. \(3 + |x - 1| = 3\)

29. **Space Shuttle** The diameter of a valve for the space shuttle must be within 0.001 mm of 5 mm. Write and solve an absolute-value equation to find the boundary values for the acceptable diameters of the valve.

30. The two numbers that are 5 units from 3 on the number line are represented by the equation \(|n - 3| = 5\). What are these two numbers? Graph the solutions.

31. Write and solve an absolute-value equation that represents two numbers \(x\) that are 2 units from 7 on a number line. Graph the solutions.

32. **Manufacturing** A quality control inspector at a bolt factory examines random bolts that come off the assembly line. Any bolt whose diameter differs by more than 0.04 mm from 6.5 mm is sent back. Write and solve an absolute-value equation to find the maximum and minimum diameters of an acceptable bolt.

33. **Construction** A brick company guarantees to fill a contractor’s order to within 5% accuracy. A contractor orders 1500 bricks. Write and solve an absolute-value equation to find the maximum and minimum number of bricks guaranteed.

34. **Multi-Step** A machine prints posters and then trims them to the correct size. The equation \(|\ell - 65.1| = 0.2\) gives the maximum and minimum acceptable lengths for the posters in inches. Does a poster with a length of 64.8 inches fall within the acceptable range? Why or why not?
Write an absolute-value equation whose solutions are graphed on the number line.

35.  
36.  
37.  
38.  

Tell whether each statement is sometimes, always, or never true. Explain.

39. An absolute-value equation has two solutions.
40. The value of $|x + 4|$ is equal to the value of $|x| + 4$.
41. The absolute value of a number is nonnegative.

42. **Temperature** A thermostat is set so that the temperature in a laboratory freezer stays within $2.5 \, ^\circ F$ of $2 \, ^\circ F$. Write and solve an absolute-value equation to find the maximum and minimum temperatures in the freezer.

43. **Recreation** To ensure safety, boaters must be aware of wind conditions while they are on the water. A particular anemometer gives a measurement of wind speed within a certain amount of the true wind speed, as shown in the table.

<table>
<thead>
<tr>
<th>Measured Wind Speed (mi/h)</th>
<th>True Wind Speed (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>15–25</td>
</tr>
<tr>
<td>22</td>
<td>17–27</td>
</tr>
<tr>
<td>24</td>
<td>19–29</td>
</tr>
<tr>
<td>26</td>
<td>21–31</td>
</tr>
<tr>
<td>28</td>
<td>23–33</td>
</tr>
<tr>
<td>30</td>
<td>25–35</td>
</tr>
</tbody>
</table>

a. Use the table to write an absolute-value equation for the minimum and maximum possible true wind speeds $t$ for the measured wind speed shown on the anemometer.

b. Solve your equation from part a. Check that the solution is correct by comparing it to the values given in the table when the measured wind speed is 24 mi/h.

c. Will your equation work for all of the values in the table? Explain.

d. Explain what your equation says about the instrument’s measurements.

44. The water pumps on a wildland fire apparatus can pump at various rates. The center of the acceptable range of pumping rates is 55 gallons per minute.

a. Write an absolute-value expression that gives the distance on the number line of a pump’s rate $r$ from 55.

b. The smallest and largest pumps have rates that differ by 45 gallons per minute from the rate at the center of the range. Write an absolute-value equation for the rates of these pumps.

c. Find the least and greatest rates that are acceptable for a wildland fire apparatus.
45. **Write About It** Do you agree with the following statement: “To solve an absolute-value equation, you need to solve two equations.” Why or why not?

46. **Critical Thinking** Is there a value of $a$ for which the equation $|x - a| = 1$ has exactly one solution? Explain.

47. Which situation could be modeled by the equation $|x - 65| = 3$?

- A) Two numbers on the number line are 65 units away from 3.
- B) The length of a carpet is 3 inches less than 65 inches.
- C) The maximum and minimum weights of wrestlers on the team are within 3 kg of 65 kg.
- D) The members of an exercise club for seniors are all between 63 and 67 years old.

48. For which of the following is $n = -3$ a solution?

- F) $|n - 1| = 2$
- G) $|n + 2| = -1$
- H) $|n - 2| = 1$
- I) $|n + 1| = 2$

49. The minimum and maximum sound levels at a rock concert are 90 decibels and 95 decibels. Which equation models this situation?

- A) $|x - 90| = 5$
- B) $|x - 92.5| = 2.5$
- C) $|x - 92.5| = 5$
- D) $|x - 95| = 2.5$

### CHALLENGE AND EXTEND

50. The perimeter of a rectangle is 100 inches. The length of the rectangle is $|2x - 4|$ inches, and the width is $x$ inches. What are the possible values of $x$? Explain.

51. Fill in the missing reasons to justify each step in solving the equation $3|2x + 1| = 21$.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $3</td>
<td>2x + 1</td>
</tr>
<tr>
<td>2. $</td>
<td>2x + 1</td>
</tr>
<tr>
<td>3. $2x + 1 = -7$ or $2x + 1 = 7$</td>
<td>3. Definition of absolute value</td>
</tr>
<tr>
<td>4. $2x = -8$ or $2x = 6$</td>
<td>4. ?</td>
</tr>
<tr>
<td>5. $x = -4$ or $x = 3$</td>
<td>5. ?</td>
</tr>
</tbody>
</table>

52. Solve $|x| = |x + 1|$. *(Hint: Consider two cases: $x \geq 0$ and $x < 0$.)*
A ratio is a comparison of two quantities by division. The ratio of $a$ to $b$ can be written $a:b$ or $\frac{a}{b}$, where $b \neq 0$. Ratios that name the same comparison are said to be equivalent.

A statement that two ratios are equivalent, such as $\frac{1}{12} = \frac{2}{24}$, is called a proportion.

A rate is a ratio of two quantities with different units, such as $\frac{34 \text{ mi}}{2 \text{ gal}}$. Rates are usually written as unit rates. A unit rate is a rate with a second quantity of 1 unit, such as $\frac{17 \text{ mi}}{1 \text{ gal}}$, or 17 mi/gal. You can convert any rate to a unit rate.
Dimensional analysis is a process that uses rates to convert measurements from one unit to another. A rate such as $\frac{12 \text{ in.}}{1 \text{ ft}}$, in which the two quantities are equal but use different units, is called a conversion factor. To convert from one set of units to another, multiply by a conversion factor.

### Example 3

#### A

A large adult male human has about 12 pints of blood. Use dimensional analysis to convert this quantity to gallons.

**Step 1** Convert pints to quarts.

$$12 \text{ pt} \cdot \frac{1 \text{ qt}}{2 \text{ pt}} = 6 \text{ qt}$$

12 pints is 6 quarts.

**Step 2** Convert quarts to gallons.

$$6 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} = \frac{6}{4} \text{ gal} = 1 \frac{1}{2} \text{ gal}$$

A large adult male human has about $1 \frac{1}{2}$ gallons of blood.

#### B

The dwarf sea horse *Hippocampus zosterae* swims at a rate of 52.68 feet per hour. Use dimensional analysis to convert this speed to inches per minute.

Use the conversion factor $\frac{12 \text{ in.}}{1 \text{ ft}}$ to convert feet to inches, and use the conversion factor $\frac{1 \text{ h}}{60 \text{ min}}$ to convert hours to minutes.

$$\frac{52.68 \text{ ft}}{1 \text{ h}} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} \cdot \frac{1 \text{ h}}{60 \text{ min}} = \frac{10.536 \text{ in.}}{1 \text{ min}}$$

The speed is 10.536 inches per minute.

Check that the answer is reasonable. The answer is about 10 in./min.

- There are 60 min in 1 h, so 10 in./min is $60 \cdot (10) = 600$ in./h.

- There are 12 in. in 1 ft, so 600 in./h is $\frac{600}{12} = 50$ ft/h. This is close to the rate given in the problem, 52.68 ft/h.

#### 3.

A cyclist travels 56 miles in 4 hours. Use dimensional analysis to convert the cyclist's speed to feet per second. Round your answer to the nearest tenth, and show that your answer is reasonable.

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called cross products. You can solve a proportion for a missing value by using the Cross Products Property.
**EXAMPLE 4**

**Solving Proportions**

Solve each proportion.

**A** \( \frac{5}{9} = \frac{3}{w} \)

\[
\frac{5}{9} \times \frac{3}{w} = \frac{9(3)}{5w} \]

\[
5w = 27
\]

\[
\frac{5w}{5} = \frac{27}{5}
\]

\[
w = \frac{27}{5}
\]

**B** \( \frac{8}{x + 10} = \frac{1}{12} \)

\[
\frac{8}{x + 10} \times \frac{1}{12} = \frac{8(12)}{1(x + 10)}
\]

\[
96 = x + 10
\]

\[
x = 86
\]

**CHECK IT OUT!**

4a. \( \frac{-5}{2} = \frac{y}{8} \)

4b. \( \frac{g + 3}{5} = \frac{7}{4} \)

---

**EXAMPLE 5**

**Scale Drawings and Scale Models**

**A** On the map, the distance from Chicago to Evanston is 0.625 in. What is the actual distance?

Write the scale as a fraction.

\[
\frac{\text{map}}{\text{actual}} \rightarrow \frac{1}{18 \text{ mi}}
\]

Let \( x \) be the actual distance.

\[
x \cdot 1 = 18(0.625)
\]

\[
x = 11.25
\]

The actual distance is 11.25 mi.

**B** The actual distance between North Chicago and Waukegan is 4 mi. What is this distance on the map? Round to the nearest tenth.

Write the scale as a fraction.

\[
\frac{\text{map}}{\text{actual}} \rightarrow \frac{1}{18 \text{ mi}}
\]

Let \( x \) be the distance on the map.

\[
\frac{1}{18} \times \frac{x}{4} = \frac{4}{18x}
\]

Since \( x \) is multiplied by 18, divide both sides by 18 to undo the multiplication.

\[
0.2 \approx x
\]

The distance on the map is about 0.2 in.

**Reading Math**

A scale written without units, such as 32:1, means that 32 units of any measure correspond to 1 unit of that same measure.

---

5. A scale model of a human heart is 16 ft long. The scale is 32:1. How many inches long is the actual heart it represents?
**THINK AND DISCUSS**

1. Explain two ways to solve the proportion \( \frac{t}{4} = \frac{3}{5} \).

2. How could you show that the answer to Example 5A is reasonable?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of each use of ratios.

---

**GUIDED PRACTICE**

1. **Vocabulary** What does it mean when two ratios form a *proportion*?

2. The ratio of the sale price of a jacket to the original price is 3:4. The original price is $64. What is the sale price?

3. **Chemistry** The ratio of hydrogen atoms to oxygen atoms in water is 2:1. If an amount of water contains 341 trillion atoms of oxygen, how many hydrogen atoms are there?

4. A computer’s fan rotates 2000 times in 40 seconds. Find the unit rate in rotations per second.

5. Twelve cows produce 224,988 pounds of milk. Find the unit rate in pounds per cow.

6. A yellow jacket can fly 4.5 meters in 9 seconds. Find the unit rate in meters per second.

7. Lydia wrote 4 1/2 pages of her science report in one hour. What was her writing rate in pages per minute?

8. A model airplane flies 18 feet in 2 seconds. What is the airplane’s speed in miles per hour? Round your answer to the nearest hundredth.

9. A vehicle uses 1 tablespoon of gasoline to drive 125 yards. How many miles can the vehicle travel per gallon? Round your answer to the nearest mile. (Hint: There are 256 tablespoons in a gallon.)

10. \( \frac{3}{z} = \frac{1}{8} \)

11. \( \frac{x}{3} = \frac{1}{5} \)

12. \( \frac{b}{4} = \frac{3}{2} \)

13. \( \frac{f + 3}{12} = \frac{7}{2} \)

14. \( -\frac{1}{5} = \frac{3}{2d} \)

15. \( \frac{3}{14} = \frac{s - 2}{21} \)

16. \( -\frac{4}{9} = \frac{7}{x} \)

17. \( \frac{3}{s - 2} = \frac{1}{7} \)

18. \( \frac{10}{h} = \frac{52}{13} \)
19. **Archaeology** Stonehenge II in Hunt, Texas, is a scale model of the ancient construction in Wiltshire, England. The scale of the model to the original is 3:5. The Altar Stone of the original construction is 4.9 meters tall. Write and solve a proportion to find the height of the model of the Altar Stone.

**PRACTICE AND PROBLEM SOLVING**

20. **Gardening** The ratio of the height of a bonsai ficus tree to the height of a full-size ficus tree is 1:9. The bonsai ficus is 6 inches tall. What is the height of a full-size ficus?

21. **Manufacturing** At one factory, the ratio of defective light bulbs produced to total light bulbs produced is about 3:500. How many light bulbs are expected to be defective when 12,000 are produced?

22. Four gallons of gasoline weigh 25 pounds. Find the unit rate in pounds per gallon.

23. Fifteen ounces of gold cost $6058.50. Find the unit rate in dollars per ounce.

24. **Biology** The tropical giant bamboo can grow 11.9 feet in 3 days. What is this rate of growth in inches per hour? Round your answer to the nearest hundredth, and show that your answer is reasonable.

25. **Transportation** The maximum speed of the Tupolev Tu-144 airliner is 694 m/s. What is this speed in kilometers per hour?

Solve each proportion.

26. \( \frac{v}{6} = \frac{1}{2} \)  
27. \( \frac{2}{\frac{5}{y}} = \frac{4}{9} \)  
28. \( \frac{2}{h} = \frac{-\frac{5}{6}}{} \)  
29. \( \frac{\frac{3}{10}}{b + \frac{7}{20}} \)  
30. \( \frac{5t}{\frac{9}{2}} = \frac{1}{2} \)  
31. \( \frac{2}{\frac{3}{q - 4}} = \frac{6}{3} \)  
32. \( \frac{x}{8} = \frac{7.5}{20} \)  
33. \( \frac{\frac{3}{k}}{\frac{45}{18}} \)  
34. \( \frac{6}{a} = \frac{15}{17} \)  
35. \( \frac{\frac{9}{2}}{x + 1} = \frac{5}{3} \)  
36. \( \frac{3}{5} = \frac{x}{100} \)  
37. \( \frac{\frac{38}{19}}{\frac{n - 5}{20}} \)

38. **Science** The image shows a dust mite as seen under a microscope. The scale of the drawing to the dust mite is 100:1. Use a ruler to measure the length of the dust mite in the image in millimeters. What is the actual length of the dust mite?

39. **Finance** On a certain day, the exchange rate was 60 U.S. dollars for 50 euro. How many U.S. dollars were 70 euro worth that day? Show that your answer is reasonable.

40. **Environmental Science** An environmental scientist wants to estimate the number of carp in a pond. He captures 100 carp, tags all of them, and releases them. A week later, he captures 85 carp and records how many have tags. His results are shown in the table. Write and solve a proportion to estimate the number of carp in the pond.

<table>
<thead>
<tr>
<th>Status</th>
<th>Number Captured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tagged</td>
<td>20</td>
</tr>
<tr>
<td>Not tagged</td>
<td>65</td>
</tr>
</tbody>
</table>
41. //ERROR ANALYSIS// Below is a bonus question that appeared on an algebra test and a student’s response.

The ratio of junior varsity members to varsity members on the track team is 3:5. There are 24 members on the team. Write a proportion to find the number of junior varsity members.

\[
\frac{3}{5} = \frac{x}{24}
\]

The student did not receive the bonus points. Why is this proportion incorrect?

42. Sports The table shows world record times for women’s races of different distances.

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>10.5</td>
</tr>
<tr>
<td>200</td>
<td>21.3</td>
</tr>
<tr>
<td>800</td>
<td>113.3</td>
</tr>
<tr>
<td>5000</td>
<td>864.7</td>
</tr>
</tbody>
</table>

a. Find the speed in meters per second for each race. Round your answers to the nearest hundredth.

b. Which race has the fastest speed? the slowest?

c. Critical Thinking Give a possible reason why the speeds are different.

43. Entertainment Lynn, Faith, and Jeremy are film animators. In one 8-hour day, Lynn rendered 203 frames, Faith rendered 216 frames, and Jeremy rendered 227 frames. How many more frames per hour did Faith render than Lynn did?

44. \[\frac{x - \frac{1}{3}}{\frac{3}{5}} = \frac{x + \frac{1}{5}}{\frac{7}{5}}\]

45. \[\frac{m}{3} = \frac{m + 4}{7}\]

46. \[\frac{1}{x - 3} = \frac{3}{x - 5}\]

47. \[\frac{a}{2} = \frac{a - 4}{30}\]

48. \[\frac{2}{y} = \frac{16}{y + 2}\]

49. \[\frac{n + 3}{5} = \frac{n - 1}{2}\]

50. \[\frac{1}{y} = \frac{1}{6y - 1}\]

51. \[\frac{2}{n} = \frac{4}{n + 3}\]

52. \[\frac{5t - 3}{-2} = \frac{t + 3}{2}\]

53. \[\frac{3}{d + 3} = \frac{4}{d + 12}\]

54. \[\frac{3x + 5}{14} = \frac{x}{3}\]

55. \[\frac{5}{2n} = \frac{8}{3n - 24}\]

56. Decorating A particular shade of paint is made by mixing 5 parts red paint with 7 parts blue paint. To make this shade, Shannon mixed 12 quarts of blue paint with 8 quarts of red paint. Did Shannon mix the correct shade? Explain.

57. Write About It Give three examples of proportions. How do you know they are proportions? Then give three nonexamples of proportions. How do you know they are not proportions?

58. a. Marcus is shopping for a new jacket. He finds one with a price tag of $120. Above the rack is a sign that says that he can take off \(\frac{1}{5}\). Find out how much Marcus can deduct from the price of the jacket.

b. What price will Marcus pay for the jacket?

c. Copy the model below. Complete it by placing numerical values on top and the corresponding fractional parts below.

\[
\begin{array}{cccc}
$0$ & ? & $48$ & ? \\
0 & \frac{1}{5} & ? & \frac{4}{5} \\
\end{array}
\]

\[
\begin{array}{cc}
\text{Sale Price} & 50\% \text{ off} \\
\end{array}
\]

\[
\begin{array}{cc}
\text{Multi-Step Test Prep} & \text{Rates, Ratios, and Proportions} \\
\end{array}
\]

\[
\begin{array}{cc}
\text{2-8} & 61 \\
\end{array}
\]
59. One day the U.S. dollar was worth approximately 100 yen. An exchange of 2500 yen was made that day. What was the value of the exchange in dollars?

A) $25       B) $400       C) $2500       D) $40,000

60. Brett walks at a speed of 4 miles per hour. He walks for 20 minutes in a straight line at this rate. Approximately what distance does Brett walk?

F) 0.06 miles       G) 1.3 miles       H) 5 miles       I) 80 miles

61. A shampoo company conducted a survey and found that 3 out of 8 people use their brand of shampoo. Which proportion could be used to find the expected number of users $n$ in a city of 75,000 people?

A) $\frac{3}{8} = \frac{75,000}{n}$       B) $\frac{3}{75,000} = \frac{n}{8}$       C) $8 = \frac{n}{75,000}$       D) $\frac{3}{8} = \frac{n}{75,000}$

62. A statue is 3 feet tall. The display case for a model of the statue can fit a model that is no more than 9 inches tall. Which of the scales below allows for the tallest model of the statue that will fit in the display case?

F) 2:1       G) 1:1       H) 1:3       I) 1:4

**CHALLENGE AND EXTEND**

63. **Geometry** Complementary angles are two angles whose measures add up to 90°. The ratio of the measures of two complementary angles is 4:5. What are the measures of the angles?

64. A customer wanted 24 feet of rope. The clerk at the hardware store used what she thought was a yardstick to measure the rope, but the yardstick was actually 2 inches too short. How many inches were missing from the customer's piece of rope?

65. **Population** The population density of Jackson, Mississippi, is 672.2 people per square kilometer. What is the population density in people per square meter? Show that your answer is reasonable. (*Hint:* There are 1000 meters in 1 kilometer. How many square meters are in 1 square kilometer?)
Objectives
Use proportions to solve problems involving geometric figures.
Use proportions and similar figures to measure objects indirectly.

Vocabulary
similar
corresponding sides
corresponding angles
indirect measurement
scale factor

Why learn this?
Proportions can be used to find the heights of tall objects, such as totem poles, that would otherwise be difficult to measure. (See Example 2.)

Similar figures have exactly the same shape but not necessarily the same size.

Corresponding sides of two figures are in the same relative position, and corresponding angles are in the same relative position. Two figures are similar if and only if the lengths of corresponding sides are proportional and all pairs of corresponding angles have equal measures.

Reading Math
- $\overline{AB}$ means segment $AB$. $AB$ means the length of $\overline{AB}$.
- $\angle A$ means angle $A$. $m\angle A$ means the measure of angle $A$.

When stating that two figures are similar, use the symbol $\sim$. For the triangles above, you can write $\triangle ABC \sim \triangle DEF$. Make sure corresponding vertices are in the same order. It would be incorrect to write $\triangle ABC \sim \triangle EFD$.

You can use proportions to find missing lengths in similar figures.

EXAMPLE 1 Finding Missing Measures in Similar Figures
Find the value of $x$ in each diagram.

A $\triangle RST \sim \triangle BCD$

$R$ corresponds to $B$, $S$ corresponds to $C$, and $T$ corresponds to $D$.

\[
\frac{5}{12} = \frac{8}{x} \quad \text{Use cross products.}
\]

\[
5x = 96
\]

\[
x = \frac{96}{5}
\]

Since $x$ is multiplied by 5, divide both sides by 5 to undo the multiplication.

$x = 19.2$

The length of $\overline{BC}$ is 19.2 ft.
Find the value of $x$ in each diagram.

**B**

$$FGHJKL \sim MNPQRS$$

$$\frac{6}{4} = \frac{x}{2} \quad NP = RQ$$

$$\frac{4x}{2} = 12 \quad GH = KJ$$

Use cross products.

$$4x = 12 \quad \text{Since } x \text{ is multiplied by } 4,$$

$$x = 3 \quad \text{divide both sides by } 4 \text{ to undo the multiplication.}$$

The length of $QR$ is 3 cm.

**CHECK IT OUT!**

1. Find the value of $x$ in the diagram if $ABCD \sim WXYZ$.

You can solve a proportion involving similar triangles to find a length that is not easily measured. This method of measurement is called **indirect measurement**. If two objects form right angles with the ground, you can apply indirect measurement using their shadows.

### EXAMPLE 2

**Measurement Application**

A totem pole casts a shadow 45 feet long at the same time that a 6-foot-tall man casts a shadow that is 3 feet long. Write and solve a proportion to find the height of the totem pole.

Both the man and the totem pole form right angles with the ground, and their shadows are cast at the same angle. You can form two similar right triangles.

$$\frac{6}{x} = \frac{3}{45}$$

$$3x = 270$$

$$x = 90$$

The totem pole is 90 feet tall.

### Helpful Hint

A height of 90 ft seems reasonable for a totem pole. If you got 900 or 9000 ft, that would not be reasonable, and you should check your work.

2a. A forest ranger who is 150 cm tall casts a shadow 45 cm long. At the same time, a nearby tree casts a shadow 195 cm long. Write and solve a proportion to find the height of the tree.

2b. A woman who is 5.5 feet tall casts a shadow 3.5 feet long. At the same time, a building casts a shadow 28 feet long. Write and solve a proportion to find the height of the building.
If every dimension of a figure is multiplied by the same number, the result is a similar figure. The multiplier is called a **scale factor**.

### Example 3

**Changing Dimensions**

**A**

Every dimension of a 2-by-4-inch rectangle is multiplied by 1.5 to form a similar rectangle. How is the ratio of the perimeters related to the ratio of corresponding sides? How is the ratio of the areas related to the ratio of corresponding sides?

<table>
<thead>
<tr>
<th></th>
<th>Rectangle A</th>
<th>Rectangle B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P</strong></td>
<td>2ℓ + 2w</td>
<td></td>
</tr>
<tr>
<td>2(2) + 2(4)</td>
<td><strong>12</strong></td>
<td>2(6) + 2(3)</td>
</tr>
<tr>
<td><strong>A</strong></td>
<td>ℓw</td>
<td></td>
</tr>
<tr>
<td>4(2)</td>
<td><strong>8</strong></td>
<td>6(3)</td>
</tr>
</tbody>
</table>

Sides: \(\frac{4}{6} = \frac{2}{3}\)  
Perimeters: \(\frac{12}{18} = \frac{2}{3}\)  
Areas: \(\frac{8}{18} = \frac{4}{9} = \left(\frac{2}{3}\right)^2\)

The ratio of the perimeters is equal to the ratio of corresponding sides. The ratio of the areas is the square of the ratio of corresponding sides.

**B**

Every dimension of a cylinder with radius 4 cm and height 6 cm is multiplied by \(\frac{1}{2}\) to form a similar cylinder. How is the ratio of the volumes related to the ratio of corresponding dimensions?

<table>
<thead>
<tr>
<th></th>
<th>Cylinder A</th>
<th>Cylinder B</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>V</strong></td>
<td>(\pi r^2 h)</td>
<td>(\pi r^2 h)</td>
</tr>
<tr>
<td>(\pi(4)^2(6))</td>
<td><strong>96\pi</strong></td>
<td>(\pi(2)^2(3))</td>
</tr>
</tbody>
</table>

Radii: \(\frac{4}{2} = \frac{2}{1} = 2\)  
Heights: \(\frac{6}{3} = \frac{2}{1} = 2\)  
Volumes: \(\frac{96\pi}{12\pi} = \frac{8}{1} = 8 = 2^3\)

The ratio of the volumes is the cube of the ratio of corresponding dimensions.

3. A rectangle has width 12 inches and length 3 inches. Every dimension of the rectangle is multiplied by \(\frac{1}{3}\) to form a similar rectangle. How is the ratio of the perimeters related to the ratio of the corresponding sides?

### THINK AND DISCUSS

1. Name some pairs of real-world items that appear to be similar figures.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In the top box, sketch and label two similar triangles. Then list the corresponding sides and angles in the bottom boxes.
GUIDED PRACTICE

1. **Vocabulary**  What does it mean for two figures to be similar?

Find the value of \( x \) in each diagram.

2. \( \triangle ABC \sim \triangle DEF \)

3. \( \triangle RST \sim \triangle WXYZ \)

4. Roger is 5 feet tall and casts a shadow 3.5 feet long. At the same time, the flagpole outside his school casts a shadow 14 feet long. Write and solve a proportion to find the height of the flagpole.

5. A rectangle has length 12 feet and width 8 feet. Every dimension of the rectangle is multiplied by \( \frac{3}{4} \) to form a similar rectangle. How is the ratio of the areas related to the ratio of corresponding sides?

PRACTICE AND PROBLEM SOLVING

Find the value of \( x \) in each diagram.

6. \( \triangle LMN \sim \triangle RST \)

7. prism \( A \sim \) prism \( B \)

8. Write and solve a proportion to find the height of the taller tree in the diagram at right.

9. A triangle has side lengths of 5 inches, 12 inches, and 15 inches. Every dimension is multiplied by \( \frac{1}{5} \) to form a new triangle. How is the ratio of the perimeters related to the ratio of corresponding sides?

10. **Hobbies**  For a baby shower gift, Heather crocheted a baby blanket whose length was 2 \( \frac{1}{2} \) feet and whose width was 2 feet. She plans to crochet a proportionally larger similar blanket for the baby’s mother. If she wants the length of the mother’s blanket to be 6 \( \frac{1}{4} \) feet, what should the width be? Show that your answer is reasonable.
11. **Real Estate** Refer to the home builder's advertisement. The family rooms in both models are rectangular. How much carpeting is needed to carpet the family room in the Weston model?

12. A rectangle has an area of 16 ft\(^2\). Every dimension is multiplied by a scale factor, and the new rectangle has an area of 64 ft\(^2\). What was the scale factor?

13. A cone has a volume of 98\(\pi\) cm\(^3\). Every dimension is multiplied by a scale factor, and the new cone has a volume of 6272\(\pi\) cm\(^3\). What was the scale factor?

Find the value of \(x\) in each diagram.

14. \(\triangle FGHJK \sim \triangle MNPQR\)

15. Cylinder \(A \sim\) Cylinder \(B\)

16. \(\triangle BCD \sim \triangle FGD\)

17. \(\triangle RST \sim \triangle QSV\)

18. A tower casts a 450 ft shadow at the same time that a 4 ft child casts a 6 ft shadow. Write and solve a proportion to find the height of the tower.

19. **Write About It** At Pizza Palace, a pizza with a diameter of 8 inches costs $6.00. The restaurant manager says that a 16-inch pizza should be priced at $12.00 because it is twice as large. Do you agree? Explain why or why not.

20. Another common application of proportion is **percents**. A percent is a ratio of a number to 100. For example, \(80\% = \frac{80}{100}\).

   a. Write 12%, 18%, 25%, 67%, and 98% as ratios.

   b. Percents can also be written as decimals. Write each of your ratios from part a as a decimal.

   c. What do you notice about a percent and its decimal equivalent?

   You will learn more about percents and their connections to proportions in upcoming lessons.
21. A lighthouse casts a shadow that is 36 meters long. At the same time, a person who is 1.5 meters tall casts a shadow that is 4.5 meters long. Write and solve a proportion to find the height of the lighthouse.

22. In the diagram, $\triangle ABC \sim \triangle DEC$. What is the distance across the river from $A$ to $B$?

23. **Critical Thinking** If every dimension of a two-dimensional figure is multiplied by $k$, by what quantity is the area multiplied?

24. A beach ball holds 800 cubic inches of air. Another beach ball has a radius that is half that of the larger ball. How much air does the smaller ball hold?

   - A 400 cubic inches
   - B 200 cubic inches
   - C 100 cubic inches
   - D 80 cubic inches

25. For two similar triangles, $\frac{SG}{MW} = \frac{GT}{WR} = \frac{TS}{RM}$. Which statement below is NOT correct?

   - F $\triangle SGT \sim \triangle MWR$
   - G $\triangle GST \sim \triangle MRW$
   - H $\triangle TGS \sim \triangle RWM$
   - I $\triangle GTS \sim \triangle WRM$

26. **Gridded Response** A rectangle has length 5 centimeters and width 3 centimeters. A similar rectangle has length 7.25 centimeters. What is the width in centimeters of this rectangle?

27. Find the values of $w$, $x$, and $y$ given that $\triangle ABC \sim \triangle DEF \sim \triangle GHJ$.

28. $\triangle RST \sim \triangle VWX$ and $\frac{RT}{VX} = b$.

   What is $\frac{\text{area of } \triangle RST}{\text{area of } \triangle VWX}$?

29. **Multi-Step** Rectangles $A$ and $B$ are similar. The area of $A$ is 30.195 cm$^2$. The length of $B$ is 6.1 cm. Each dimension of $B$ is $\frac{2}{3}$ the corresponding dimension of $A$. What is the perimeter of $B$?
Objectives
Analyze and compare measurements for precision and accuracy. Choose an appropriate level of accuracy when reporting measurements.

Vocabulary
precision
accuracy
tolerance

Who uses this?
Chemists must understand precision and accuracy when weighing or mixing specific amounts of chemicals. (See Example 2.)

When you measure an object, you must use an instrument that will give an appropriate measurement. A scale to measure the mass of a person may show mass to the nearest kilogram. A scale to measure chemicals in a lab may show mass to the nearest milligram.

Precision is the level of detail in a measurement and is determined by the smallest unit or fraction of a unit that you can reasonably measure. Sometimes, the instrument determines the precision of a measurement. At other times, measurements are rounded to a specified precision.

A scale that shows the mass of an object to the nearest milligram is more precise than a scale that shows the mass of an object to the nearest kilogram, because a milligram is a smaller unit of measure than a kilogram. Likewise, a scale that shows the mass of an object as 24.23 grams is more precise than a scale that shows the mass of the same object as 24.2 grams.

Example 1
Comparing Precision of Measurements
Choose the more precise measurement in each pair.

A 3.4 kg; 3421 g
3.4 kg Nearest tenth of a kilogram
3421 g Nearest gram
A gram is smaller than a tenth of a kilogram, so 3421 g is more precise.

B 3.4 cm; 3.43 cm
3.4 cm Nearest tenth of a centimeter
3.43 cm Nearest hundredth of a centimeter
A hundredth of a centimeter is smaller than a tenth of a centimeter, so 3.43 cm is more precise.

C 3 ft; 36 in.
3 ft Nearest foot
36 in. Nearest inch
An inch is smaller than a foot, so 36 in. is more precise.
Choose the more precise measurement in each pair.

1a. 2 lb; 17 oz  
1b. 7.85 m; 7.8 m  
1c. 6 kg; 6000 g

A precise measurement is only useful if the measurement is also **accurate**. The **accuracy** of a measurement is the closeness of a measured value to the actual or true value. Two measurement tools may measure to the same precision, but not have the same accuracy. Similarly, using a more precise measuring instrument will not necessarily give a more accurate measurement.

**EXAMPLE 2**

Comparing Precision and Accuracy

Sam is a technician in a pharmaceutical lab. Each week, she must test the scales in the lab to make sure they are accurate. She uses a standard mass that is *exactly* 5.000 grams and gets the following results:

<table>
<thead>
<tr>
<th>Scale 1</th>
<th>Scale 2</th>
<th>Scale 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.01 g</td>
<td>5.033 g</td>
<td>4.98 g</td>
</tr>
</tbody>
</table>

a. Which scale is the most precise?

Scales 1 and 3 measure to the nearest hundredth of a gram.

Scale 2 measures to the nearest thousandth of a gram.

Because a thousandth of a gram is smaller than a hundredth of a gram, Scale 2 is the most precise.

b. Which scale is the most accurate?

For each scale, find the absolute value of the difference of the standard mass and the scale reading.

- Scale 1: \(|5.000 - 5.01| = 0.01\)
- Scale 2: \(|5.000 - 5.033| = 0.033\)
- Scale 3: \(|5.000 - 4.98| = 0.02\)

Because 0.01 < 0.02 < 0.033, Scale 1 is the most accurate.

2. A standard mass of 16 ounces is used to test three postal scales. The results are shown below.

<table>
<thead>
<tr>
<th>Scale A</th>
<th>Scale B</th>
<th>Scale C</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.3 oz</td>
<td>15.8 oz</td>
<td>16.07 oz</td>
</tr>
</tbody>
</table>

a. Which scale is the most precise? **C**

b. Which scale is the most accurate? **C**

When you measure a group of objects that are expected to be similar, you may find that there are variations from the expected value. **Tolerance** describes the amount by which a measurement is permitted to vary from a specified value. Tolerance is often expressed as a range of values, such as 5 mm ± 0.3 mm, which is equivalent to 4.7 mm–5.3 mm.
Using a Specified Tolerance

Acme Nuts & Bolts is manufacturing a bolt to use in an airplane. The length of the bolt should be 50 mm, with a tolerance of 0.5 mm (50 mm ± 0.5 mm). A batch of bolts had the lengths shown in the table. Do all of the bolts measure within the specified tolerance? If not, which bolt(s) are not within the specified tolerance?

<table>
<thead>
<tr>
<th>Bolt</th>
<th>Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>49.8</td>
</tr>
<tr>
<td>B</td>
<td>50.4</td>
</tr>
<tr>
<td>C</td>
<td>49.5</td>
</tr>
<tr>
<td>D</td>
<td>50.1</td>
</tr>
<tr>
<td>E</td>
<td>49.4</td>
</tr>
<tr>
<td>F</td>
<td>50.0</td>
</tr>
</tbody>
</table>

50 – 0.5 = 49.5  \[50 \text{ mm} \pm 0.5 \text{ mm} \text{ means that the bolts must be between 49.5 and 50.5 mm.}\]
50 + 0.5 = 50.5

Bolt E measures 49.4 mm, so it is not within the specified tolerance.

3. A lacrosse ball must weigh 5.25 oz ± 0.25 oz. The weights of the lacrosse balls in one box are given in the table. Do all of the lacrosse balls weigh within the specified tolerance? If not, which lacrosse ball(s) are not within the specified tolerance? no; C

<table>
<thead>
<tr>
<th>Ball</th>
<th>Weight (oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5.41</td>
</tr>
<tr>
<td>B</td>
<td>5.23</td>
</tr>
<tr>
<td>C</td>
<td>5.54</td>
</tr>
<tr>
<td>D</td>
<td>5.33</td>
</tr>
<tr>
<td>E</td>
<td>5.21</td>
</tr>
</tbody>
</table>

Tolerance can also be expressed as a percent. A measurement written as 5 mm ± 5% means that the measurement can be greater or less than 5 mm by an amount equal to 5% of 5 mm, or 0.25 mm. Therefore, the measurement can have a range of 4.75 mm–5.25 mm.

Using Tolerance Expressed as a Percent

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

4.1 in. ± 5%  
475 m ± 2.5%  
85 mg ± 0.5%

4a. 3.89 cm–4.31 cm  
4b. 463.12 m–486.88 m  
4c. 84.57 mg–85.43 mg
THINK AND DISCUSS
1. Explain the difference between precision and accuracy.
2. Describe a situation where the expected size of an object might be specified as 10 in. ± 0.5 in.
3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of when that characteristic of measurement would be important.

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. A ruler that can measure length to a smaller unit than another ruler is said to be more ______?______. (precise or accurate) **precise**

2. A scale that gives a mass closer to the true mass of an object than another scale of the exact same type is said to be more ______?______. (precise or accurate)**accurate**

Choose the more precise measurement in each pair.

3. 4 mL; 4.3 mL
4. 7 m; 6.8 m
5. 2.4 mg; 2.37 mg
6. 7 lb; 6.5 lb
7. 47 ft; 47.3 ft
8. 14 oz; 13.9 oz

9. Sarah is comparing five different scales using a standard mass that is exactly 10 grams. Her results are shown below.

```
Scale 1          Scale 2          Scale 3          Scale 4          Scale 5
9.9 g           10 g            10 g            10 g            10 g
```

a. Which scale is the most precise? **1**
b. Which scale is the most accurate? **1**

10. A group of students compare the odometer readings on their bicycle computers after riding their bikes on a one-mile track. Their odometer readings are shown in the table. Whose odometer is the most precise? Whose is the most accurate? **Rasheed's; Jen's**

<table>
<thead>
<tr>
<th>Student</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jen</td>
<td>1.01</td>
</tr>
<tr>
<td>Bill</td>
<td>0.97</td>
</tr>
<tr>
<td>Rasheed</td>
<td>0.989</td>
</tr>
<tr>
<td>Sasha</td>
<td>1.02</td>
</tr>
</tbody>
</table>
11. **Sports** A basketball for men’s college games must have a mass of 595.5 ± 28.5 grams. Several basketballs are tested. Their masses are shown in the table. Do all of the basketballs fall within the specified tolerance? If not, which basketball(s) do not fall within the specified tolerance? **no; ball 4**

<table>
<thead>
<tr>
<th>Basketball</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (g)</td>
<td>617.5</td>
<td>567.5</td>
<td>608</td>
<td>624.5</td>
<td>593.5</td>
</tr>
</tbody>
</table>

12. **Sports** A basketball for men’s college games must bounce 51.5 ± 2.5 in. when dropped from a height of 6 feet. The bounce heights of several basketballs when dropped from a height of 6 feet are shown in the graph. Do all of the basketballs fall within the specified tolerance? If not, which basketball(s) do not have a bounce height within the specified tolerance? **no; balls 3 and 4**

13. Write the possible range of each measurement. Round to the nearest hundredth if necessary.
   - 13. 50 lb ± 2% 49 lb–51 lb
   - 14. 100 yd ± 0.5% 100.5 yd
   - 15. 70 kg ± 3% 67.9 kg–72.1 kg
   - 16. 400 L ± 6% 376 L–424 L
   - 17. 250 mm ± 4% 240 mm–260 mm
   - 18. 100 yd ± 0.5% 99.5 yd–100.5 yd
   - 19. 25 cm ± 4% 24 cm–26 cm

**PRACTICE AND PROBLEM SOLVING**

Choose the more precise measurement in each pair.
   - 19. 4.33 g; 4337 mg
   - 20. 11 ft; 122 in.
   - 21. 6 tons; 11,000 lb
   - 22. 3 c; 2 pt
   - 23. 67 mm; 6.83 cm
   - 24. 4.5 km; 3 mi
   - 25. 12 cm; 0.0127 m
   - 26. 7.23 lb; 115 oz

27. Maria is trying to beat the school record for the 400-meter dash. Her friends timed her using the stopwatch functions in their cell phones. The official track timer, which is highly accurate, reported that she ran the race in 51.12 seconds. Her friends recorded the times shown in the table.
   a. Who recorded the most precise time? **Chandra**
   b. Who recorded the most accurate time? **Lucy**

28. Anael cut several boards to build a deck. The boards must be 100 in. ± 0.25 in. Her measurements of the boards after cutting them are shown in the graph. Which boards, if any, can she not use? **board 4**
Write the possible range of each measurement. Round to the nearest hundredth if necessary.

29. 45 lb ± 2%  
   44.1 lb–45.9 lb
30. 3 m ± 5%  
   2.85 m–3.15 m
31. 37 °C ± 1.5%  
   36.44 °C–37.56 °C
32. 750 kg ± 3%  
   727.5 kg–772.5 kg
33. 30 ft ± 4%  
   28.8 ft–31.2 ft
34. 550 mL ± 8%  
   506 mL–594 mL
35. 0.2 cm ± 5%  
   0.19 cm–0.21 cm
36. 0.25 kg ± 10%  
   0.23 kg–0.28 kg

Round each measurement to the specified precision.

37. 5456.3 mi to the nearest mile  
   5456 mi
38. 3.627 m to the nearest hundredth of a meter  
   3.63 m
39. 119.8 ft to the nearest ten feet  
   120 ft
40. 62.301 cg to the nearest tenth of a centigram  
   62.3 cg
41. 5,721 mg to the nearest kilogram  
   6 kg
42. 0.4586 km to the nearest meter  
   0.46 km

Choose the more precise measurement in each pair. If they are equally precise, write “neither.”

43. 16.270 liters; 16,453.2 mL  
   16.45 L
44. 437 cm; 437 mm  
   437 cm
45. 0.265 cm; 260 mm  
   0.26 cm
46. 5.20 kg; 5200.0 mg  
   5.2 kg
47. 55 yd; 165 ft  
   55 yd
48. 67 min; 1.1 h  
   1.1 h
49. 33 mg; 0.033 g  
   33 mg
50. 42.7 cm; 427.0 mm  
   42.7 cm
51. 475.0 mL; 0.475 L  
   475.0 mL

Rewrite each specified tolerance as a percent.

52. 100 m ± 2%  
   2 m
53. 50 g ± 4%  
   2 g
54. 240 ft ± 5%  
   12 ft
55. 750 kg ± 2%  
   15 kg
56. 25 in. ± 0.25 in. ± 1%  
   25 in.
57. 425 lb ± 8.5 lb ± 2%  
   425 lb
58. 60 oz ± 1.5 oz ± 2.5%  
   60 oz
59. 175 km ± 5.25 km ± 3%  
   175 km
60. Postcards that do not fit in the U.S. Postal Service’s automatic sorting machines require additional postage for mailing. The machine will accept postcards whose length is between 5 and 6 inches and whose width is between 3 1/2 and 4 1/4 inches. Write these requirements as tolerances.

61. For women’s collegiate competition, a basketball’s circumference, mass, and bounce height must fall within given tolerance levels of regulation measurements. The table shows these tolerance levels as well as measurements taken on five different basketballs. Which basketball meets all of the specified tolerances? ball 4

<table>
<thead>
<tr>
<th>Tolerance</th>
<th>Circumference (mm)</th>
<th>Mass (g)</th>
<th>Bounce Height (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>730.56 ± 6.5</td>
<td>538.5 ± 28.5</td>
<td>1358.5 ± 63.5</td>
</tr>
<tr>
<td>Basketball #1</td>
<td>729.8</td>
<td>509.3</td>
<td>1343.4</td>
</tr>
<tr>
<td>Basketball #2</td>
<td>723.5</td>
<td>529.8</td>
<td>1299.8</td>
</tr>
<tr>
<td>Basketball #3</td>
<td>734.2</td>
<td>542.6</td>
<td>1293.5</td>
</tr>
<tr>
<td>Basketball #4</td>
<td>725.5</td>
<td>528.0</td>
<td>1364.5</td>
</tr>
<tr>
<td>Basketball #5</td>
<td>740.0</td>
<td>555.9</td>
<td>1407.4</td>
</tr>
</tbody>
</table>

62. Linda wants to purchase a new sofa. Before buying the sofa, Linda must measure her doorway to make sure that the sofa will fit through the door. The sofa manufacturer says that the sofa measures 39 inches from front to back. What level of precision would you recommend Linda measure to? Explain.

63. Yusuf measured a board and determined that it was 125.5 centimeters long. He then cut the board into eight equal pieces. His calculator shows that 125.5 ÷ 8 = 15.6875. Is it reasonable for Yusuf to record the length of the 8 smaller boards as 15.6875 centimeters? Explain why or why not.
64. The mass of a crystal is 0.9728 grams. What is the mass of the crystal to the nearest milligram?
   - A 1 milligram
   - B 9.73 milligrams
   - C 973 milligrams
   - D 972.8 milligrams

65. A piece used to assemble a computer must be 1.4 millimeters ± 0.02 millimeters in diameter. Which of the following measurements does NOT meet the specified tolerance?
   - F 1.420 millimeters
   - G 1.402 millimeters
   - H 1.382 millimeters
   - I 1.378 millimeters

66. Which measurement is most precise?
   - A 475.3 milliliters
   - B 475 milliliters
   - C 0.475 liter
   - D 0.5 liter

**CHALLENGE AND EXTEND**

**Percent accuracy or percent error** indicates how far a measurement is from the true value. An instrument that has 1.5% accuracy means that the measured value is within 1.5% of the true value.

67. A scale shows that a standard mass of exactly 5.000 grams has a mass of 5.002 grams. What is the percent accuracy of the scale? **0.04%**

68. A car odometer is accurate to within 0.5%. The odometer records the distance from Charlotte, North Carolina, to Orlando, Florida, as 525.3 miles. What is the range of possible values for the actual mileage? **522.7 mi–527.9 mi**

69. **Astronomy** A scientist measures the distance to the moon using a method that has a percent error of 0.02%. He finds that the distance at a particular time is 384,403 kilometers. What is the range of possible values for the actual distance? **384,326 km–384,480 km**
Vocabulary

accuracy
algebraic expression
constant
conversion factor
corresponding angles
corresponding sides
cross products
dimensional analysis
equation
evaluate

formula
identity
indirect measurement
literal equation
numerical expression
precision
proportion
rate
equation
evaluate

solution of an equation
scale factor
scale model
similar
tolerance
unit rate
variable

Complete the sentences below with vocabulary words from the list above.

1. A formula is a type of a(n) _____.
2. A(n) _____ is used to compare two quantities by division.
3. A(n) _____ is a value that does not change.

2-1 Variables and Expressions

EXAMPLES

Barbara has saved $d$ dollars for a $65 sweater. Write an expression for the amount of money she still needs to buy the sweater.

$65 - d$ Think: $d$ dollars less than the price of the sweater.

Evaluate $b - a$ for $a = 7$ and $b = 15$.

$b - a = 15 - 7$ Substitute the values for the variables.

$= 8$

EXERCISES

4. Grapes cost $1.99 per pound. Write an expression for the cost of $g$ pounds of grapes.

5. Today’s temperature is 3 degrees warmer than yesterday’s temperature $t$. Write an expression for today’s temperature.

Evaluate each expression for $p = 5$ and $q = 1$.

6. $qp$
7. $p ÷ q$
8. $q + p$

9. Each member of the art club will make the same number of posters to advertise their club. They will make 150 posters total. Write an expression for how many posters each member will make if there are $m$ members. Find how many posters each member will make if there are 5, 6, and 10 members.
2-2 Solving Equations by Adding or Subtracting

**EXAMPLES**

Solve each equation. Check your answer.

10. \(b - 16 = 20\)

- Add 16 to both sides.
  \[b = 20 + 16\]
  \[b = 36\]

11. \(4 + x = 2\)

- Subtract 4 from both sides.
  \[x = 2 - 4\]
  \[x = -2\]

12. \(9 + a = -12\)

- Subtract 9 from both sides.
  \[a = -12 - 9\]
  \[a = -21\]

13. \(-7 + y = 11\)

- Add 7 to both sides.
  \[y = 11 + 7\]
  \[y = 18\]

14. \(z - \frac{1}{4} = \frac{7}{8}\)

- Add \(\frac{1}{4}\) to both sides.
  \[z = \frac{7}{8} + \frac{1}{4}\]
  \[z = \frac{7}{8} + \frac{2}{8}\]
  \[z = \frac{9}{8}\]

15. \(w + \frac{2}{3} = 3\)

- Subtract \(\frac{2}{3}\) from both sides.
  \[w = 3 - \frac{2}{3}\]
  \[w = \frac{9}{3} - \frac{2}{3}\]
  \[w = \frac{7}{3}\]

16. Robin needs 108 signatures for her petition. So far, she has 27. Write and solve an equation to determine how many more signatures she needs.

\[108 - 27 = x\]

\[81 = x\]

**EXERCISES**

Solve each equation. Check your answer.

10. \(b - 16 = 20\)

11. \(4 + x = 2\)

12. \(9 + a = -12\)

13. \(-7 + y = 11\)

14. \(z - \frac{1}{4} = \frac{7}{8}\)

15. \(w + \frac{2}{3} = 3\)

16. Robin needs 108 signatures for her petition. So far, she has 27. Write and solve an equation to determine how many more signatures she needs.

2-3 Solving Equations by Multiplying or Dividing

**EXAMPLES**

Solve each equation.

17. \(35 = 5x\)

- Divide both sides by 5.
  \[x = \frac{35}{5}\]
  \[x = 7\]

18. \(-3n = 10\)

- Divide both sides by -3.
  \[n = \frac{-10}{-3}\]
  \[n = \frac{10}{3}\]

19. \(-30 = n - 3\)

- Add 3 to both sides.
  \[n = -30 + 3\]
  \[n = -27\]

20. \(5y = 0\)

- Divide both sides by 5.
  \[y = \frac{0}{5}\]
  \[y = 0\]

21. \(-4.6r = 9.2\)

- Divide both sides by -4.6.
  \[r = \frac{9.2}{-4.6}\]
  \[r = -2\]

22. \(x = -18.5\)

**EXERCISES**

Solve each equation. Check your answer.

17. \(35 = 5x\)

18. \(-3n = 10\)

19. \(-30 = n - 3\)

20. \(5y = 0\)

21. \(-4.6r = 9.2\)

22. \(x = -18.5\)

2-4 Solving Two-Step and Multi-Step Equations

**EXAMPLE**

Solve \(\frac{3x}{5} - \frac{x}{4} + \frac{1}{2} = \frac{6}{5}\).

- Find the least common denominator (LCD), which is 20.
  \[\frac{3x}{5} \cdot \frac{4}{4} - \frac{x}{4} \cdot \frac{5}{5} + \frac{1}{2} \cdot \frac{10}{10} = \frac{6}{5} \cdot \frac{4}{4}\]
  \[\frac{12x}{20} - \frac{5x}{20} + \frac{10}{20} = \frac{24}{20}\]
  \[\frac{7x}{20} + \frac{10}{20} = \frac{24}{20}\]
  \[\frac{7x}{20} = \frac{14}{20}\]
  \[x = \frac{2}{2}\]

- Combine like terms.

\[\frac{7x}{20} = \frac{14}{20}\]

\[x = \frac{2}{2}\]

**EXERCISES**

Solve each equation. Check your answer.

23. \(4t - 13 = 57\)

24. \(5 - 2y = 15\)

25. \(\frac{k}{5} - 6 = 2\)

26. \(\frac{5}{6}f - \frac{3}{4}f + \frac{3}{4} = \frac{1}{2}\)

27. \(7x - 19x = 6\)

28. \(4 + 3a - 6 = 43\)

29. If \(8n + 22 = 70\), find the value of \(3n\).

30. If \(0 = 6n - 36\), find the value of \(n - 5\).

31. The sum of the measures of two angles is 180°. One angle measures \(3a\) and the other angle measures \(2a - 25\). Find \(a\). Then find the measure of each angle.
2-5 Solving Equations with Variables on Both Sides

**Example**

- Solve \( x + 7 = 12 + 3x - 7x \).
  \[
  x + 7 = 12 + 3x - 7x
  \]
  
  \[
  x + 7 = 12 - 4x
  \]
  
  Combine like terms.

- Solve \( 42x + 6 = 21 \).

- Solve \( 7y - 5 = 14 \).

- Solve \( 3y + 4 = 31 \).

- Solve \( 12 = x - 5.4 \).

- Solve \( g + 6 + 12 = 14 \).

2-6 Solving for a Variable

**Example**

- Solve \( A = P + Prt \) for \( r \).
  \[
  A = P + Prt
  \]
  
  \[
  A - P = Prt
  \]
  
  \[
  \frac{A - P}{P} = rt
  \]
  
  \[
  \frac{A - P}{P} = r
  \]

2-7 Solving Absolute-Value Equations

**Examples**

- Solve \( 3|y + 4| = 30 \).
  \[
  3|y + 4| = 30
  \]
  
  Divide both sides by 3.

- Solve \( 3|y + 4| = 30 \) for \( y \):
  \[
  3|y + 4| = 30
  \]
  
  \[
  \frac{3|y + 4|}{3} = \frac{30}{3}
  \]
  
  \[
  |y + 4| = 10
  \]
  
  Case 1
  \[
  y + 4 = 10
  \]
  
  \[
  y = 6
  \]
  
  Case 2
  \[
  y + 4 = -10
  \]
  
  \[
  y = -14
  \]

2-8 Solving Equations with Variables on Both Sides

**Example**

- Solve \( 4x + 2 = 3x \).

- Solve \( -3r - 8 = -5r - 12 \).

- Solve \( -a - 3 + 7 = 3a \).

- Solve \( -(x - 4) = 2x + 6 \).

- Solve \( \frac{2}{3} n = 4n - \frac{10}{3}n - \frac{1}{2} \).

- Solve \( n = 0.4(7 + 2t) = 0.4t + 1.4 \).

2-9 Solving for a Variable

**Example**

- Solve \( C = \frac{360}{n} \) for \( n \).

- Solve \( S = \frac{n}{2}(a + t) \) for \( a \).

2-10 Solving Absolute-Value Equations

**Examples**

- Solve \( |x + 6| = 21 \).

- Solve \( 7|y - 5| = 14 \).

- Solve \( 3|y| + 4 = 31 \).

- Solve \( 12 = |x - 5.4| \).

- Solve \( |g + 6| + 12 = 14 \).

- Solve \( |x| = \frac{5}{7} \).

2-11 Jason is driving his car at 55 mi/h. He needs to keep his car within 5 mi/h of his current speed. Write and solve an absolute-value equation to find Jason’s maximum and minimum speeds.
Choose the more precise measurement in each pair.

59. 1 ft; 12 in.

60. 37 g; 37.0 g

61. 550 cm; 5.5 km

62. 1.5 L; 1 L

Write the possible range of each measurement. Round to the nearest hundredth if necessary.

63. 500 lb ± 4%

64. 20 oz ± 0.5%

65. 75 kg ± 3%

66. 1035 mm ± 10%

Choose the more precise measurement in each pair.

56. Find the value of $x$ in the diagram.

$\triangle ABC \sim \triangle DEF$

57. A tree casts a shadow that is 14 ft long at the same time that a nearby 2-foot-tall pole casts a shadow that is 1.75 ft long. How tall is the tree?

58. A circle has a radius of 9 inches. The radius is multiplied by $\frac{2}{3}$ to form a second circle. How is the ratio of the areas related to the ratio of the radii?
Evaluate each expression for \( a = 2, b = 3, \) and \( c = 6. \)

1. \( c - a \)  
2. \( ab \)  
3. \( c \div a \)  
4. \( \frac{c}{b} \)  
5. \( b - a \)

6. Give two ways write \( n - 5 \) in words.

7. Nate runs 8 miles each week. Write an expression for the number of miles he runs in \( n \) weeks. Find the number of miles Nate runs in 5 weeks.

Solve each equation.

8. \( y - 7 = 2 \)

9. \( x + 12 = 19 \)

10. \( -5 + z = 8 \)

11. \( 9x = 72 \)

12. \( \frac{m}{-8} = -2.5 \)

13. \( \frac{7}{8} a = 42 \)

14. \( 15 = 3 - 4x \)

15. \( \frac{2a}{3} + \frac{1}{5} = \frac{7}{6} \)

16. \( 8 - (b - 2) = 11 \)

17. \( -2x + 4 = 5 - 3x \)

18. \( 3(q - 2) + 2 = 5q - 7 - 2q \)

19. \( 5z = -3(z + 7) \)

Solve for the indicated variable.

20. \( r - 2s = 14 \) for \( s \)

21. \( V = \frac{1}{3} bh \) for \( b \)

22. \( P = 2(\ell + w) \) for \( \ell \)

Solve each equation.

23. \( |x - 14| = 21 \)

24. \( 3|x| + 5 = 8 \)

25. \( |2\nu| = 6 \)

26. Twenty-five students use 120 sheets of paper. Find the unit rate in sheets per student.

27. Nutritionists recommend that teenagers consume 1300 milligrams of calcium per day. Use dimensional analysis to convert this rate to grams per year.

Solve each proportion.

28. \( \frac{5}{4} = \frac{x}{12} \)

29. \( \frac{8}{2z} = \frac{15}{60} \)

30. \( \frac{x + 10}{10} = \frac{18}{12} \)

31. The scale on a map is 1 inch : 500 miles. If two cities are 875 miles apart, how far apart are they on the map?

Find the value of \( x \) in each diagram. Round your answer to the nearest tenth.

32. \( \triangle EFG \sim \triangle RTS \)

33. \( HJKL \sim WXYZ \)

Choose the more precise measurement in each pair.

34. 6.5 oz; 6 oz

35. 16 oz; 1 lb

36. 3525 m; 3.5 km

Write the possible range of each measurement.

37. 25 ft ± 1%

38. 400 lb ± 4%

39. 250 cm ± 0.5%
For a **Good Cause**

You can use the concepts in this chapter to plan for a fund-raising event. Inequalities help you determine how to reach your fund-raising goals.
Study Strategy: Use Your Notes Effectively

Taking notes helps you arrange, organize, and process information from your textbook and class lectures. In addition to taking notes, you need to use your notes before and after class effectively.

Step 1: Before Class
- Review your notes from the last class.
- Then preview the next lesson and write down any questions you have.

Step 2: During Class
- Write down main ideas.
- If you miss something, leave a blank and keep taking notes. Fill in any holes later.
- Use diagrams and abbreviations. Make sure you will understand any abbreviations later.

Step 3: After Class
- Fill in the holes you left during class.
- Highlight or circle the most important ideas, such as vocabulary, formulas, or procedures.
- Use your notes to quiz yourself.

Try This

1. Look at the next lesson in your textbook. Write down some questions you have about the material in that lesson. Leave space between each question so that you can write the answers during the next class.

2. Look at the notes you took during the last class. List three ways you can improve your note-taking skills.
Graphing and Writing Inequalities

Objectives
Identify solutions of inequalities in one variable.
Write and graph inequalities in one variable.

Vocabulary
inequality
solution of an inequality

Who uses this?
Members of a crew team can use inequalities to be sure they fall within a range of weights. (See Example 4.)

The athletes on a lightweight crew team must weigh 165 pounds or less. The acceptable weights for these athletes can be described using an inequality.

An inequality is a statement that two quantities are not equal. The quantities are compared by using one of the following signs:

\[
\begin{align*}
A & < B & \text{A is less than B.} \\
A & > B & \text{A is greater than B.} \\
A & \leq B & \text{A is less than or equal to B.} \\
A & \geq B & \text{A is greater than or equal to B.} \\
A & \neq B & \text{A is not equal to B.}
\end{align*}
\]

A solution of an inequality is any value of the variable that makes the inequality true.

EXAMPLE 1
Identifying Solutions of Inequalities

Describe the solutions of \(3 + x < 9\) in words.

Test values of \(x\) that are positive, negative, and 0.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-2.75)</th>
<th>0</th>
<th>5.99</th>
<th>6</th>
<th>6.01</th>
<th>6.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3 + x)</td>
<td>0.25</td>
<td>3</td>
<td>8.99</td>
<td>9</td>
<td>9.01</td>
<td>9.1</td>
</tr>
<tr>
<td>(3 + x \leq 9)</td>
<td>0.25 (\leq) 9</td>
<td>3 (\leq) 9</td>
<td>8.99 (\leq) 9</td>
<td>9 (\leq) 9</td>
<td>9.01 (\leq) 9</td>
<td>9.1 (\leq) 9</td>
</tr>
<tr>
<td>Solution?</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

When the value of \(x\) is a number less than 6, the value of \(3 + x\) is less than 9.
When the value of \(x\) is 6, the value of \(3 + x\) is equal to 9.
When the value of \(x\) is a number greater than 6, the value of \(3 + x\) is greater than 9.

The solutions of \(3 + x < 9\) are numbers less than 6.

CHECK IT OUT!
1. Describe the solutions of \(2p > 8\) in words.
An inequality like $3 + x < 9$ has too many solutions to list. You can use a graph on a number line to show all the solutions.

The solutions are shaded and an arrow shows that the solutions continue past those shown on the graph. To show that an endpoint is a solution, draw a solid circle at the number. To show that an endpoint is not a solution, draw an empty circle.

### Graphing Inequalities

<table>
<thead>
<tr>
<th>WORDS</th>
<th>ALGEBRA</th>
<th>GRAPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>All real numbers less than 5</td>
<td>$x &lt; 5$</td>
<td><img src="image1" alt="Graph" /></td>
</tr>
<tr>
<td>All real numbers greater than $-1$</td>
<td>$x &gt; -1$</td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td>All real numbers less than or equal to $\frac{1}{2}$</td>
<td>$x \leq \frac{1}{2}$</td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td>All real numbers greater than or equal to 0</td>
<td>$x \geq 0$</td>
<td><img src="image4" alt="Graph" /></td>
</tr>
</tbody>
</table>

### Example 2 Graphing Inequalities

Graph each inequality.

**A** $b < -1.5$

Draw an empty circle at $-1.5$. Shade all the numbers less than $-1.5$ and draw an arrow pointing to the left.

**B** $r \geq 2$

Draw a solid circle at 2. Shade all the numbers greater than 2 and draw an arrow pointing to the right.

### Check It Out!

Graph each inequality.

2a. $c > 2.5$  
2b. $2^2 - 4 \geq w$  
2c. $m \leq -3$

### Student to Student Graphing Inequalities

To know which direction to shade a graph, I write inequalities with the variable on the left side of the inequality symbol. I know that the symbol has to point to the same number after I rewrite the inequality.

For example, I write $4 < y$ as $y > 4$.

Now the inequality symbol points in the direction that I should draw the shaded arrow on my graph.
**Example 3**

**Writing an Inequality from a Graph**

Write the inequality shown by each graph.

**A**

Use any variable. The arrow points to the right, so use either $>$ or $\geq$.

The empty circle at 4.5 means that 4.5 is not a solution, so use $>.$

$h > 4.5$

**B**

Use any variable. The arrow points to the left, so use either $<$ or $\leq$.

The solid circle at $-3$ means that $-3$ is a solution, so use $\leq$.

$m \leq -3$

**Example 4**

**Sports Application**

The members of a lightweight crew team can weigh no more than 165 pounds each. Define a variable and write an inequality for the acceptable weights of the team members. Graph the solutions.

Let $w$ represent the weights that are allowed.

Athletes may weigh no more than 165 pounds.

$w \leq 165$

Stop the graph at 0 because a person’s weight must be a positive number.

4. A store’s employees earn at least $8.25 per hour. Define a variable and write an inequality for the amount the employees may earn per hour. Graph the solutions.

**Think and Discuss**

1. Compare the solutions of $x > 2$ and $x \geq 2$.

2. Get organized Copy and complete the graphic organizer. Draw a graph in the first row and write the correct inequality in the second row.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &gt; 1$</td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. Vocabulary How is a solution of an inequality like a solution of an equation?

Describe the solutions of each inequality in words.

2. \( g - 5 \geq 6 \)  
3. \(-2 < h + 1\)
4. \( 20 > 5t \)
5. \( 5 - x \leq 2 \)

Graph each inequality.

6. \( x < -5 \)
7. \( c \geq 3 \frac{1}{2} \)
8. \( (4 - 2)^3 > m \)
9. \( p \geq \sqrt{17} + 8 \)

Write the inequality shown by each graph.

10. \( \ldots \)
11. \( \ldots \)
12. \( \ldots \)
13. \( \ldots \)
14. \( \ldots \)
15. \( \ldots \)

Define a variable and write an inequality for each situation. Graph the solutions.

16. There must be at least 20 club members present in order to hold a meeting.
17. A trainer advises an athlete to keep his heart rate under 140 beats per minute.

PRACTICE AND PROBLEM SOLVING

Describe the solutions of each inequality in words.

18. \(-2t > -8\)
19. \(0 > w - 2\)
20. \(3k > 9\)
21. \(\frac{1}{2}b \leq 6\)

Graph each inequality.

22. \(7 < x\)
23. \(t \leq -\frac{1}{2}\)
24. \(d > 4(5 - 8)\)
25. \(t \leq 3^2 - 2^2\)

Write the inequality shown by each graph.

26. \(\ldots\)
27. \(\ldots\)
28. \(\ldots\)
29. \(\ldots\)
30. \(\ldots\)
31. \(\ldots\)

Define a variable and write an inequality for each situation. Graph the solutions.

32. The maximum speed allowed on Main Street is 25 miles per hour.
33. Applicants must have at least 5 years of experience.
Write each inequality in words.
34. \( x > 7 \)  
35. \( h < -5 \)  
36. \( d \leq 23 \)  
37. \( r \geq -2 \)

Write each inequality with the variable on the left. Graph the solutions.
38. \( 19 < g \)  
39. \( 17 \geq p \)  
40. \( 10 < e \)  
41. \( 0 < f \)

Define a variable and write an inequality for each situation. Graph the solutions.
42. The highest temperature ever recorded on Earth was \( 135.9 \) °F at Al Aziziyah, Libya, on September 13, 1922.
43. Businesses with profits less than \$10,000 per year will be shut down.
44. You must be at least 46 inches tall to ride a roller coaster at an amusement park.
45. Due to a medical condition, a hiker can hike only in areas with an elevation no more than 5000 feet above sea level.

Write a real-world situation that could be described by each inequality.
46. \( x \geq 0 \)  
47. \( x < 10 \)  
48. \( x \leq 12 \)  
49. \( x > 8.5 \)

Match each inequality with its graph.
50. \( x \geq 5 \)  
51. \( x < 5 \)  
52. \( x > 5 \)  
53. \( x \leq 5 \)

54. \(/[ERROR ANALYSIS/]\) Two students graphed the inequality \( 4 > b \). Which graph is incorrect? Explain the error.

55. a. Mirna earned \$125 baby-sitting during the spring break. She needs to save \$90 for the German Club trip. She wants to spend the remainder of the money shopping. Write an inequality to show how much she can spend.
b. Graph the inequality you wrote in part a.
c. Mirna spends \$15 on a bracelet. Write an inequality to show how much money she has left to spend.
56. **Critical Thinking** Graph all positive integer solutions of the inequality \( x < 5 \).

57. **Write About It** Explain how to write an inequality that is modeled by a graph. What characteristics do you look for in the graph?

58. **Write About It** You were told in the lesson that the phrase “no more than” means “less than or equal to” and the phrase “at least” means “greater than or equal to.”
   a. What does the phrase “at most” mean?
   b. What does the phrase “no less than” mean?

59. Which is NOT a solution of the inequality \( 5 - 2x \geq -3 \)?
   - A. 0
   - B. 2
   - C. 4
   - D. 5

60. Which is NOT a solution of the inequality \( 3 - x < 2 \)?
   - E. 1
   - F. 2
   - G. 3
   - H. 4

61. Which graph represents the solutions of \(-2 \leq 1 - t\)?
   - A
   - B
   - C
   - D

**CHALLENGE AND EXTEND**

Describe the values for \( x \) and \( y \) that make each inequality true.

62. \( x + y \leq |x + y| \)  
63. \( x^2 < xy \)  
64. \( x - y \geq y - x \)

Complete each statement. Write < or >.

65. If \( a > b \), then \( b \square a \).
66. If \( x > y \) and \( y > z \), then \( x \square z \).

67. Name a value of \( x \) that makes the statement \( 0.35 < x < 1.27 \) true.

68. Is \( \frac{5}{6} \) a solution of \( x < 1 \)? How many solutions of \( x < 1 \) are between 0 and 1?

69. **Write About It** Explain how to graph all the solutions of \( x \neq 5 \).
**Objectives**
Solve one-step inequalities by using addition.
Solve one-step inequalities by using subtraction.

**Who uses this?**
You can use inequalities to determine how many more photos you can take. (See Example 2.)

Tenea has a cell phone that also takes pictures. After taking some photos, Tenea can use a one-step inequality to determine how many more photos she can take.

Solving one-step inequalities is much like solving one-step equations. To solve an inequality, you need to isolate the variable using the properties of inequality and inverse operations.

**Properties of Inequality**

### Addition and Subtraction

<table>
<thead>
<tr>
<th>WORDS</th>
<th>NUMBERS</th>
<th>ALGEBRA</th>
</tr>
</thead>
</table>
| **Addition** | You can add the same number to both sides of an inequality, and the statement will still be true. | $3 < 8$
| | | $3 + 2 < 8 + 2$
| | | $5 < 10$
| | | $a < b$
| | | $a + c < b + c$
| **Subtraction** | You can subtract the same number from both sides of an inequality, and the statement will still be true. | $9 < 12$
| | | $9 - 5 < 12 - 5$
| | | $4 < 7$
| | | $a < b$
| | | $a - c < b - c$

These properties are also true for inequalities that use the symbols $>$, $\geq$, and $\leq$.

**Example 1**

Using Addition and Subtraction to Solve Inequalities

Solve each inequality and graph the solutions.

**A**

$x + 9 < 15$

Since 9 is added to $x$, subtract 9 from both sides to undo the addition.

$x < 6$

**B**

$d - 3 > -6$

Since 3 is subtracted from $d$, add 3 to both sides to undo the subtraction.

$d > -3$

Use an inverse operation to “undo” the operation in an inequality. If the inequality contains addition, use subtraction to undo the addition.
Solve each inequality and graph the solutions.

\[ C \quad 0.7 \geq n - 0.4 \]

Since 0.4 is subtracted from \( n \), add 0.4 to both sides to undo the subtraction.

\[ \begin{align*}
+ 0.4 & \quad + 0.4 \\
1.1 & \geq n \\
\end{align*} \]

\[ n \leq 1.1 \]

\[ \begin{align*}
\text{Graph:} & \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \\
1.1 & \end{align*} \]

\[ \text{Check It Out!} \]

Solve each inequality and graph the solutions.

\[ \begin{align*}
1a. \quad s + 1 & \leq 10 \\
1b. \quad 2 \frac{1}{2} & > -3 + t \\
1c. \quad q - 3.5 & < 7.5 \\
\end{align*} \]

Since there can be an infinite number of solutions to an inequality, it is not possible to check all the solutions. You can check the endpoint and the direction of the inequality symbol.

\[ \begin{align*}
\text{The solutions of } x + 9 & < 15 \text{ are given by } x < 6. \\
\text{Step 1 Check the endpoint.} & \\
\text{Substitute 6 for } x \text{ in the related equation } x + 9 = 15. \\
The endpoint should be a solution of the equation. & \\
x + 9 & < 15 \\
4 + 9 & < 15 \\
13 & < 15 & \checkmark \\
\end{align*} \]

\[ \text{Step 2 Check the inequality symbol.} \]

\[ \begin{align*}
\text{Substitute a number less than 6 for } x \text{ in the original inequality. The number you choose should be a solution of the inequality.} \\
x + 9 & < 15 \\
6 + 9 & < 15 \\
15 & < 15 & \checkmark \\
\end{align*} \]

\[ \begin{align*}
\text{EXAMPLE 2 Problem Solving Application} \\
\text{The memory in Tenea's camera phone allows her to take up to 20 pictures. Tenea has already taken 16 pictures. Write, solve, and graph an inequality to show how many more pictures Tenea could take.} \\
\end{align*} \]

1. **Understand the Problem**

   The answer will be an inequality and a graph that show all the possible numbers of pictures that Tenea can take.

   List the important information:
   - Tenea can take up to, or at most, 20 pictures.
   - Tenea has taken 16 pictures already.

2. **Make a Plan**

   Write an inequality.

   Let \( p \) represent the remaining number of pictures Tenea can take.

   \[ \begin{align*}
   \text{Number taken} & \quad + \quad \text{Number remaining} & \quad \text{is at most} & \quad 20 \text{ pictures.} \\
   16 & \quad + \quad p & \quad \leq & \quad 20 \\
   \end{align*} \]
3. **Solve**

\[ 16 + p \leq 20 \]

Since 16 is added to \( p \), subtract 16 from both sides to undo the addition.

\[ \begin{align*}
-16 & \quad -16 \\
\hline
p & \leq 4
\end{align*} \]

It is not reasonable for Tenea to take a negative or fractional number of pictures, so graph the nonnegative integers less than or equal to 4. Tenea could take 0, 1, 2, 3, or 4 more pictures.

4. **Look Back**

**Check**

Check the endpoint, 4. Check a number less than 4.

\[
\begin{align*}
16 + p &= 20 \\
16 + 4 &= 20 \\
20 &\leq 20 \\
20 &\checkmark
\end{align*}
\]

Adding 0, 1, 2, 3, or 4 more pictures will not exceed 20.

2. The Recommended Dietary Allowance (RDA) of iron for a female in Sarah’s age group (14–18 years) is 15 mg per day. Sarah has consumed 11 mg of iron today. Write and solve an inequality to show how many more milligrams of iron Sarah can consume without exceeding the RDA.

**EXAMPLE 3**

**Sports Application**

Josh can bench press 220 pounds. He wants to bench press at least 250 pounds. Write and solve an inequality to determine how many more pounds Josh must lift to reach his goal. Check your answer.

Let \( p \) represent the number of additional pounds Josh must lift.

\[
\begin{align*}
220 \text{ pounds} &\quad \text{plus} &\quad \text{additional pounds} &\quad \text{is at least} &\quad 250 \text{ pounds.} \\
\hline
220 &\quad + &\quad p &\quad \geq &\quad 250 \\
\end{align*}
\]

\[ 220 + p \geq 250 \]

Since 220 is added to \( p \), subtract 220 from both sides to undo the addition.

\[ -220 \quad -220 \]

\[ p \geq 30 \]

**Check**

Check the endpoint, 30. Check a number greater than 30.

\[
\begin{align*}
220 + p &= 250 \\
220 + 30 &= 250 \\
250 &\geq 250 \\
250 &\checkmark
\end{align*}
\]

Josh must lift at least 30 additional pounds to reach his goal.

3. **What if...?** Josh has reached his goal of 250 pounds and now wants to try to break the school record of 282 pounds. Write and solve an inequality to determine how many more pounds Josh needs to break the school record. Check your answer.
THINK AND DISCUSS

1. Show how to check your solution to Example 1B.

2. Explain how the Addition and Subtraction Properties of Inequality are like the Addition and Subtraction Properties of Equality.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an inequality that you must use the specified property to solve. Then solve and graph the inequality.

GUIDED PRACTICE

Solve each inequality and graph the solutions.

1. \(12 < p + 6\)
2. \(w + 3 \geq 4\)
3. \(-5 + x \leq -20\)
4. \(z - 2 > -11\)
5. Health For adults, the maximum safe water temperature in a spa is 104 °F. The water temperature in Bill’s spa is 102 °F. The temperature is increased by \(t\) °F. Write, solve, and graph an inequality to show the values of \(t\) for which the water temperature is still safe.

6. Consumer Economics A local restaurant will deliver food to your house if the purchase amount of your order is at least $25.00. The total for part of your order is $17.95. Write and solve an inequality to determine how much more you must spend for the restaurant to deliver your order.

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

7. \(a - 3 \geq 2\)
8. \(2.5 > q - 0.8\)
9. \(-45 + x < -30\)
10. \(r + \frac{1}{4} \leq \frac{3}{4}\)

11. Engineering The maximum load for a certain elevator is 2000 pounds. The total weight of the passengers on the elevator is 1400 pounds. A delivery man who weighs 243 pounds enters the elevator with a crate of weight \(w\). Write, solve, and graph an inequality to show the values of \(w\) that will not exceed the weight limit of the elevator.

12. Transportation The gas tank in Mindy’s car holds at most 15 gallons. She has already filled the tank with 7 gallons of gas. She will continue to fill the tank with \(g\) gallons more. Write and solve an inequality that shows all values of \(g\) that Mindy can add to the car’s tank.

Write an inequality to represent each statement. Solve the inequality and graph the solutions.

13. Ten less than a number \(x\) is greater than 32.
14. A number \(n\) increased by 6 is less than or equal to 4.
15. A number \(r\) decreased by 13 is at most 15.
Health

Special-effects contact lenses are sometimes part of costumes for movies. All contact lenses should be worn under an eye doctor’s supervision.

24. Use the inequality \( s + 12 \geq 20 \) to fill in the missing numbers.
   a. \( s \geq \) 
   b. \( s + \) \( \geq 30 \)
   c. \( s - 8 \geq \) 

25. **Health** A particular type of contact lens can be worn up to 30 days in a row. Alex has been wearing these contact lenses for 21 days. Write, solve, and graph an inequality to show how many more days Alex could wear his contact lenses.

Solve each inequality and graph the solutions.

16. \( x + 4 \leq 2 \)
17. \(-12 + q > 39 \)
18. \( x + \frac{3}{5} < 7 \)
19. \( 4.8 \geq p + 4 \)
20. \(-12 \leq x - 12 \)
21. \( 4 < 206 + c \)
22. \( y - \frac{1}{3} > \frac{2}{3} \)
23. \( x + 1.4 \geq 1.4 \)

26. \( 1 \leq x - 2 \)
27. \( 8 > x - (-5) \)
28. \( x + 6 > 9 \)
29. \(-4 \geq x - 7 \)

30. **Estimation** Is \( x < 10 \) a reasonable estimate for the solutions to the inequality \( 11.879 + x < 21.709 \)? Explain your answer.

31. **Sports** At the Seattle Mariners baseball team’s home games, there are 45,611 seats in the four areas listed in the table. Suppose all the suite level and club level seats during a game are filled. Write and solve an inequality to determine how many people \( p \) could be sitting in the other types of seats.

<table>
<thead>
<tr>
<th>Type of Seat</th>
<th>Number of Seats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main bowl</td>
<td>24,399</td>
</tr>
<tr>
<td>Upper bowl</td>
<td>16,022</td>
</tr>
<tr>
<td>Club level</td>
<td>4,254</td>
</tr>
<tr>
<td>Suite level</td>
<td>936</td>
</tr>
</tbody>
</table>

32. **Critical Thinking** Recall that in Chapter 3 a balance scale was used to model solving equations. Describe how a balance scale could model solving inequalities.

33. **Critical Thinking** Explain why \( x + 4 \geq 6 \) and \( x - 4 \geq -2 \) have the same solutions.

34. **Write About It** How do the solutions of \( x + 2 \geq 3 \) differ from the solutions of \( x + 2 > 3 \)? How do the graphs of the solutions differ?

35. a. Daryl finds that the distance from Columbus, Ohio, to Washington, D.C., is 411 miles. What is the round-trip distance?

b. Daryl can afford to drive a total of 1000 miles. Write an inequality to show the number of miles \( m \) he can drive while in Washington, D.C.

c. Solve the inequality and graph the solutions on a number line. Show that your answer is reasonable.
36. Which is a reasonable solution of $4.7367 + p < 20.1784$?

<p>| | | | | | |</p>
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<thead>
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<tbody>
<tr>
<td>A</td>
<td>15</td>
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<tr>
<td>B</td>
<td>16</td>
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<tr>
<td>C</td>
<td>24</td>
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<tr>
<td>D</td>
<td>25</td>
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</tbody>
</table>

37. Which statement can be modeled by $x + 3 \leq 12$?

- F: Sam has 3 bottles of water. Together, Sam and Dave have at most 12 bottles of water.
- G: Jennie sold 3 cookbooks. To earn a prize, Jennie must sell at least 12 cookbooks.
- H: Peter has 3 baseball hats. Peter and his brothers have fewer than 12 baseball hats.
- J: Kathy swam 3 laps in the pool this week. She must swim more than 12 laps.

38. Which graph represents the solutions of $p + 3 < 1$?

<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<tr>
<td>A</td>
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<td>D</td>
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</tbody>
</table>

39. Which inequality does NOT have the same solutions as $n + 12 \leq 26$?

- F: $n \leq 14$
- G: $n + 6 \leq 20$
- H: $10 \geq n - 4$
- J: $n - 12 \leq 14$

**CHALLENGE AND EXTEND**

Solve each inequality and graph the solutions.

40. $6 - \frac{9}{10} \geq \frac{4}{5} + x$
41. $r - \frac{12}{5} \leq \frac{7}{10}$
42. $6 \frac{2}{3} + m > \frac{7}{6}$

Determine whether each statement is sometimes, always, or never true. Explain.

43. $a + b > a - b$
44. If $a > c$, then $a + b > c + b$.
45. If $a > b$ and $c > d$, then $a + c > b + d$.
46. If $x + b > c$ and $x > 0$ have the same solutions, what is the relationship between $b$ and $c$?
**Objectives**

Solve one-step inequalities by using multiplication.
Solve one-step inequalities by using division.

**Who uses this?**

You can solve an inequality to determine how much you can buy with a certain amount of money. (See Example 3.)

Remember, solving inequalities is similar to solving equations. To solve an inequality that contains multiplication or division, undo the operation by dividing or multiplying both sides of the inequality by the same number.

The rules below show the properties of inequality for multiplying or dividing by a positive number. The rules for multiplying or dividing by a negative number appear later in this lesson.

**Properties of Inequality**

<table>
<thead>
<tr>
<th>Multiplication and Division by Positive Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>WORDS</strong></td>
</tr>
<tr>
<td>Multiplication</td>
</tr>
<tr>
<td>You can multiply both sides of an inequality by the same <strong>positive</strong> number, and the statement will still be true.</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Division</td>
</tr>
<tr>
<td>You can divide both sides of an inequality by the same <strong>positive</strong> number, and the statement will still be true.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

These properties are also true for inequalities that use the symbols >, ≥, and ≤.

**Example 1**

Multiplying or Dividing by a Positive Number

Solve each inequality and graph the solutions.

\[
\begin{align*}
A & \quad 3x > -27 \\
\frac{3x}{3} & > \frac{-27}{3} \\
\end{align*}
\]

Since \( x \) is multiplied by 3, divide both sides by 3 to undo the multiplication.

\[
\begin{align*}
& \quad \frac{3x}{3} > \frac{-27}{3} \\
& \quad x > -9 \\
& \quad -9 \\
& \quad -10 -8 -6 -4 -2 0 2 4 6 8 10
\end{align*}
\]
Solve each inequality and graph the solutions.

\[ \frac{2}{3} r < 6 \]

Since \( r \) is multiplied by \( \frac{2}{3} \), multiply both sides by the reciprocal of \( \frac{2}{3} \).

\[ \frac{3}{2} \left( \frac{2}{3} r \right) < \frac{3}{2} (6) \]

\[ r < 9 \]

---

CHECK IT OUT!

Solve each inequality and graph the solutions.

1a. \( 4k > 24 \)

1b. \( -50 \geq 5q \)

1c. \( \frac{3}{4} g > 27 \)

What happens when you multiply or divide both sides of an inequality by a negative number?

Look at the number line below.

\[
\begin{align*}
2 &< 6 \\
-2 &< -6 & \text{Multiply both sides by } -1.
\end{align*}
\]

\[
\begin{align*}
6 &> -2 \\
-6 &< 2 & \text{Multiply both sides by } -1.
\end{align*}
\]

Notice that when you multiply (or divide) both sides of an inequality by a negative number, you must reverse the inequality symbol. This means there is another set of properties of inequality for multiplying or dividing by a negative number.

Properties of Inequality

Know it! Note

**Multiplication and Division by Negative Numbers**

<table>
<thead>
<tr>
<th>WORDS</th>
<th>NUMBERS</th>
<th>ALGEBRA</th>
</tr>
</thead>
</table>
| **Multiplication** | If you multiply both sides of an inequality by the same negative number, you must reverse the inequality symbol for the statement to still be true. | \[ 8 > 4 \]
| | \[ 8(-2) < 4(-2) \] | \[ -16 < -8 \] | If \( a > b \) and \( c < 0 \), then \( ac < bc \). |
| **Division** | If you divide both sides of an inequality by the same negative number, you must reverse the inequality symbol for the statement to still be true. | \[ 12 > 4 \]
| | \[ \frac{12}{-4} < \frac{4}{-4} \] | \[ -3 < -1 \] | If \( a > b \) and \( c < 0 \), then \( \frac{a}{c} < \frac{b}{c} \). |

These properties are also true for inequalities that use the symbols \( <, \geq, \text{ and } \leq \).
**EXAMPLE 2**

*Multiplying or Dividing by a Negative Number*

Solve each inequality and graph the solutions.

**A** 

\[-8x > 72\]

\[-8x < 72\]

\[\frac{-8x}{-8} < \frac{72}{-8}\]

\[x < -9\]

*Since x is multiplied by -8, divide both sides by -8. Change > to <.*

**B**

\[-3 \leq \frac{x}{-5}\]

\[-5(-3) \geq -5\left(\frac{x}{-5}\right)\]

\[15 \geq x \text{ (or } x \leq 15)\]

*Since x is divided by -5, multiply both sides by -5. Change \leq to \geq.*

---

**EXAMPLE 3**

*Consumer Application*

Ryan has a $16 gift card for a health store where a smoothie costs $2.50 with tax. What are the possible numbers of smoothies that Ryan can buy?

Let \(s\) represent the number of smoothies Ryan can buy.

\[
\begin{align*}
\text{\$2.50} & \times \text{ number of smoothies} & \text{is at most} & \text{\$16.00.} \\
2.50 & \times s & \leq 16.00 \\
\end{align*}
\]

\[
\frac{2.50s}{2.50} \leq \frac{16.00}{2.50} \\
\]

*Since s is multiplied by 2.50, divide both sides by 2.50. The symbol does not change. Ryan can buy only a whole number of smoothies.*

\[s \leq 6.4\]

Ryan can buy 0, 1, 2, 3, 4, 5, or 6 smoothies.

**THINK AND DISCUSS**

1. Compare the Multiplication and Division Properties of Inequality and the Multiplication and Division Properties of Equality.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each cell, write and solve an inequality.

<table>
<thead>
<tr>
<th>Solving Inequalities by Using Multiplication and Division</th>
<th>By a Positive Number</th>
<th>By a Negative Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divide</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiply</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**CHECK IT OUT!**

2a. \(10 \geq -x\)  

2b. \(4.25 > -0.25h\)
GUIDED PRACTICE

Solve each inequality and graph the solutions.

1. $3b > 27$
2. $-40 \geq 8b$
3. $\frac{d}{3} > 6$
4. $24d \leq 6$
5. $1.1m \leq 1.21$
6. $\frac{2}{3}k > 6$
7. $9s > -18$
8. $\frac{4}{5} \geq \frac{r}{2}$

See Example 1

9. $-2x < -10$
10. $\frac{b}{2} \geq 8$
11. $-3.5n < 1.4$
12. $4 > -8g$

See Example 2

13. $\frac{d}{-6} < \frac{1}{2}$
14. $-10h \geq -6$
15. $12 > \frac{t}{-6}$
16. $-\frac{1}{2}m \geq -7$

See Example 3

17. Travel Tom saved $550 to go on a school trip. The cost for a hotel room, including tax, is $80 per night. What are the possible numbers of nights Tom can stay at the hotel?

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

18. $10 < 2t$
19. $\frac{1}{3}j \leq 4$
20. $-80 < 8c$
21. $21 > 3d$
22. $\frac{w}{4} \geq -2$
23. $\frac{h}{4} \leq \frac{2}{7}$
24. $6y < 4.2$
25. $12c \leq -144$
26. $\frac{4}{5}x \geq \frac{2}{5}$
27. $6b \geq \frac{3}{5}$
28. $-25 > 10p$
29. $\frac{b}{8} \leq -2$
30. $-9a > 81$
31. $\frac{1}{2} < \frac{r}{-3}$
32. $-6p > 0.6$
33. $\frac{y}{-4} > -\frac{1}{2}$
34. $-\frac{1}{6}f < 5$
35. $-2.25t < -9$
36. $24 \leq -10w$
37. $-11z > 121$
38. $\frac{3}{5} < \frac{f}{-5}$
39. $-k \geq 7$
40. $-2.2b < -7.7$
41. $16 \geq \frac{4}{3}p$
42. Camping The rope Roz brought with her camping gear is 54 inches long. Roz needs to cut shorter pieces of rope that are each 18 inches long. What are the possible number of pieces Roz can cut?

43. $-8x < 24$
44. $3t \leq 24$
45. $\frac{1}{4}x < 5$
46. $\frac{4}{5}p \geq -24$
47. $54 \leq -9p$
48. $3t > -\frac{1}{2}$
49. $-\frac{3}{4}b > -\frac{3}{2}$
50. $216 > 3.6r$

Write an inequality for each statement. Solve the inequality and graph the solutions.

51. The product of a number and 7 is not less than 21.
52. The quotient of $h$ and $-6$ is at least 5.
53. The product of $-\frac{4}{5}$ and $b$ is at most $-16$.
54. Ten is no more than the quotient of $t$ and 4.

55. Write About It Explain how you know whether to reverse the inequality symbol when solving an inequality.

56. Geometry The area of a rectangle is at most 21 square inches. The width of the rectangle is 3.5 inches. What are the possible measurements for the length of the rectangle?
Orangutans weigh about 3.5 pounds at birth. As adults, female orangutans can weigh as much as 110 pounds, and male orangutans can weigh up to 300 pounds.

Animals

Solve each inequality and match the solution to the correct graph.

57. \(-0.5t \geq 1.5\)  
   A.  
   \[\begin{array}{cccccccc}
   -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
   \end{array}\]

58. \(\frac{1}{9}t \leq -3\)  
   B.  
   \[\begin{array}{cccccccc}
   -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
   \end{array}\]

59. \(-13.5 \leq -4.5t\)  
   C.  
   \[\begin{array}{cccccccc}
   -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
   \end{array}\]

60. \(\frac{t}{-6} \leq -\frac{1}{2}\)  
   D.  
   \[\begin{array}{cccccccc}
   -45 & -36 & -27 & -18 & -9 & 0 & 9 \\
   \end{array}\]

61. **Animals** A wildlife shelter is home to birds, mammals, and reptiles. If cat chow is sold in 20 lb bags, what is the least number of bags of cat chow needed for one year at this shelter?

<table>
<thead>
<tr>
<th>Food Consumed at a Wildlife Shelter per Week</th>
<th>Amount of Food (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grapes</td>
<td>4</td>
</tr>
<tr>
<td>Mixed seed</td>
<td>10</td>
</tr>
<tr>
<td>Peanuts</td>
<td>5</td>
</tr>
<tr>
<td>Cat chow</td>
<td>10</td>
</tr>
<tr>
<td>Kitten chow</td>
<td>5</td>
</tr>
</tbody>
</table>

62. **Education** In order to earn an A in a college math class, a student must score no less than 90% of all possible points. One semester, a student with 567 points earned an A in the class. Write an inequality to show the numbers of points possible.

63. **Critical Thinking** Explain why you cannot solve an inequality by multiplying both sides by zero.

64. /// **ERROR ANALYSIS///** Two students have different answers for a homework problem. Which answer is incorrect? Explain the error.

A.  
   \[\begin{array}{cccccccc}
   9m & \geq & -27 \\
   \frac{9m}{9} & \geq & \frac{-27}{9} \\
   m & \leq & -3 \\
   \end{array}\]

B.  
   \[\begin{array}{cccccccc}
   9m & \geq & -27 \\
   \frac{9m}{9} & \geq & \frac{-27}{9} \\
   m & \geq & -3 \\
   \end{array}\]

65. Jan has a budget of $800 for catering. The catering company charges $12.50 per guest. Write and solve an inequality to show the numbers of guests Jan can invite.

66. a. The Swimming Club can spend a total of $250 for hotel rooms for its spring trip. One hotel costs $75 per night. Write an inequality to find the number of rooms the club can reserve at this hotel. Let \(n\) be the number of rooms.

b. Solve the inequality you wrote in part a. Graph the solutions on a number line. Make sure your answer is reasonable.

c. Another hotel offers a rate of $65 per night. Does this allow the club to reserve more rooms? Explain your reasoning.

100  Chapter 3  Inequalities
67. Which inequality does NOT have the same solutions as \(- \frac{2}{3}y > 4\)?

(A) \(12 < -2y\)  
(B) \(\frac{y}{2} < -12\)  
(C) \(-\frac{3}{4}y > \frac{9}{2}\)  
(D) \(-3y > 18\)

68. The solutions of which inequality are NOT represented by the following graph?

(F) \(\frac{x}{2} \geq -2\)  
(G) \(-5x \geq 20\)  
(H) \(3x \geq -12\)  
(I) \(-7x \leq 28\)

69. Which inequality can be used to find the number of 39-cent stamps you can purchase for $4.00?

(A) \(0.39s \geq 4.00\)  
(B) \(0.39s \leq 4.00\)  
(C) \(\frac{s}{0.39} \leq 4.00\)  
(D) \(\frac{4.00}{0.39} \leq s\)

70. Short Response Write three different inequalities that have the same solutions as \(x > 4\). Show your work and explain each step.

CHALLENGE AND EXTEND

Solve each inequality.

71. \(2 \frac{1}{3} \leq -\frac{5}{6}g\)  
72. \(\frac{2x}{3} < 8.25\)  
73. \(2 \frac{5}{8}m > \frac{7}{10}\)  
74. \(3 \frac{3}{5}f \geq 14 \frac{2}{5}\)

75. Estimation What is the greatest possible integer solution of the inequality \(3.806x < 19.902\)?

76. Critical Thinking The Transitive Property of Equality states that if \(a = b\) and \(b = c\), then \(a = c\). Is there a Transitive Property of Inequality using the symbol \(<\)? Give an example to support your answer.

77. Critical Thinking The Symmetric Property of Equality states that if \(a = b\), then \(b = a\). Is there a Symmetric Property of Inequality? Give an example to support your answer.
Objective
Solve inequalities that contain more than one operation.

Who uses this?
Contestants at a county fair can solve an inequality to find how many pounds a prize-winning pumpkin must weigh. (See Example 3.)

At the county fair, contestants can enter contests that judge animals, recipes, crops, art projects, and more. Sometimes an average score or average weight is used to determine the winner of the blue ribbon. A contestant can use a multi-step inequality to determine what score or weight is needed in order to win.

Inequalities that contain more than one operation require more than one step to solve. Use inverse operations to undo the operations in the inequality one at a time.

**Example 1**

**Solving Multi-Step Inequalities**

Solve each inequality and graph the solutions.

**A**

\[ 160 + 4f \leq 500 \]

\[
\begin{align*}
160 + 4f & \leq 500 \\
160 & \quad -160 \\
4f & \quad \leq 340 \\
\frac{4f}{4} & \quad \leq \frac{340}{4} \\
f & \quad \leq 85
\end{align*}
\]

**B**

\[ 7 - 2t \leq 21 \]

\[
\begin{align*}
7 - 2t & \leq 21 \\
7 & \quad -7 \\
-2t & \quad \leq 14 \\
\frac{-2t}{-2} & \quad \geq \frac{14}{-2} \\
t & \quad \geq -7
\end{align*}
\]

**Check It Out!**

Solve each inequality and graph the solutions.

1a. \( -12 \geq 3x + 6 \)

1b. \( \frac{x + 5}{-2} > 3 \)

1c. \( \frac{1 - 2n}{3} \geq 7 \)
To solve more complicated inequalities, you may first need to simplify the expressions on one or both sides by using the order of operations, combining like terms, or using the Distributive Property.

**EXAMPLE 2**

**Simplifying Before Solving Inequalities**

Solve each inequality and graph the solutions.

**A**

\[-4 + (-8) < -5c - 2\]

Combine like terms. Since 2 is subtracted from \(-5c\), add 2 to both sides to undo the subtraction.

\[-10 < -5c\]

Since \(c\) is multiplied by \(-5\), divide both sides by \(-5\) to undo the multiplication. Change < to >.

\[2 > c\] (or \(c < 2\))

**B**

\[-3(3 - x) < 4^2\]

Distribute \(-3\) on the left side.

\[-9 + 3x < 16\]

Simplify the right side.

\[3x < 25\]

Since \(-9\) is added to \(3x\), add 9 to both sides to undo the addition.

\[x < 8 \frac{1}{3}\]

**C**

\[\frac{4}{5}x + \frac{1}{2} > \frac{3}{5}\]

Multiply both sides by 10, the LCD of the fractions.

\[8x + 5 > 6\]

Since 5 is added to \(8x\), subtract 5 from both sides to undo the addition.

\[x > \frac{1}{8}\]

Solve each inequality and graph the solutions.

**2a.** \(2m + 5 > 5^2\)

**2b.** \(3 + 2(x + 4) > 3\)

**2c.** \(\frac{5}{8} < \frac{3}{8}x - \frac{1}{4}\)
**Example 3**

**Gardening Application**

To win the blue ribbon for the Heaviest Pumpkin Crop at the county fair, the average weight of John's two pumpkins must be greater than 819 lb. One of his pumpkins weighs 887 lb. What is the least number of pounds the second pumpkin could weigh in order for John to win the blue ribbon?

Let $p$ represent the weight of the second pumpkin. The average weight of the pumpkins is the sum of each weight divided by 2.

\[
\frac{887 + p}{2} > 819
\]

Since 887 + $p$ is divided by 2, multiply both sides by 2 to undo the division.

\[
2 \left( \frac{887 + p}{2} \right) > 2(819)
\]

\[
887 + p > 1638
\]

Since 887 is added to $p$, subtract 887 from both sides to undo the addition.

\[
\frac{887 - 887}{p} > \frac{1638 - 887}{751}
\]

\[
p > 751
\]

The second pumpkin must weigh more than 751 pounds.

**Check**

Check the endpoint, 751. Check a number greater than 751.

<table>
<thead>
<tr>
<th>Check the endpoint, 751.</th>
<th>Check a number greater than 751.</th>
</tr>
</thead>
</table>
| \[
\frac{887 + p}{2} = 819
\] | \[
\frac{887 + p}{2} > 819
\] |
| \[
\frac{887 + 751}{2} = 819
\] | \[
\frac{887 + 755}{2} > 819
\] |
| \[
\frac{1638}{2} = 819
\] | \[
\frac{1642}{2} > 819
\] |
| 819 819 ✓ | 821 > 819 ✓ |

**Check it Out!**

3. The average of Jim's two test scores must be at least 90 to make an A in the class. Jim got a 95 on his first test. What scores can Jim get on his second test to make an A in the class?

---

**Think and Discuss**

1. The inequality $v \geq 25$ states that 25 is the ___?___. (value of $v$, minimum value of $v$, or maximum value of $v$)

2. Describe two sets of steps for solving the inequality $\frac{x + 5}{3} > 7$.

3. **Get Organized** Copy and complete the graphic organizer.

**Know It! Note**

Solving Multi-Step Equations and Inequalities

- How are they alike?
- How are they different?
GUIDED PRACTICE

Solve each inequality and graph the solutions.

1. $2m + 1 > 13$
2. $2d + 21 \leq 11$
3. $6 \leq -2x + 2$
4. $4c - 7 > 5$
5. $\frac{4 + x}{3} > -4$
6. $1 < 0.2x - 0.7$
7. $\frac{3 - 2x}{3} \leq 7$
8. $2x + 5 \geq 2$

9. $4(x + 2) > 6$
10. $\frac{1}{4}x + \frac{2}{3} < \frac{3}{4}$
11. $4 - x + 6^2 \geq 21$
12. $4 - x > 3(4 - 2)$
13. $0.2(x - 10) > -1.8$
14. $3(j + 41) \leq 35$

15. Business A sales representative is given a choice of two paycheck plans. One choice includes a monthly base pay of $300 plus 10% commission on his sales. The second choice is a monthly salary of $1200. For what amount of sales would the representative make more money with the first plan?

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

16. $4r - 9 > 7$
17. $3 \leq 5 - 2x$
18. $\frac{w + 3}{2} > 6$
19. $11w + 99 < 77$
20. $9 \geq \frac{1}{2}v + 3$
21. $-4x - 8 > 16$
22. $8 - \frac{2}{3}z \leq 2$
23. $f + 2 \frac{1}{2} < -2$
24. $\frac{3n - 8}{5} \geq 2$
25. $-5 > -5 - 3w$
26. $10 > \frac{5 - 3p}{2}$
27. $2v + 1 > 2 \frac{1}{3}$
28. $4(x + 3) > -24$
29. $4 > x - 3(x + 2)$
30. $-18 \geq 33 - 3h$
31. $-2 > 7x - 2(x - 4)$
32. $9 - (9)^2 > 10x - x$
33. $2a - (-3)^2 \geq 13$
34. $6 - \frac{x}{3} + 1 > \frac{2}{3}$
35. $12(x - 3) + 2x > 6$
36. $15 \geq 19 + 2(q - 18)$
37. Communications One cell phone company offers a plan that costs $29.99 and includes unlimited night and weekend minutes. Another company offers a plan that costs $19.99 and charges $0.35 per minute during nights and weekends. For what numbers of night and weekend minutes does the second company’s plan cost more than the first company’s plan?

Solve each inequality and graph the solutions.

38. $-12 > -4x - 8$
39. $5x + 4 \leq 14$
40. $\frac{2}{3}x - 5 > 7$
41. $x - 3x > 2 - 10$
42. $5 - x - 2 > 3$
43. $3 < 2x - 5(x + 3)$
44. $\frac{1}{6} - \frac{2}{3}m \geq \frac{1}{4}$
45. $4 - (r - 2) > 3 - 5$
46. $0.3 - 0.5n + 1 \geq 0.4$
47. $6^2 > 4(x + 2)$
48. $-4 - 2n + 4n > 7 - 2^2$
49. $\frac{1}{4}(p - 10) \geq 6 - 4$
50. Use the inequality $-4t - 8 \leq 12$ to fill in the missing numbers.
   a. $t \geq$ 4
   b. $t + 4 \geq$ 16
   c. $t -$ 1 \geq 0
   d. $t + 10 \geq$ 4
   e. $3t \geq$ 12
   f. $t \geq$ -5
Write an inequality for each statement. Solve the inequality and graph the solutions.

51. One-half of a number, increased by 9, is less than 33.

52. Six is less than or equal to the sum of 4 and \(-2x\).

53. The product of 4 and the sum of a number and 12 is at most 16.

54. The sum of half a number and two-thirds of the number is less than 14.

Solve each inequality and match the solution to the correct graph.

55. \(4x - 9 \geq 7\) A. 

56. \(-6 \geq 3(x - 2)\) B. 

57. \(-2x - 6 \geq -4 + 2\) C. 

58. \(\frac{1}{2} - \frac{1}{3}x \leq \left(\frac{2}{3} + \frac{1}{3}\right)^2\) D. 

59. **Entertainment** A digital video recorder (DVR) records television shows on an internal hard drive. To use a DVR, you need a subscription with a DVR service company. Two companies advertise their charges for a DVR machine and subscription service.

For what numbers of months will a consumer pay less for the machine and subscription at Easy Electronics than at Cable Solutions?

60. **Geometry** The area of the triangle shown is less than 55 square inches.
   a. Write an inequality that can be used to find \(x\).
   b. Solve the inequality you wrote in part a.
   c. What is the maximum height of the triangle?

61. a. A band wants to create a CD of their last concert. They received a donation of $500 to cover the cost. The total cost is $350 plus $3 per CD. Complete the table to find a relationship between the number of CDs and the total cost.

<table>
<thead>
<tr>
<th>Number</th>
<th>Process</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>350 + 3</td>
<td>353</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Write an equation for the cost \(c\) of the CDs based on the number of CDs \(n\).

c. Write an inequality that can be used to determine how many CDs can be made with the $500 donation. Solve the inequality and determine how many CDs the band can have made from the $500 donation.
62. **Critical Thinking** What is the least whole number that is a solution of

\[ 4r - 4.9 > 14.95 \] ?

63. **Write About It** Describe two sets of steps to solve \(2(x + 3) > 10\).

---

64. **Test Prep** What are the solutions of \(3y > 2x + 4\) when \(y = 6\)?

- A. \(7 > x\)
- B. \(x > 7\)
- C. \(x > 11\)
- D. \(11 > x\)

65. Cecilia has $30 to spend at a carnival. Admission costs $5.00, lunch will cost $6.00, and each ride ticket costs $1.25. Which inequality represents the number of ride tickets \(x\) that Cecilia can buy?

- F. \(30 - (5 - 6) + 1.25x \leq 30\)
- G. \(5 + 6 + 1.25x \leq 30\)
- H. \(30 - (5 + 6) \leq 1.25x\)
- J. \(30 + 1.25x \leq 5 + 6\)

66. Which statement is modeled by \(2p + 5 < 11\)?

- A. The sum of 5 and 2 times \(p\) is at least 11.
- B. Five added to the product of 2 and \(p\) is less than 11.
- C. Two times \(p\) plus 5 is at most 11.
- D. The product of 2 and \(p\) added to 5 is 11.

67. **Gridded Response** A basketball team scored 8 points more in its second game than in its first. In its third game, the team scored 42 points. The total number of points scored in the three games was more than 150. What is the least number of points the team might have scored in its second game?

---

**CHALLENGE AND EXTEND**

Solve each inequality and graph the solutions.

68. \(3(x + 2) - 6x + 6 \leq 0\)  
69. \(-18 > -(2x + 9) - 4 + x\)  
70. \(\frac{2 + x}{2} - (x - 1) > 1\)

Write an inequality for each statement. Graph the solutions.

71. \(x\) is a positive number.  
72. \(x\) is a negative number.

73. \(x\) is a nonnegative number.  
74. \(x\) is not a positive number.

75. \(x\) times negative 3 is positive.  
76. The opposite of \(x\) is greater than 2.
Solving Inequalities with Variables on Both Sides

Objective
Solve inequalities that contain variable terms on both sides.

Who uses this?
Business owners can use inequalities to find the most cost-efficient services. (See Example 2.)

Some inequalities have variable terms on both sides of the inequality symbol. You can solve these inequalities like you solved equations with variables on both sides.

Use the properties of inequality to “collect” all the variable terms on one side and all the constant terms on the other side.

Example 1
Solving Inequalities with Variables on Both Sides

Solve each inequality and graph the solutions.

A  \( x < 3x + 8 \)

\[
\begin{align*}
    x &< 3x + 8 \\
    -x &< -2x + 8 \\
    0 &< 2x + 8 \\
    -8 &< 2x \\
    -4 &< x \quad \text{(or } x > -4) \\
\end{align*}
\]

B  \( 6x - 1 \leq 3.5x + 4 \)

\[
\begin{align*}
    6x - 1 &\leq 3.5x + 4 \\
    -6x &\leq -2.5x + 4 \\
    -1 &\leq -2.5x \\
    -0.4 &\leq -x \\
    -2.5 &\geq x \\
\end{align*}
\]

Check It Out!
Solve each inequality and graph the solutions.

1a.  \( 4x \geq 7x + 6 \)

1b.  \( 5t + 1 < -2t - 6 \)
**Example 2**  

**Business Application**  

The *Daily Info* charges a fee of $650 plus $80 per week to run an ad. The *People’s Paper* charges $145 per week. For how many weeks will the total cost at *Daily Info* be less expensive than the cost at *People’s Paper*?  

Let \( w \) be the number of weeks the ad runs in the paper.  

Let us be the number of weeks the ad runs in the paper.  

<table>
<thead>
<tr>
<th>Daily Info fee</th>
<th>plus</th>
<th>$80 per week</th>
<th>times</th>
<th>number of weeks</th>
<th>is less expensive than</th>
<th>People’s Paper charge per week</th>
<th>times</th>
<th>number of weeks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$650</td>
<td></td>
<td>$80</td>
<td>( w )</td>
<td>$145 \times )</td>
<td>$80 \times )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
650 + 80w < 145 \times w
\]

\[
\begin{align*}
650 & + 80w < 145w \\
-80w & -80w \\
650 & < 65w \\
\frac{650}{65} & < \frac{65w}{65} \\
10 & < w
\end{align*}
\]

The total cost at *Daily Info* is less than the cost at *People’s Paper* if the ad runs for more than 10 weeks.  

**Check it Out!**  

2. A-Plus Advertising charges a fee of $24 plus $0.10 per flyer to print and deliver flyers. Print and More charges $0.25 per flyer. For how many flyers is the cost at A-Plus Advertising less than the cost at Print and More?  

You may need to simplify one or both sides of an inequality before solving it. Look for like terms to combine and places to use Distributive Property.  

**Example 3**  

**Simplifying Each Side Before Solving**  

Solve each inequality and graph the solutions.  

**A** \( 6(1 - x) < 3x \)  

\[
6(1 - x) < 3x
\]

\[
\begin{align*}
6(1) & - 6x < 3x \\
6 & - 6x < 3x \\
+ 6x & + 6x \\
6 & < 9x \\
\frac{6}{9} & < \frac{9x}{9} \\
\frac{2}{3} & < x
\end{align*}
\]

Distribute 6 on the left side of the inequality.  

Add 6x to both sides so that the coefficient of \( x \) is positive.  

Since \( x \) is multiplied by 9, divide both sides by 9 to undo the multiplication.  

---

3-5 Solving Inequalities with Variables on Both Sides  

109
Solve each inequality and graph the solutions.

**B** 1.6x ≤ −0.2x + 0.9

\[
1.6x \leq -0.2x + 0.9 \\
+ 0.2x \quad + 0.2x \\
1.8x \leq 0.9 \\
1.8x \leq \frac{0.9}{1.8} \\
x \leq \frac{1}{2}
\]

**Solution:** Since −0.2x is added to 0.9, subtract −0.2x from both sides. Subtracting −0.2x is the same as adding 0.2x. Since x is multiplied by 1.8, divide both sides by 1.8 to undo the multiplication.

---

Solve each inequality and graph the solutions. Check your answer.

3a. 5(2 − r) ≥ 3(r − 2)  
3b. 0.5x − 0.3 + 1.9x < 0.3x + 6

Some inequalities are true no matter what value is substituted for the variable. For these inequalities, all real numbers are solutions.

Some inequalities are false no matter what value is substituted for the variable. These inequalities have no solutions.

If both sides of an inequality are fully simplified and the same variable term appears on both sides, then the inequality has all real numbers as solutions or it has no solutions. Look at the other terms in the inequality to decide which is the case.

---

**Example 4**

**All Real Numbers as Solutions or No Solutions**

Solve each inequality.

**A** x + 5 ≥ x + 3

\[
x + 5 \geq x + 3
\]

The same variable term (x) appears on both sides. Look at the other terms.

For any number x, adding 5 will always result in a greater number than adding 3.

All values of x make the inequality true.

All real numbers are solutions.

**B** 2(x + 3) < 5 + 2x

\[
2x + 6 < 5 + 2x \\
\text{Distribute 2 on the left side.}
\]

The same variable term (2x) appears on both sides. Look at the other terms.

For any number 2x, adding 6 will never result in a lesser number than adding 5.

No values of x make the inequality true.

There are no solutions.

---

**Check It Out!**

Solve each inequality.

4a. 4(y − 1) ≥ 4y + 2  
4b. x − 2 < x + 1
THINK AND DISCUSS

1. Explain how you would collect the variable terms to solve the inequality
   \[ 5c - 4 > 8c + 2. \]

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example of an inequality of the indicated type.

GUIDED PRACTICE

Solve each inequality and graph the solutions.

1. \[ 2x > 4x - 6 \]
2. \[ 7y + 1 \leq y - 5 \]
3. \[ 27x + 33 > 58x - 29 \]
4. \[ -3r < 10 - r \]
5. \[ 5c - 4 > 8c + 2 \]
6. \[ 4.5x - 3.8 \geq 1.5x - 2.3 \]

7. **School** The school band will sell pizzas to raise money for new uniforms. The supplier charges $100 plus $4 per pizza. If the band members sell the pizzas for $7 each, how many pizzas will they have to sell to make a profit?

Solve each inequality and graph the solutions.

8. \[ 5(4 + x) \leq 3(2 + x) \]
9. \[ -4(3 - p) > 5(p + 1) \]
10. \[ 2(6 - x) < 4x \]
11. \[ 4x > 3(7 - x) \]
12. \[ \frac{1}{2}f + \frac{3}{4} \geq \frac{1}{4}f \]

Solve each inequality.

13. \[ 7 + 4b \geq 3b \]
14. \[ 2x - 2 \leq -2(1 - x) \]
15. \[ 4(y + 1) < 4y + 2 \]
16. \[ 4v + 1 < 4v - 7 \]
17. \[ b - 4 \geq b - 6 \]
18. \[ 3(x - 5) > 3x \]
19. \[ 2k + 7 \geq 2(k + 14) \]

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

20. \[ 3x \leq 5x + 8 \]
21. \[ 9y + 3 > 4y - 7 \]
22. \[ 1.5x - 1.2 < 3.1x - 2.8 \]
23. \[ 7 + 4b \geq 3b \]
24. \[ 7 - 5t < 4t - 2 \]
25. \[ 2.8m - 5.2 > 0.8m + 4.8 \]

26. **Geometry** For what values of \( x \) is the area of the rectangle greater than the area of the triangle?

```
   12
  
  x + 2

   10
  
  x + 16
```
Solve each inequality and graph the solutions.

27. $4(2 - x) \leq 5(x - 2)$
28. $-3(n + 4) < 6(1 - n)$
29. $9(w + 2) \leq 12w$
30. $4.5 + 1.3t > 3.8t - 3$
31. $\frac{1}{2}r + \frac{2}{3} \geq \frac{1}{3}r$
32. $2(4 - n) < 3n - 7$

Solve each inequality.

33. $3(2 - x) < -3(x - 1)$
34. $7 - y > 5 - y$
35. $3(10 + z) \leq 3z + 36$
36. $-5(k - 1) \geq 5(2 - k)$
37. $4(x - 1) \leq 4x$
38. $3(v - 9) \geq 15 + 3v$

Solve each inequality and graph the solutions.

39. $3t - 12 > 5t + 2$
40. $-5(y + 3) - 6 < y + 3$
41. $3x + 9 - 5x < x$
42. $18 + 9p > 12p - 31$
43. $2(x - 5) < -3x$
44. $-\frac{2}{5}x \leq \frac{4}{5} - \frac{3}{5}x$
45. $-2(x - 7) - 4 - x < 8x + 32$
46. $-3(2r - 4) \geq 2(5 - 3r)$
47. $-7x - 10 + 5x \geq 3(x + 4) + 8$
48. $-\frac{1}{3}(n + 8) + \frac{1}{3}n \leq 1 - n$

49. Recreation A red kite is 100 feet off the ground and is rising at 8 feet per second. A blue kite is 180 feet off the ground and is rising at 5 feet per second. How long will it take for the red kite to be higher than the blue kite? Round your answer to the nearest second.

50. Education The table shows the enrollment in Howard High School and Phillips High School for three school years.

<table>
<thead>
<tr>
<th>School Enrollment</th>
<th>Year 1</th>
<th>Year 2</th>
<th>Year 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Howard High School</td>
<td>1192</td>
<td>1188</td>
<td>1184</td>
</tr>
<tr>
<td>Phillips High School</td>
<td>921</td>
<td>941</td>
<td>961</td>
</tr>
</tbody>
</table>

a. How much did the enrollment change each year at Howard?
b. Use the enrollment in year 1 and your answer from part a to write an expression for the enrollment at Howard in any year $x$.
c. How much did the enrollment change each year at Phillips?
d. Use the enrollment in year 1 and your answer from part c to write an expression for the enrollment at Phillips in any year $x$.
e. Assume that the pattern in the table continues. Use your expressions from parts b and d to write an inequality that can be solved to find the year in which the enrollment at Phillips High School will be greater than the enrollment at Howard High School. Solve your inequality and graph the solutions.

51. a. The school orchestra is creating a CD of their last concert. The total cost is $400 + 4.50 per CD. Write an expression for the cost of creating the CDs based on the number of CDs $n$.
b. The orchestra plans to sell the CDs for $12. Write an expression for the amount the orchestra earns from the sale of $n$ CDs.
c. In order for the orchestra to make a profit, the amount they make selling the CDs must be greater than the cost of creating the CDs. Write an inequality that can be solved to find the number of CDs the orchestra must sell in order to make a profit. Solve your inequality.
Write an inequality to represent each relationship. Solve your inequality.

52. Four more than twice a number is greater than two-thirds of the number.

53. Ten less than five times a number is less than six times the number decreased by eight.

54. The sum of a number and twenty is less than four times the number decreased by one.

55. Three-fourths of a number is greater than or equal to five less than the number.

56. Entertainment Use the table to determine how many movies you would have to rent for Video View to be less expensive than Movie Place.

<table>
<thead>
<tr>
<th>Membership Fee ($)</th>
<th>Cost per Rental ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Movie Place</td>
<td>None</td>
</tr>
<tr>
<td>Video View</td>
<td>19.99</td>
</tr>
</tbody>
</table>

57. Geometry In an acute triangle, all angles measure less than 90°. Also, the sum of the measures of any two angles is greater than the measure of the third angle. Can the measures of an acute triangle be \( x \), \( x - 1 \), and \( 2x \)? Explain.

58. Write About It Compare the steps you would follow to solve an inequality to the steps you would follow to solve an equation.

59. Critical Thinking How can you tell just by looking at the inequality \( x > x + 1 \) that it has no solutions?

60. ERROR ANALYSIS Two students solved the inequality \( 5x < 3 - 4x \). Which is incorrect? Explain the error.

61. If \( a - b > a + b \), which statement is true?
   
   A. The value of \( a \) is positive.  
   B. The value of \( b \) is positive.  
   C. The value of \( a \) is negative.  
   D. The value of \( b \) is negative.

62. If \( -a < b \), which statement is always true?
   
   F. \( a < b \)  
   G. \( a > b \)  
   H. \( a < -b \)  
   J. \( a > -b \)

63. Which is a solution of the inequality \( 7(2 - x) > 4(x - 2) \)?
   
   A. \( -2 \)  
   B. \( 2 \)  
   C. \( 4 \)  
   D. \( 7 \)

64. Which is the graph of \( -5x < -2x - 6 \)?
   
   F.  
   G.  
   H.  
   J.
65. **Short Response** Write a real-world situation that could be modeled by the inequality \(7x + 4 > 4x + 13\). Explain how the inequality relates to your situation.

**Challenge and Extend**

Solve each inequality.

66. \(\frac{2}{2} + 2x \geq \frac{5}{2} + 2 \frac{1}{2}x\)

67. \(1.6x - 20.7 > 6.3x - (-2.2x)\)

68. \(1.3x - 7.5x < 8.5x - 29.4\)

69. \(-4w + \frac{-8 - 37}{9} \leq \frac{75 - 3}{9} + 3w\)

70. Replace the square and circle with numbers so that the inequality has all real numbers as solutions. \(□ - 2x < □ - 2x\)

71. Replace the square and circle with numbers so that the inequality has no solutions. \(□ - 2x < □ - 2x\)

72. **Critical Thinking** Explain whether there are any numbers that can replace the square and circle so that the inequality has all real numbers as solutions. \(□ + 2x < □ + x\)

---

**Career Path**

**Q:** What math classes did you take in high school?
**A:** Algebra 1, Geometry, and Algebra 2

**Q:** What math classes have you taken since high school?
**A:** I have taken a basic accounting class and a business math class.

**Q:** How do you use math?
**A:** I use math to estimate how much food I need to buy. I also use math when adjusting recipe amounts to feed large groups of people.

**Q:** What are your future plans?
**A:** I plan to start my own catering business. The math classes I took will help me manage the financial aspects of my business.
Use with Solving Compound Inequalities

Truth Tables and Compound Statements

A compound statement is formed by combining two or more simple statements. A compound statement is either true or false depending on whether its simple statements are true or false.

Activity 1

- Let $P$ be “Cindy is at least 17 years old.”
- Let $Q$ be “Cindy has a driver’s license.”

<table>
<thead>
<tr>
<th>If...</th>
<th>then $P$ is</th>
<th>and $Q$ is</th>
<th>so $P$ AND $Q$ is</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cindy is 18 years old. Cindy has a driver’s license.</td>
<td>True</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>Cindy is 17 years old. Cindy does not have a driver’s license.</td>
<td>True</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>Cindy is 16 years old. Cindy has a driver’s license.</td>
<td>False</td>
<td>True</td>
<td>False</td>
</tr>
<tr>
<td>Cindy is 15 years old. Cindy does not have a driver’s license.</td>
<td>False</td>
<td>False</td>
<td>False</td>
</tr>
</tbody>
</table>

$P$ AND $Q$ is true when __________ ? __________.

Try This

For each pair of simple statements, tell whether $P$ AND $Q$ is true or false.

1. $P$: Many birds can fly; $Q$: A zebra is an animal.

Activity 2

- Let $P$ be “Paul plays tennis.”
- Let $Q$ be “Paul has brown eyes.”

<table>
<thead>
<tr>
<th>If...</th>
<th>then $P$ is</th>
<th>and $Q$ is</th>
<th>so $P$ OR $Q$ is</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paul plays tennis. Paul has brown eyes.</td>
<td>True</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>Paul plays tennis. Paul has green eyes.</td>
<td>True</td>
<td>False</td>
<td>True</td>
</tr>
<tr>
<td>Paul does not play tennis. Paul has brown eyes.</td>
<td>False</td>
<td>True</td>
<td>True</td>
</tr>
<tr>
<td>Paul does not play tennis. Paul has green eyes.</td>
<td>False</td>
<td>False</td>
<td>False</td>
</tr>
</tbody>
</table>

$P$ OR $Q$ is true when __________ ? __________.

Try This

For each pair of simple statements, tell whether $P$ OR $Q$ is true or false.

2. $P$: The number 12 is even; $Q$: The number 12 is a composite number.
Solving Compound Inequalities

Objectives
Solve compound inequalities in one variable.
Graph solution sets of compound inequalities in one variable.

Vocabulary
compound inequality
intersection
union

Who uses this?
A lifeguard can use compound inequalities to describe the safe pH levels in a swimming pool. (See Example 1.)

The inequalities you have seen so far are simple inequalities. When two simple inequalities are combined into one statement by the words AND or OR, the result is called a compound inequality.

<table>
<thead>
<tr>
<th>WORDS</th>
<th>ALGEBRA</th>
<th>GRAPH</th>
</tr>
</thead>
<tbody>
<tr>
<td>All real numbers greater than 2 AND less than 6</td>
<td>(x &gt; 2) AND (x &lt; 6)</td>
<td>(2 &lt; x &lt; 6)</td>
</tr>
<tr>
<td>All real numbers greater than or equal to 2 AND less than or equal to 6</td>
<td>(x \geq 2) AND (x \leq 6)</td>
<td>(2 \leq x \leq 6)</td>
</tr>
<tr>
<td>All real numbers less than 2 OR greater than 6</td>
<td>(x &lt; 2) OR (x &gt; 6)</td>
<td></td>
</tr>
<tr>
<td>All real numbers less than or equal to 2 OR greater than or equal to 6</td>
<td>(x \leq 2) OR (x \geq 6)</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1**

Chemistry Application

A water analyst recommends that the pH level of swimming pool water be between 7.2 and 7.6 inclusive. Write a compound inequality to show the pH levels that are within the recommended range. Graph the solutions.

Let \(p\) be the pH level of swimming pool water.

<table>
<thead>
<tr>
<th>pH level</th>
<th>is less than or equal to</th>
<th>7.2</th>
</tr>
</thead>
</table>

\[7.2 \leq p \leq 7.6\]
The free chlorine level in a pool should be between 1.0 and 3.0 parts per million inclusive. Write a compound inequality to show the levels that are within this range. Graph the solutions.

In this diagram, oval $A$ represents some integer solutions of $x < 10$, and oval $B$ represents some integer solutions of $x > 0$. The overlapping region represents numbers that belong in both ovals. Those numbers are solutions of both $x < 10$ and $x > 0$.

You can graph the solutions of a compound inequality involving AND by using the idea of an overlapping region. The overlapping region is called the intersection and shows the numbers that are solutions of both inequalities.

**Example 2**

Solving Compound Inequalities Involving AND

Solve each compound inequality and graph the solutions.

**A**

$4 \leq x + 2 \leq 8$

Write the compound inequality using AND.

$-2 \leq x \leq 6$

Solve each simple inequality.

Graph $2 \leq x$.

Graph $x \leq 6$.

Graph the intersection by finding where the two graphs overlap.

**B**

$-5 \leq 2x + 3 < 9$

Since 3 is added to $2x$, subtract 3 from each part of the inequality.

$-8 \leq 2x < 6$

Since $x$ is multiplied by 2, divide each part of the inequality by 2.

$-4 \leq x < 3$

Graph $-4 \leq x$.

Graph $x < 3$.

Graph the intersection by finding where the two graphs overlap.

**Check It Out!**

Solve each compound inequality and graph the solutions.

2a. $-9 < x - 10 < -5$

2b. $-4 \leq 3n + 5 < 11$
In this diagram, circle $A$ represents some integer solutions of $x < 0$, and circle $B$ represents some integer solutions of $x > 10$. The combined shaded regions represent numbers that are solutions of either $x < 0$ or $x > 10$.

You can graph the solutions of a compound inequality involving OR by using the idea of combining regions. The combined regions are called the **union** and show the numbers that are solutions of either inequality.

### Example 3

**Solving Compound Inequalities Involving OR**

Solve each compound inequality and graph the solutions.

**A**

\[-4 + a > 1 \text{ OR } -4 + a < -3\]

\[-4 + a > 1 \text{ OR } -4 + a < -3\]

\[
\begin{align*}
+4 & \quad +4 \\
\hline
a > & \quad 5 \text{ OR } a < \quad 1
\end{align*}
\]

Solve each simple inequality.

Graph $a > 5$.

Graph $a < 1$.

Graph the union by combining the regions.

**B**

\[2x \leq 6 \text{ OR } 3x > 12\]

\[2x \leq 6 \text{ OR } 3x > 12\]

\[
\begin{align*}
\frac{2x}{2} & \quad \frac{3x}{3} \\
\hline
x \leq & \quad 3 \text{ OR } x > \quad 4
\end{align*}
\]

Solve each simple inequality.

Graph $x \leq 3$.

Graph $x > 4$.

Graph the union by combining the regions.

### Check It Out!

Solve each compound inequality and graph the solutions.

**3a.** $2 + r < 12 \text{ OR } r + 5 > 19$

**3b.** $7x \geq 21 \text{ OR } 2x < -2$

Every solution of a compound inequality involving AND must be a solution of both parts of the compound inequality. If no numbers are solutions of *both* simple inequalities, then the compound inequality has no solutions.

The solutions of a compound inequality involving OR are not always two separate sets of numbers. There may be numbers that are solutions of both parts of the compound inequality.
EXAMPLE 4  Writing a Compound Inequality from a Graph

Write the compound inequality shown by each graph.

A

-2 -1 0 1 2 3 4 5 6 7 8

The shaded portion of the graph is not between two values, so the compound inequality involves OR.

On the left, the graph shows an arrow pointing left, so use either < or \( \leq \).
The solid circle at -1 means -1 is a solution, so use \( \leq \).
\( x \leq -1 \)

On the right, the graph shows an arrow pointing right, so use either > or \( \geq \).
The solid circle at 7 means 7 is a solution, so use \( \geq \).
\( x \geq 7 \)

The compound inequality is \( x \leq -1 \) OR \( x \geq 7 \).

B

-1 0 1 2 3 4 5 6 7 8

The shaded portion of the graph is between the values 0 and 6, so the compound inequality involves AND.

The shaded values are to the right of 0, so use > or \( \geq \).
The solid circle at 0 means 0 is a solution, so use \( \geq \).
\( x \geq 0 \)

The shaded values are to the left of 6, so use < or \( \leq \).
The empty circle at 6 means 6 is not a solution, so use <.
\( x < 6 \)

The compound inequality is \( x \geq 0 \) AND \( x < 6 \).

The compound inequality in Example 48 can also be written with the variable between the two endpoints.

\( 0 \leq x < 6 \)

THINK AND DISCUSS

1. Describe how to write the compound inequality \( y > 4 \) AND \( y \leq 12 \) without using the joining word AND.

2. GET ORGANIZED  Copy and complete the graphic organizers. Write three solutions in each of the three sections of the diagram. Then write each of your nine solutions in the appropriate column or columns of the table.

\[
\begin{array}{c|c|c}
\text{A} & \text{B} \\
\hline
x > 5 & x < 10 \\
\hline
x > 5 \text{ AND } x < 10 & x > 5 \text{ OR } x < 10 \\
\end{array}
\]

3-6 Solving Compound Inequalities 119
GUIDED PRACTICE

1. **Vocabulary** The graph of a(n) ___ ? ___ shows all values that are solutions to both simple inequalities that make a compound inequality. (union or intersection)

2. **Biology** An iguana needs to live in a warm environment. The temperature in a pet iguana's cage should be between 70 °F and 95 °F inclusive. Write a compound inequality to show the temperatures that are within the recommended range. Graph the solutions.

Solve each compound inequality and graph the solutions.

3. \[-3 < x + 2 < 7\]
4. \[5 \leq 4x + 1 \leq 13\]
5. \[2 < x + 2 < 5\]
6. \[11 < 2x + 3 < 21\]
7. \[x + 2 < -6 \text{ OR } x + 2 > 6\]
8. \[r - 1 < 0 \text{ OR } r - 1 > 4\]
9. \[n + 2 < 3 \text{ OR } n + 3 > 7\]
10. \[x - 1 < -1 \text{ OR } x - 5 > -1\]

Write the compound inequality shown by each graph.

11. \[-6 \ldots -3 -2 -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4\]
12. \[-5 -4 -3 -2 -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5\]
13. \[0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10\]
14. \[-10 -8 -6 -4 -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10\]

PRACTICE AND PROBLEM SOLVING

15. **Meteorology** One layer of Earth's atmosphere is called the stratosphere. At one point above Earth's surface the stratosphere extends from an altitude of 16 km to an altitude of 50 km. Write a compound inequality to show the altitudes that are within the range of the stratosphere. Graph the solutions.

Solve each compound inequality and graph the solutions.

16. \[-1 < x + 1 < 1\]
17. \[1 \leq 2n - 5 \leq 7\]
18. \[-2 < x - 2 < 2\]
19. \[5 < 3x - 1 < 17\]
20. \[x - 4 < -7 \text{ OR } x + 3 > 4\]
21. \[2x + 1 < 1 \text{ OR } x + 5 > 8\]
22. \[x + 1 < 2 \text{ OR } x + 5 > 8\]
23. \[x + 3 < 0 \text{ OR } x - 2 > 0\]

Write the compound inequality shown by each graph.

24. \[\quad -3 -2 -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7\]
25. \[\quad -3 -2 -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7\]
26. \[\quad -10 -8 -6 -4 -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10\]
27. \[\quad -5 -4 -3 -2 -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5\]

28. **Music** A typical acoustic guitar has a range of three octaves. When the guitar is tuned to “concert pitch,” the range of frequencies for those three octaves is between 82.4 Hz and 659.2 Hz inclusive. Write a compound inequality to show the frequencies that are within the range of a typical acoustic guitar. Graph the solutions.
29. Jenna’s band is going to record a CD at a recording studio. They will pay $225 to use the studio for one day and $80 per hour for sound technicians. Jenna has $200 and can reasonably expect to raise up to an additional $350 by taking pre-orders for the CDs.
   a. Explain how the inequality $200 \leq 225 + 80n \leq 550$ can be used to find the number of hours Jenna and her band can afford to use the studio and sound technicians.
   b. Solve the inequality. Are there any numbers in the solution set that are not reasonable in this situation?
   c. Suppose Jenna raises $350 in pre-orders. How much more money would she need to raise if she wanted to use the studio and sound technicians for 6 hours?

Write and graph a compound inequality for the numbers described.
30. all real numbers between $-6$ and 6
31. all real numbers less than or equal to 2 and greater than or equal to 1
32. all real numbers greater than 0 and less than 15
33. all real numbers between $-10$ and 10 inclusive

34. Transportation The cruise-control function on Georgina’s car should keep the speed of the car within 3 mi/h of the set speed. Write a compound inequality to show the acceptable speeds $s$ if the set speed is 55 mi/h. Graph the solutions.

35. Chemistry Water is not a liquid if its temperature is above 100 °C or below 0 °C. Write a compound inequality for the temperatures $t$ when water is not a liquid.

Solve each compound inequality and graph the solutions.
36. $5 \leq 4b - 3 \leq 9$
37. $-3 < x - 1 < 4$
38. $r + 2 < -2$ OR $r - 2 > 2$
39. $2a - 5 < -5$ OR $3a - 2 > 1$
40. $x - 4 \geq 5$ AND $x - 4 \leq 5$
41. $n - 4 < -2$ OR $n + 1 > 6$

42. Sports The ball used in a soccer game may not weigh more than 16 ounces or less than 14 ounces at the start of the match. After $1 \frac{1}{2}$ ounces of air was added to a ball, the ball was approved for use in a game. Write and solve a compound inequality to show how much the ball might have weighed before the air was added.

43. Meteorology Tornado damage is rated using the Fujita scale shown in the table. A tornado has a wind speed of 200 miles per hour. Write and solve a compound inequality to show how many miles per hour the wind speed would need to increase for the tornado to be rated “devastating” but not “incredible.”

<table>
<thead>
<tr>
<th>Fujita Tornado Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>F0</td>
</tr>
<tr>
<td>F1</td>
</tr>
<tr>
<td>F2</td>
</tr>
<tr>
<td>F3</td>
</tr>
<tr>
<td>F4</td>
</tr>
<tr>
<td>F5</td>
</tr>
</tbody>
</table>

44. Give a real-world situation that can be described by a compound inequality. Write the inequality that describes your situation.

45. Write About It How are the graphs of the compound inequality $x < 3$ AND $x < 7$ and the compound inequality $x < 3$ OR $x < 7$ different? How are the graphs alike? Explain.
46. **Critical Thinking**  If there is no solution to a compound inequality, does the compound inequality involve OR or AND? Explain.

47. Which of the following describes the solutions of $-x + 1 > 2$ OR $x - 1 > 2$?
   - **A** all real numbers greater than 1 or less than 3
   - **B** all real numbers greater than 3 or less than 1
   - **C** all real numbers greater than $-1$ or less than 3
   - **D** all real numbers greater than 3 or less than $-1$

48. Which of the following is a graph of the solutions of $x - 3 < 2$ AND $x + 3 > 2$?

49. Which compound inequality is shown by the graph?
   - **A** $x \leq 2$ OR $x > 5$
   - **B** $x < 2$ OR $x \geq 5$
   - **C** $x \leq 2$ OR $x \geq 5$
   - **D** $x \geq 2$ OR $x > 5$

50. Which of the following is a solution of $x + 1 \geq 3$ AND $x + 1 \leq 3$?

**CHALLENGE AND EXTEND**

Solve and graph each compound inequality.

51. $2c - 10 < 5 - 3c < 7c$
52. $5p - 10 < p + 6 < 3p$
53. $2s \leq 18 - s$ OR $5s \geq s + 36$
54. $9 - x \geq 5x$ OR $20 - 3x \leq 17$

55. Write a compound inequality that represents all values of $x$ that are NOT solutions to $x < -1$ OR $x > 3$.

56. For the compound inequality $x + 2 \geq a$ AND $x - 7 \leq b$, find values of $a$ and $b$ for which the only solution is $x = 1$. 
Triangle Inequality

For any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

The sides of this triangle are labeled $a$, $b$, and $c$. You can use the Triangle Inequality to write three statements about the triangle.

$$a + b > c \quad a + c > b \quad b + c > a$$

Unless all three of the inequalities are true, the lengths $a$, $b$, and $c$ cannot form a triangle.

Example 1

Can three side lengths of 25 cm, 15 cm, and 5 cm form a triangle?

- a. $25 + 15 > 5$ True
- b. $25 + 5 > 15$ True
- c. $15 + 5 > 25$ False

One of the inequalities is false, so the three lengths will not make a triangle. The situation is shown in the figure to the right.

Example 2

Two sides of a triangle measure 8 ft and 10 ft. What is the range of lengths of the third side?

Start by writing three statements about the triangle. Use $x$ for the unknown side length.

- a. $8 + 10 > x \quad 18 > x$ The third side must be shorter than 18 ft.
- b. $8 + 10 > x \quad 18 > x$ The third side must be longer than 2 ft.
- c. $8 + x > 10 \quad x > 2$ This provides no new useful information.

From part a, the third side must be shorter than 18 ft. And from part b, it must be longer than 2 ft. An inequality showing this is $2 < x < 18$.

Try This

Decide whether the three lengths given can form a triangle. If not, explain.

1. 14 ft, 30 ft, 10 ft
2. 11 cm, 8 cm, 17 cm
3. $6\frac{1}{2}$ yd, 3 yd, $2\frac{3}{4}$ yd

Write a compound inequality for the range of lengths of the third side of each triangle.

4. 7 in.
5. 8.2 ft
6. 18 m
Solving Absolute-Value Inequalities

**Objective**
Solve inequalities in one variable involving absolute-value expressions.

**Why learn this?**
You can solve an absolute-value inequality to determine the safe range for the pressure of a fire extinguisher. (See Example 3.)

When an inequality contains an absolute-value expression, it can be rewritten as a compound inequality. The inequality \(|x| < 5\) describes all real numbers whose distance from 0 is less than 5 units. The solutions are all numbers between \(-5\) and 5, so \(|x| < 5\) can be rewritten as \(-5 < x < 5\) or as \(x > -5\) AND \(x < 5\).

**Absolute-Value Inequalities Involving <**

<table>
<thead>
<tr>
<th>WORDS</th>
<th>NUMBERS</th>
</tr>
</thead>
</table>
| The inequality \(|x| < a\) (when \(a > 0\)) asks, “What values of \(x\) have an absolute value less than \(a\)?” The solutions are numbers between \(-a\) and \(a\). | \(|x| < 5\)  
\(-5 < x < 5\)  
\(x > -5\) AND \(x < 5\) |

**Example 1**

Solve each inequality and graph the solutions.

A \(|x| + 3 < 12\)

\[|x| + 3 < 12\]

\[|x| < 9\]

\[x > -9\] AND \(x < 9\)

B \(|x + 4| \leq 2\)

\[x + 4 \geq -2\] AND \(x + 4 \leq 2\)

\[x \geq -6\] AND \(x \leq -2\)

**Helpful Hint**
Just as you do when solving absolute-value equations, you first isolate the absolute-value expression when solving absolute-value inequalities.

Since 3 is added to \(|x|\), subtract 3 from both sides to undo the addition.

Write as a compound inequality.

Write as a compound inequality.

Write as a compound inequality.
Solve each inequality and graph the solutions.

1a. \(2|x| \leq 6\)

1b. \(|x + 3| - 4.5 \leq 7.5\)

The inequality \(|x| > 5\) describes all real numbers whose distance from 0 is greater than 5 units. The solutions are all numbers less than \(-5\) or greater than \(5\). The inequality \(|x| > 5\) can be rewritten as the compound inequality \(x < -5\) OR \(x > 5\).

### Absolute-Value Inequalities Involving >

<table>
<thead>
<tr>
<th>WORDS</th>
<th>NUMBERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>The inequality (</td>
<td>x</td>
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</table>

<table>
<thead>
<tr>
<th>GRAPH</th>
<th>ALGEBRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) units (\quad) (a) units</td>
<td>(</td>
</tr>
</tbody>
</table>

The same properties are true for inequalities that use the symbol \(\geq\).

### Example 2

Solve each inequality and graph the solutions.

\[A \quad |x| - 20 > -13\]
\[|x| - 20 > -13\]
\[+20 \quad +20\]
\[|x| > 7\]
\[x < -7\] OR \(x > 7\)

\[B \quad |x - 8| + 5 \geq 11\]
\[|x - 8| + 5 \geq 11\]
\[-5 \quad -5\]
\[|x - 8| \geq 6\]
\[x - 8 \leq -6\] OR \(x - 8 \geq 6\)

\[2a. \quad |x| + 10 \geq 12\]

\[2b. \quad |x + 2\frac{1}{2}| + \frac{1}{2} \geq 4\]
**Example 3**

Safety Application

Some fire extinguishers contain pressurized water. The water pressure should be 162.5 psi (pounds per square inch), but it is acceptable for the pressure to differ from this value by at most 12.5 psi. Write and solve an absolute-value inequality to find the range of acceptable pressures.

Graph the solutions.

Let \( p \) represent the actual water pressure of a fire extinguisher.

The difference between \( p \) and the ideal pressure is at most 12.5 psi.

\[
|p - 162.5| \leq 12.5
\]

\[
p - 162.5 \geq -12.5 \quad \text{AND} \quad p - 162.5 \leq 12.5 \quad \text{Solve the two inequalities.}
\]

\[
p \geq 150 \quad \text{AND} \quad p \leq 175
\]

The range of acceptable pressures is \( 150 \leq p \leq 175 \).

**Example 4**

Special Cases of Absolute-Value Inequalities

Solve each inequality.

- **4A** \( |x - 6| + 7 > 2 \)
  \[
  |x - 6| + 7 > 2 \\
  - 7 - 7 \\
  |x - 6| > -5 
  
  Subtract 7 from both sides. Absolute-value expressions are always nonnegative. Therefore, the statement is true for all values of \( x \). All real numbers are solutions.

- **4B** \( |x + 12| - 5 \leq -6 \)
  \[
  |x + 12| - 5 \leq -6 \\
  + 5 + 5 \\
  |x + 12| \leq -1 
  
  Add 5 to both sides. Absolute-value expressions are always nonnegative. Therefore, the statement is false for all values of \( x \). The inequality has no solutions.

Solve each inequality.

- **4a** \( |x| - 9 \geq -11 \)
- **4b** \( 4|x - 3.5| \leq -8 \)
THINK AND DISCUSS

1. Describe how the solutions of $7|x| \leq 21$ are different from the solutions of $7|x| < 21.$

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of the indicated type of absolute-value inequality and then solve.

GUIDED PRACTICE

Solve each inequality and graph the solutions.

SEE EXAMPLE 1
1. $|x| - 5 \leq -2$
2. $|x + 1| - 7.8 < 6.2$
3. $|3x| + 2 < 8$
4. $4|x| \leq 20$
5. $|x - 5| + 1 < 2$
6. $|x + \frac{1}{2}| - \frac{1}{2} \leq 3\frac{1}{2}$

SEE EXAMPLE 2
7. $|x| - 6 > 16$
8. $|x| + 2.9 > 8.6$
9. $2|x| \geq 8$
10. $|x + 2| > 7$
11. $|x - 3| + 2 \geq 4$
12. $|x + 5| - 4\frac{1}{2} \geq 7\frac{1}{2}$

SEE EXAMPLE 3
13. Nutrition A nutritionist recommends that an adult male consume 55 grams of fat per day. It is acceptable for the fat intake to differ from this amount by at most 25 grams. Write and solve an absolute-value inequality to find the range of fat intake that is acceptable. Graph the solutions.

SEE EXAMPLE 4
14. $|x| + 8 \leq 2$
15. $|x + 3| < -5$
16. $|x + 4| \geq -8$
17. $|x - 5| + \frac{1}{3} > -1$
18. $|3x| + 7 > 2$
19. $|x - 7| + 3.5 \leq 2$

PRACTICE AND PROBLEM SOLVING

Solve each inequality and graph the solutions.

20. $|x| + 6 \leq 10$
21. $|x - 3| < 1$
22. $|x - 2| - 8 \leq -3$
23. $|5x| < 15$
24. $|x - 2.4| + 4 \leq 6.4$
25. $4 + |x + 3| < 7$
26. $|x - 1| > 2$
27. $6|x| \geq 60$
28. $|x - 4| + 3 > 8$
29. $2|x + 2| \geq 16$
30. $3 + |x - 4| > 4$
31. $|x - \frac{1}{2}| + 9 > 10\frac{1}{2}$
32. The thermostat for a sauna is set to 175 °F, but the actual temperature of the sauna may vary by as much as 12 °F. Write and solve an absolute-value inequality to find the range of possible temperatures. Graph the solutions.

Solve each inequality.

33. $12 + |x| \leq 10$
34. $|x + \frac{3}{5}| - 2 > -4$
35. $|x + 1| + 5 \geq 4$
36. $|4x| - 3 < -6$
37. $3|x - 4| \leq -9$
38. $2|x| + 9 \geq 9$
Tell whether each statement is sometimes, always, or never true. Explain.
39. The value of $|x + 1|$ is greater than $-5$.
40. The value of $|x - 7|$ is less than 0.
41. An absolute-value inequality has all real numbers as solutions.

Write and solve an absolute-value inequality for each expression. Graph the solutions on a number line.
42. All numbers whose absolute value is less than or equal to 15
43. All numbers less than or equal to 3 units from 2 on the number line
44. All numbers at least 2 units from 8 on the number line

Write an absolute-value inequality for each graph.
45.
46.
47.
48.

49. **Multi-Step** The frequency of a sound wave determines its pitch. The human ear can detect a wide range of frequencies, from 20 Hz (very low notes) to 20,000 Hz (very high notes).
   a. What frequency is at the middle of the range?
   b. Write an absolute-value inequality for the range of frequencies the human ear can detect.

50. **Biology** The diagram shows the temperature range at which several fish species can survive. For each species, write an absolute-value inequality that gives the range of temperatures at which it can survive.

51. **Entertainment** On a game show, a contestant must guess a secret two-digit number. The secret number is 23. Write an absolute-value inequality that shows that the contestant's guess is more than 12 numbers away from the secret number.

52. The manager of a band recommends that the band sell its CDs for $8.75. The band decides to sell the CDs for $p$ dollars.
   a. Write an absolute-value expression that tells how far the band's price is from the recommended price.
   b. The band wants the price of its CD to be no more than $1.25 from the recommended price. Write an absolute-value inequality that gives the range of possible prices for the CD.
   c. Solve the inequality. Write the solution as a compound inequality.
53. **Critical Thinking** For which values of \( k \) does the inequality \(|x| + 1 < k\) have no solutions? Explain.

54. **Write About It** Describe how to use an absolute-value inequality to find all the values on a number line that are within 5 units of \(-6\).

55. What is the solution of the inequality \(3 + |x + 4| < 6?\)
   - (A) \(-13 < x < 5\)
   - (B) \(-7 < x < -1\)
   - (C) \(-6 < x < -2\)
   - (D) \(1 < x < 7\)

56. A thermometer gives temperature readings that may be inaccurate by at most 2 °F. The actual temperature is 75 °F. Which absolute-value inequality describes the range of temperatures that may be shown on the thermometer?
   - (F) \(|x - 75| \leq 2\)
   - (G) \(|x + 75| \leq 2\)
   - (H) \(|x - 75| \geq 2\)
   - (I) \(|x + 75| \geq 2\)

57. The inequality \(|w - 156| \leq 3\) describes the weights of members of a wrestling team. Which statement is NOT true?
   - (A) All of the team members weigh no more than 159 pounds.
   - (B) A team member may weigh 152 pounds.
   - (C) Every member of the team is at most 3 pounds away from 156 pounds.
   - (D) There are no team members who weigh 160 pounds.

**CHALLENGE AND EXTEND**

Write an absolute-value inequality for each graph.

58.

59.

60. **Critical Thinking** Fill in the missing reasons to justify each step in solving \(|2x - 6| + 5 \leq 7\).

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (</td>
<td>2x - 6</td>
</tr>
<tr>
<td>2. (</td>
<td>2x - 6</td>
</tr>
<tr>
<td>3. (2x - 6 \geq -2) AND (2x - 6 \leq 2)</td>
<td>Definition of absolute value</td>
</tr>
<tr>
<td>4. (2x \geq 4) AND (2x \leq 8)</td>
<td>_<em><strong><strong><strong><strong><strong><strong>?_</strong></strong></strong></strong></strong></strong></em></td>
</tr>
<tr>
<td>5. (x \geq 2) AND (x \leq 4)</td>
<td>_<em><strong><strong><strong><strong><strong><strong>?_</strong></strong></strong></strong></strong></strong></em></td>
</tr>
</tbody>
</table>
Graph each inequality.

6. \( x > -3 \)
7. \( p \leq 4 \)
8. \( -1 > t \)
9. \( r \geq 9.5 \)
10. \( 2(3 - 5) < k \)
11. \( w < 3 \)

Write the inequality shown by each graph.

12. \( x > -3 \)
13. \( p \leq 4 \)
14. \( -1 > t \)
15. \( r \geq 9.5 \)
16. \( 2(3 - 5) < k \)
17. \( w < 3 \)

Write the inequality shown by each graph.

12. \( x > -3 \)
13. \( p \leq 4 \)
14. \( -1 > t \)
15. \( r \geq 9.5 \)
16. \( 2(3 - 5) < k \)
17. \( w < 3 \)

Write an inequality for the situation and graph the solutions.

Applicants for a driver’s permit must be at least 16 years old.

\[ a \geq 16 \]

Define a variable and write an inequality for each situation. Graph the solutions.

15. The temperature must be at least 72 °F.
16. No more than 12 students were present.
17. It takes less than 30 minutes to complete the lab activity.
3-2 Solving Inequalities by Adding or Subtracting

**Examples**

Solve each inequality and graph the solutions.

- **x + 6 > 2**
  
  Since 6 is added to x, subtract 6 from both sides.
  
  \[
  x + 6 > 2 \\
  -6 \\n  \underline{-6} \\
  x > -4
  \]

- **n - 1.3 < 3.2**
  
  Since 1.3 is subtracted from x, add 1.3 to both sides.
  
  \[
  n - 1.3 < 3.2 \\
  +1.3 \\n  \underline{+1.3} \\
  n < 4.5
  \]

**Exercises**

Solve each inequality and graph the solutions.

18. \( r + 3 < 10 \)
19. \( k - 7 \leq -5 \)
20. \( -1 < m + 4 \)
21. \( x + 2.3 \geq 6.8 \)
22. \( -1 < m + 4 \)
23. \( 4 > a - 1 \)
24. \( h - \frac{1}{4} < \frac{3}{4} \)
25. \( 5 > 7 + v \)

26. Tammy wants to run at least 10 miles per week. So far this week, she ran 4.5 miles. Write and solve an inequality to determine how many more miles Tammy must run this week to reach her goal.

27. Rob has a gift card for $50. So far, he has selected a shirt that costs $32. Write and solve an inequality to determine the additional amount Rob could spend without exceeding the gift card limit.

3-3 Solving Inequalities by Multiplying or Dividing

**Examples**

- Solve \( \frac{p}{3} \leq 6 \) and graph the solutions.
  
  Since \( p \) is divided by \( -3 \), multiply both sides by \( -3 \).
  
  \[
  \frac{p}{3} \leq 6 \\
  -3 \cdot \frac{p}{3} \geq -3 \cdot 6 \\
  p \geq -18 \\
  \]

- Pizzas cost $5.50 each. What are the possible numbers of pizzas that can be purchased with $30? Let \( n \) represent the number of pizzas that can be purchased.
  
  \[
  \text{\$5.50} \times \text{\# of pizzas} \leq \text{\$30.} \\
  \\
  5.50n \leq 30 \\
  \]
  
  \[
  \frac{5.50n}{5.50} \leq \frac{30}{5.50} \\
  n \leq 5 \frac{5}{11}
  \]

Only a whole number of pizzas can be purchased, so 0, 1, 2, 3, 4, or 5 pizzas can be purchased.

**Exercises**

Solve each inequality and graph the solutions.

28. \( 3a \leq 15 \)
29. \( -18 < 6t \)
30. \( \frac{p}{4} > 2 \)
31. \( \frac{2}{5} x \leq -10 \)
32. \( -3n < -18 \)
33. \( \frac{g}{-2} > 6 \)
34. \( -2k < 14 \)
35. \( -3 > \frac{1}{3} r \)
36. \( 27 < -9h \)
37. \( -0.4g > -1 \)
38. Notebooks cost $1.39 each. What are the possible numbers of notebooks that can be purchased with $10?
39. A senior class is selling lanyards as a fundraiser. The profit for each lanyard is $0.75. Write and solve an inequality to determine the number of lanyards the class must sell to make a profit of at least $250.
3-4 Solving Two-Step and Multi-Step Inequalities

**Examples**

Solve each inequality and graph the solutions.

- **18 + 3t > -12**
  
  $18 + 3t > -12$
  
  $-18$ $18$
  
  $3t > -30$
  
  $\frac{3t}{3} > \frac{-30}{3}$
  
  $t > -10$

- **$3^2 - 5 \leq 2(1 + x)$**
  
  $3^2 - 5 \leq 2(1 + x)$
  
  Simplify the left side using order of operations.
  
  $9 - 5 \leq 2(1 + x)$
  
  $4 \leq 2(1 + x)$
  
  Distribute 2 on the right side.
  
  $4 \leq 2 + 2x$
  
  $4 \leq 2x$
  
  Subtract 2 from both sides.
  
  $2 \leq x$
  
  Divide both sides by 2.
  
  $1 \leq x$

**Exercises**

Solve each inequality and graph the solutions.

40. $3x + 4 < 19$
41. $7 \leq 2t - 5$
42. $\frac{m + 3}{2} > -4$
43. $2(x + 5) < 8$
44. $-4(2 - 5) > (-3)^2 - h$
45. $\frac{1}{5}x + \frac{1}{2} > \frac{4}{5}$
46. $0.5b - 2 \leq 4$
47. $\frac{1}{3}y - \frac{1}{2} > \frac{2}{3}$
48. $6 - 0.2n < 9$

49. Carl’s Cable Company charges $55 for monthly service plus $4 for each pay-per-view movie. Teleview Cable Company charges $110 per month with no fee for movies. For what number of movies is the cost of Carl’s Cable Company less than the cost of Teleview?

3-5 Solving Inequalities with Variables on Both Sides

**Examples**

- **Solve $b + 16 < 3b$ and graph the solutions.**
  
  $b + 16 > 3b$
  
  Subtract $b$ from both sides so that the coefficient of $b$ is positive.
  
  $16 > 2b$
  
  Divide both sides by 2.
  
  $8 > b$

- **Solve the inequality $3(1 + k) > 4 + 3k$.**
  
  $3 + 3k > 4 + 3k$
  
  Distribute 3 on the left side.
  
  The same variable term ($3k$) appears on both sides.
  
  For any number $3k$, adding $3$ will never result in a greater number than adding $4$.
  
  There are no solutions.

**Exercises**

Solve each inequality and graph the solutions.

50. $5 + 2m < -3m$
51. $y \leq 6 + 4y$
52. $4c - 7 > 9c + 8$
53. $-3(2 - q) \geq 6(q + 1)$
54. $2(5 - x) < 3x$
55. $3.5t - 1.8 < 1.6t + 3.9$

Solve each inequality.

56. $d - 2 < d - 4$
57. $2(1 - x) > -2(1 + x)$
58. $4(1 - p) < 4(2 + p)$
59. $3w + 1 > 3(w - 1)$
60. $5(4 - k) < 5k$
61. $3(c + 1) > 3c + 5$
62. Hanna has a savings account with a balance of $210 and deposits $16 per month. Faith has a savings account with a balance of $175 and deposits $20 per month. Write and solve an inequality to determine the number of months Hanna’s account balance will be greater than Faith’s account balance.
3-6 Solving Compound Inequalities

**Examples**

Solve each compound inequality and graph the solutions.

-3 < c + 5 ≤ 11  
  Since 5 is added to c, subtract 5 from each part of the inequality.

\[-3 < c + 5 \leq 6\]

\[-8 < c \leq 6\]

Graph \(c > -8\) and \(c \leq 6\).

-2 + t ≥ 2 OR t + 3 < 1  
  Solve the simple inequalities.

\[\frac{-2 + t}{t - 4} \geq 2\ OR \ t < -3\]

Graph \(t \geq 4\) and \(t < -2\).

**Exercises**

Solve each compound inequality and graph the solutions.

63. -4 < t + 6 < 10  \hspace{1em} 64. -8 < k - 2 ≤ 5

65. -3 + r > 4 OR r + 1 < -1

66. 2 > n + 3 > 5

67. 12 ≥ p + 7 > 5

68. 3 < s + 9 OR 1 > s - 4

69. One day, the high temperature was 84 °F and the low temperature was 68 °F. Write a compound inequality to represent the day’s temperatures.

70. The table shows formulas for the recommended heart rate range for a person who is \(a\) years old. Write and solve a compound inequality to determine the heart rate range for a 16-year-old person.

<table>
<thead>
<tr>
<th>Recommended Heart Rate Range</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.5 \times (220 - a))</td>
<td></td>
<td>(0.9 \times (220 - a))</td>
</tr>
</tbody>
</table>

3-7 Solving Absolute-Value Inequalities

**Examples**

Solve each inequality and graph the solutions.

\[|x| + 4 < 9\]

\[|x| + 4 < 9\]

Subtract 4 from both sides.

\[|x| < 5\]

\(x > -5\) AND \(x < 5\)  
Write as a compound inequality.

\[-7 \leq -3 < 1 \leq 5\]

\[|x - 3| + 7 ≥ 13\]

\[|x - 3| \geq 6\]

Subtract 7 from both sides.

\[|x - 3| \geq 6\]

\(x - 3 \leq -6\) OR \(x - 3 \geq 6\)  
Solve the two inequalities.

\[-6 \leq x \leq 9\]

**Exercises**

Solve each inequality and graph the solutions.

71. \(|x| - 7 \leq 15\)

72. \(|x + 4| > 8\)

73. \(6|x| \leq 24\)

74. \(|x + 9| + 11 < 20\)

75. \(3|x| \geq 9\)

76. \(4|2x| < 24\)

Solve the inequality.

77. \(|x| - 5.4 > 8.5\)

78. \(|5.2 + x| < 7.3\)

79. \(|x - 7| + 10 \geq 12\)

80. \(14|x| - 15 \geq 41\)

81. \(|x - \frac{1}{2}| + 4 \leq \frac{5}{2}\)

82. \(|x + 5.5| - 6.4 \leq 4.9\)

83. The water depth for a pool is set to 6 ft, but the actual depth of the pool may vary by as much as 4 in. Write and solve an absolute-value inequality to find the range of possible water depths in inches. Graph the solutions.
Describe the solutions of each inequality in words.

1. \(-6 \leq m\)
2. \(3t > 12\)
3. \(-x \geq 2\)
4. \(2 + b \leq 10\)

Graph each inequality.

5. \(b > -3\)
6. \(2.5 < c\)
7. \(y \leq -\sqrt{25}\)
8. \(3 - (4 + 7) \geq h\)

Write the inequality shown by each graph.

9.

10.

Write an inequality for the situation and graph the solutions.

11. Madison must run a mile in no more than 9 minutes to qualify for the race.

12. \(d - 5 > -7\)
13. \(f + 4 < -3\)
14. \(4.5 \geq s + 3.2\)
15. \(g + (-2) \leq 9\)

16. Students need at least 75 hours of volunteer service to meet their graduation requirement. Samir has already completed 48 hours. Write and solve an inequality to determine how many more hours he needs to complete.

17. \(-2c \leq 2\)
18. \(3 > \frac{k}{2}\)
19. \(\frac{4}{5}x \leq -8\)
20. \(\frac{b}{3} > -7\)

21. Marco needs to buy premium gasoline for his car. He has $20 in his wallet. Write and solve an inequality to determine how many gallons of gas Marco can buy.

<table>
<thead>
<tr>
<th>Gasoline Prices ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regular</td>
</tr>
<tr>
<td>2.05</td>
</tr>
</tbody>
</table>

22. \(3x - 8 < 4\)
23. \(-2(c - 3) > 4\)
24. \(5 \leq \frac{3}{4}n - 2^4\)
25. \(3 - 2a \leq -15 + (-9)\)

26. \(2k - 6 > 3k + 2\)
27. \(2(5 - f) \leq f + 12\)
28. \(\frac{3}{2}d \leq -\frac{1}{2}d + 6\)

29. \(-1 \leq x - 3 < 3\)
30. \(t + 7 < 3\) OR \(t - 1 > 4\)
31. \(4 \leq d - 2 < 5\)

32. The driving school instructor has asked Lina to stay within 2 miles of the posted speed limits. The current road has a speed limit of 45 mi/h. Write a compound inequality to show Lina's acceptable speeds \(s\).

33. \(|x - 3| + 7 < 17\)
34. \(6|x| + 4 \geq 16\)
35. \(|x + 12| \leq 23\)
Foundations for Geometry

4-1 Understanding Points, Lines, and Planes
LAB Explore Properties Associated with Points
4-2 Measuring and Constructing Segments
4-3 Measuring and Constructing Angles
4-4 Midpoint and Distance in the Coordinate Plane
4-5 Introduction to Coordinate Proof
4-6 Lines and Angles
LAB Explore Parallel Lines and Transversals
4-7 Angles Formed by Parallel Lines and Transversals
4-8 Proving Lines Parallel
LAB Construct Parallel Lines
4-9 Perpendicular Lines
LAB Construct Perpendicular Lines

Chapter Focus
- Use the correct terminology for basic geometric figures.
- Apply basic formulas in and out of the coordinate plane.

Picture This!
Many geometric concepts and shapes may be used in creating works of art. Unique designs can be made using only points, lines, planes, or circles.

Learn It Online
Chapter Project Online
Reading Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

As you read through an example problem, pay attention to the notes in the margin. These notes highlight key information about the concept and will help you to avoid common mistakes.

The Glossary is found in the back of your textbook. Use it when you need a definition of an unfamiliar word or phrase.

The Index is located at the end of your textbook. If you need to locate the page where a particular concept is explained, use the Index to find the corresponding page number.

The Problem-Solving Handbook is found in the back of your textbook. These pages review strategies that can help you solve real-world problems.

Try This

Use your textbook for the following problems.

1. Use the index to find the page where right angle is defined.

2. What formula does the Know-It Note on the first page of the lesson Midpoint and Distance in the Coordinate Plane refer to?

3. Use the glossary to find the definition of congruent segments.
The most basic figures in geometry are undefined terms, which cannot be defined by using other figures. The undefined terms point, line, and plane are the building blocks of geometry.

**Undefined Terms**

<table>
<thead>
<tr>
<th>TERM</th>
<th>NAME</th>
<th>DIAGRAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A point names a location and has no size. It is represented by a dot.</td>
<td>A capital letter point ( P )</td>
<td><img src="image1.png" alt="Diagram of a point" /></td>
</tr>
<tr>
<td>A line is a straight path that has no thickness and extends forever.</td>
<td>A lowercase letter or two points on the line ( \ell ) ( XY ) or ( YX )</td>
<td><img src="image2.png" alt="Diagram of a line" /></td>
</tr>
<tr>
<td>A plane is a flat surface that has no thickness and extends forever.</td>
<td>A script capital letter or three points not on a line plane ( R ) or plane ( ABC )</td>
<td><img src="image3.png" alt="Diagram of a plane" /></td>
</tr>
</tbody>
</table>

Points that lie on the same line are collinear. \( K, L, \) and \( M \) are collinear. \( K, L, \) and \( N \) are noncollinear. Points that lie in the same plane are coplanar. Otherwise they are noncoplanar.

**Example 1**

**Naming Points, Lines, and Planes**

Refer to the design in the roof of Beijing’s National Stadium.

A Name four coplanar points. \( K, L, M, \) and \( N \) all lie in plane \( R \).

B Name three lines. \( AB, BC, \) and \( CA \).

1. Use the diagram to name two planes.
### DEFINITION

<table>
<thead>
<tr>
<th>Name</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Segment</strong>, or line segment, is the part of a line consisting of two points and all points between them.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Endpoint</strong> is a point at one end of a segment or the starting point of a ray.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Ray</strong> is a part of a line that starts at an endpoint and extends forever in one direction.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>Opposite rays</strong> are two rays that have a common endpoint and form a line.</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

### Example 2: Drawing Segments and Rays

Draw and label each of the following.

- **A** a segment with endpoints $U$ and $V$
- **B** opposite rays with a common endpoint $Q$

2. Draw and label a ray with endpoint $M$ that contains $N$.

### Example 3: Identifying Points and Lines in a Plane

Name a line that passes through two points.

There is exactly one line $n$ passing through $G$ and $H$.

3. Name a plane that contains three noncollinear points.
Recall that a system of equations is a set of two or more equations containing two or more of the same variables. The coordinates of the solution of the system satisfy all equations in the system. These coordinates also locate the point where all the graphs of the equations in the system intersect.

An intersection is the set of all points that two or more figures have in common. The next two postulates describe intersections involving lines and planes.

**Postulates**

**Intersection of Lines and Planes**

4-1-4 If two lines intersect, then they intersect in exactly one point.

4-1-5 If two planes intersect, then they intersect in exactly one line.

Use a dashed line to show the hidden parts of any figure that you are drawing. A dashed line will indicate the part of the figure that is not seen.

**Example 4**

Representing Intersections

Sketch a figure that shows each of the following.

**A** A line intersects a plane, but does not lie in the plane.

**B** Two planes intersect in one line.

4. Sketch a figure that shows two lines intersect in one point in a plane, but only one of the lines lies in the plane.

**Think and Discuss**

1. Explain why any two points are collinear.
2. Which postulate explains the fact that two straight roads cannot cross each other more than once?
3. Explain why points and lines may be coplanar even when the plane containing them is not drawn.
4. Name all the possible lines, segments, and rays for the points A and B. Then give the maximum number of planes that can be determined by these points.

**5. GET ORGANIZED** Copy and complete the graphic organizer below. In each box, name, describe, and illustrate one of the undefined terms.
GUIDED PRACTICE
Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. Give an example from your classroom of three collinear points.
2. Make use of the fact that endpoint is a compound of end and point and name the endpoint of \( \overrightarrow{ST} \).

Use the figure to name each of the following.
3. five points
4. two lines
5. two planes
6. point on \( \overrightarrow{BD} \)

Draw and label each of the following.
7. a segment with endpoints \( M \) and \( N \)
8. a ray with endpoint \( F \) that passes through \( G \)

Use the figure to name each of the following.
9. a line that contains \( A \) and \( C \)
10. a plane that contains \( A, D, \) and \( C \)

Sketch a figure that shows each of the following.
11. three coplanar lines that intersect in a common point
12. two lines that do not intersect

PRACTICE AND PROBLEM SOLVING
Use the figure to name each of the following.
13. three collinear points
14. four coplanar points
15. a plane containing \( E \)

Draw and label each of the following.
16. a line containing \( X \) and \( Y \)
17. a pair of opposite rays that both contain \( R \)

Use the figure to name each of the following.
18. two points and a line that lie in plane \( \mathcal{J} \)
19. two planes that contain \( \ell \)

Sketch a figure that shows each of the following.
20. a line that intersects two nonintersecting planes
21. three coplanar lines that intersect in three different points
22. Name an object at the archaeological site shown that is represented by each of the following.
   a. a point
   b. a segment
   c. a plane

Draw each of the following.
23. plane \( \mathcal{H} \) containing two lines that intersect at \( M \)
24. \( \overline{ST} \) intersecting plane \( \mathcal{M} \) at \( R \)

Use the figure to name each of the following.
25. the intersection of \( \overline{TV} \) and \( \overline{US} \)
26. the intersection of \( \overline{US} \) and plane \( R \)
27. the intersection of \( \overline{TU} \) and \( \overline{UV} \)

Write the postulate that justifies each statement.
28. The line connecting two dots on a sheet of paper lies on the same sheet of paper as the dots.
29. If two ants are walking in straight lines but in different directions, their paths cannot cross more than once.
30. Critical Thinking Is it possible to draw three points that are noncoplanar? Explain.

Tell whether each statement is sometimes, always, or never true. Support your answer with a sketch.
31. If two planes intersect, they intersect in a straight line.
32. If two lines intersect, they intersect at two different points.
33. \( \overline{AB} \) is another name for \( \overline{BA} \).
34. If two rays share a common endpoint, then they form a line.
35. Art Pointillism is a technique in which tiny dots of complementary colors are combined to form a picture. Which postulate ensures that a line connecting two of these points also lies in the plane containing the points?
36. Probability Three of the labeled points are chosen at random. What is the probability that they are collinear?
37. Campers often use a cooking stove with three legs. Which postulate explains why they might prefer this design to a stove that has four legs?
38. Write About It Explain why three coplanar lines may have zero, one, two, or three points of intersection. Support your answer with a sketch.
39. Which of the following is a set of noncollinear points?

- A. \( P, R, T \)
- B. \( Q, R, S \)
- C. \( P, Q, R \)
- D. \( S, T, U \)

40. What is the greatest number of intersection points four coplanar lines can have?

- A. 6
- B. 4
- C. 0
- D. 2

41. Two flat walls meet in the corner of a classroom. Which postulate best describes this situation?

- A. Through any three noncollinear points there is exactly one plane.
- B. If two points lie in a plane, then the line containing them lies in the plane.
- C. If two lines intersect, then they intersect in exactly one point.
- D. If two planes intersect, then they intersect in exactly one line.

42. **Gridded Response** What is the greatest number of planes determined by four noncollinear points?

**CHALLENGE AND EXTEND**

Use the table for Exercises 43–45.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Number of Points</th>
<th>Maximum Number of Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

43. What is the maximum number of segments determined by 4 points?

44. **Multi-Step** Extend the table. What is the maximum number of segments determined by 10 points?

45. Write a formula for the maximum number of segments determined by \( n \) points.

46. **Critical Thinking** Explain how rescue teams could use two of the postulates from this lesson to locate a distress signal.
**Explore Properties Associated with Points**

The two endpoints of a segment determine its length. Other points on the segment are *between* the endpoints. Only one of these points is the *midpoint* of the segment. In this lab, you will use geometry software to measure lengths of segments and explore properties of points on segments.

**Activity**

1. Construct a segment and label its endpoints $A$ and $C$.

2. Create point $B$ on $\overline{AC}$.

3. Measure the distances from $A$ to $B$ and from $B$ to $C$. Use the Calculate tool to calculate the sum of $AB$ and $BC$.

4. Measure the length of $\overline{AC}$. What do you notice about this length compared with the measurements found in Step 3?

5. Drag point $B$ along $\overline{AC}$. Drag one of the endpoints of $\overline{AC}$. What relationships do you think are true about the three measurements?

6. Construct the midpoint of $\overline{AC}$ and label it $M$.

7. Measure $\overline{AM}$ and $\overline{MC}$. What relationships do you think are true about the lengths of $\overline{AC}$, $\overline{AM}$, and $\overline{MC}$? Use the Calculate tool to confirm your findings.

8. How many midpoints of $\overline{AC}$ exist?

**Try This**

1. Repeat the activity with a new segment. Drag each of the points in your figure (the endpoints, the point on the segment, and the midpoint). Write down any relationships you observe about the measurements.

2. Create a point $D$ not on $\overline{AC}$. Measure $\overline{AD}$, $\overline{DC}$, and $\overline{AC}$. Does $\overline{AD} + \overline{DC} = \overline{AC}$? What do you think has to be true about $D$ for the relationship to always be true?
Why learn this?
You can measure a segment to calculate the distance between two locations. Maps of a race are used to show the distance between stations on the course. (See Example 4.)

A ruler can be used to measure the distance between two points. A point corresponds to one and only one number on the ruler. This number is called a coordinate. The following postulate summarizes this concept.

Postulate 4-2-1 Ruler Postulate

The points on a line can be put into a one-to-one correspondence with the real numbers.

The distance between any two points is the absolute value of the difference of the coordinates. If the coordinates of points A and B are a and b, then the distance between A and B is |a − b| or |b − a|. The distance between A and B is also called the length of AB, or |AB|.

\[ AB = |a - b| = |b - a| \]

EXAMPLE 1 Finding the Length of a Segment

Find each length.

A. DC
\[ DC = |4.5 - 2| = |2.5| = 2.5 \]

B. EF
\[ EF = |-4 - (-1)| = |-3| = 3 \]

Find each length.

1a. XY
1b. XZ

Congruent segments are segments that have the same length. In the diagram, \(PQ = RS\), so you can write \(PQ \cong RS\). This is read as “segment PQ is congruent to segment RS.” Tick marks are used in a figure to show congruent segments.
You can make a sketch or measure and draw a segment. These may not be exact. A **construction** is a way of creating a figure that is more precise. One way to make a geometric construction is to use a compass and straightedge.

### Construction Congruent Segment

Construct a segment congruent to $\overline{AB}$.

1. Draw $\ell$. Choose a point on $\ell$ and label it $C$.
2. Open the compass to distance $AB$. Place the point of the compass at $C$ and make an arc through $\ell$. Find the point where the arc and $\ell$ intersect and label it $D$.

$CD \cong AB$

### Example 2

**Copying a Segment**

Sketch, draw, and construct a segment congruent to $\overline{MN}$.

**Step 1** Estimate and sketch.
- Estimate the length of $\overline{MN}$ and sketch $\overline{PQ}$ approximately the same length.

**Step 2** Measure and draw.
- Use a ruler to measure $\overline{MN}$. $MN$ appears to be 3.1 cm. Use a ruler and draw $\overline{XY}$ to have length 3.1 cm.

**Step 3** Construct and compare.
- Use a compass and straightedge to construct $\overline{ST}$ congruent to $\overline{MN}$.

A ruler shows that $\overline{PQ}$ and $\overline{XY}$ are approximately the same length as $\overline{MN}$, but $\overline{ST}$ is precisely the same length.

2. Sketch, draw, and construct a segment congruent to $\overline{JK}$.

In order for you to say that a point $B$ is **between** two points $A$ and $C$, all three of the points must lie on the same line, and $AB + BC = AC$.

### Postulate 4-2-2 Segment Addition Postulate

If $B$ is between $A$ and $C$, then $AB + BC = AC$.  

---

**Know it!**

Note
**EXAMPLE 3**

**Using the Segment Addition Postulate**

**A**

B is between A and C, \( AC = 14 \), and \( BC = 11.4 \). Find \( AB \).

\[
AC = AB + BC \quad \text{Seg. Add. Post.}
\]

\[
14 = AB + 11.4 \quad \text{Substitute 14 for \( AC \) and 11.4 for \( BC \).}
\]

\[
-11.4 \quad \text{ Subtract 11.4 from both sides.}
\]

\[
2.6 = AB \quad \text{Simplify.}
\]

**B**

S is between \( R \) and \( T \). Find \( RT \).

\[
RT = RS + ST \quad \text{Seg. Add. Post.}
\]

\[
4x = (2x + 7) + 28 \quad \text{Substitute the given values.}
\]

\[
4x = 2x + 35 \quad \text{Simplify.}
\]

\[
-2x \quad \text{ Subtract 2x from both sides.}
\]

\[
x = 35 \quad \text{Simplify.}
\]

\[
\frac{2x}{2} = \frac{35}{2} \quad \text{Divide both sides by 2.}
\]

\[
x = \frac{35}{2}, \text{ or } 17.5 \quad \text{Simplify.}
\]

\[
RT = 4x \quad \text{Substitute 17.5 for \( x \).}
\]

\[
= 4 \left(17.5\right) = 70
\]

---

**CHECK IT OUT!**

3a. \( Y \) is between \( X \) and \( Z \), \( XZ = 3 \), and \( XY = 1 \frac{1}{2} \). Find \( YZ \).

3b. \( E \) is between \( D \) and \( F \). Find \( DF \).

The **midpoint** \( M \) of \( \overline{AB} \) is the point that **bisects**, or divides, the segment into two congruent segments. If \( M \) is the midpoint of \( \overline{AB} \), then \( AM = MB \). So if \( AB = 6 \), then \( AM = 3 \) and \( MB = 3 \).

---

**EXAMPLE 4**

**Recreation Application**

The map shows the route for a race. You are 365 m from drink station \( R \) and 2 km from drink station \( S \). The first-aid station is located at the midpoint of the two drink stations. How far are you from the first-aid station?

Let your current location be \( X \) and the location of the first-aid station be \( Y \).

\[
XR + RS = XS \quad \text{Seg. Add. Post.}
\]

\[
365 + RS = 2000 \quad \text{Substitute 365 for \( XR \) and 2000 for \( XS \).}
\]

\[
-365 \quad \text{ Subtract 365 from both sides.}
\]

\[
RS = 1635 \quad \text{Simplify.}
\]

\[
RY = 817.5 \quad \text{\( Y \) is the mdpt. of \( RS \), so \( RY = \frac{1}{2} RS \).}
\]

\[
XY = XR + RY \quad \text{Substitute 365 for \( XR \) and 817.5 for \( RY \).}
\]

\[
= 365 + 817.5 = 1182.5 \text{ m}
\]

You are 1182.5 m from the first-aid station.

---

**CHECK IT OUT!**

4. What is the distance to a drink station located at the midpoint between your current location and the first-aid station?
A **segment bisector** is any ray, segment, or line that intersects a segment at its midpoint. It divides the segment into two equal parts at its midpoint.

### Construction Segment Bisector

1. Draw \( XY \) on a sheet of paper.
2. Fold the paper so that \( Y \) is on top of \( X \).
3. Unfold the paper. The line represented by the crease bisects \( XY \). Label the midpoint \( M \).

\[ XM = MY \]

### Example 5

**Using Midpoints to Find Lengths**

\( B \) is the midpoint of \( \overline{AC} \), \( AB = 5x \), and \( BC = 3x + 4 \). Find \( AB, BC, \) and \( AC \).

**Step 1** Solve for \( x \).

\[
AB = BC \quad B \text{ is the mdpt. of } \overline{AC}.
\]

\[
5x = 3x + 4 \quad \text{Substitute } 5x \text{ for } AB \text{ and } 3x + 4 \text{ for } BC.
\]

\[
-3x - 3x \\
2x = 4 \quad \text{Subtract } 3x \text{ from both sides.}
\]

\[
2x = 4 \\
\frac{2x}{2} = \frac{4}{2} \quad \text{Simplify.}
\]

\[
x = 2 \quad \text{Divide both sides by } 2.
\]

**Step 2** Find \( AB, BC, \) and \( AC \).

\[
AB = 5x \\
BC = 3x + 4 \\
AC = AB + BC
\]

\[
= 5(2) = 10 \\
= 3(2) + 4 = 10 \\
= 10 + 10 = 20
\]

**Think and Discuss**

1. Suppose \( R \) is the midpoint of \( \overline{ST} \). Explain how \( SR \) and \( ST \) are related.

2. **Get Organized** Copy and complete the graphic organizer.

   Make a sketch and write an equation to describe each relationship.

<table>
<thead>
<tr>
<th>Sketch</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. Line ℓ bisects XY at M and divides XY into two equal parts. Name a pair of congruent segments.

2. ___ is the amount of space between two points on a line. It is always expressed as a nonnegative number. (distance or midpoint)

Find each length.
3. AB
4. BC

5. Sketch, draw, and construct a segment congruent to RS.

6. B is between A and C, AC = 15.8, and AB = 9.9. Find BC.

7. Find MP.

8. Travel If a picnic area is located at the midpoint between Sacramento and Oakland, find the distance to the picnic area from the road sign.

9. Multi-Step K is the midpoint of JL, JL = 4x − 2, and JK = 7. Find x, KL, and JL.

10. E bisects DF, DE = 2y, and EF = 8y − 3. Find DE, EF, and DF.

PRACTICE AND PROBLEM SOLVING

Find each length.
11. DB
12. CD

13. Sketch, draw, and construct a segment twice the length of AB.


15. Find MN.

16. Sports During a football game, a quarterback standing at the 9-yard line passes the ball to a receiver at the 24-yard line. The receiver then runs with the ball halfway to the 50-yard line. How many total yards (passing plus running) did the team gain on the play?

17. Multi-Step E is the midpoint of DF, DE = 2x + 4, and EF = 3x − 1. Find DE, EF, and DF.

18. Q bisects PR, PQ = 3y, and PR = 42. Find y and QR.
Use the diagram for Exercises 20–23.

20. \(GD = 4 \frac{2}{3}\). Find \(GH\).

21. \(CD \cong DF\), \(E\) bisects \(DF\), and \(CD = 14.2\). Find \(EF\).

22. \(GH = 4x - 1\), and \(DH = 8\). Find \(x\).

23. \(GH\) bisects \(CF\), \(CF = 2y - 2\), and \(CD = 3y - 11\). Find \(CD\).

Tell whether each statement is sometimes, always, or never true. Support each of your answers with a sketch.

24. Two segments that have the same length must be congruent.

25. If \(M\) is between \(A\) and \(B\), then \(M\) bisects \(AB\).

26. If \(Y\) is between \(X\) and \(Z\), then \(X\), \(Y\), and \(Z\) are collinear.

27. **ERROR ANALYSIS** Below are two statements about the midpoint of \(AB\). Which is incorrect? Explain the error.

28. **Carpentry** A carpenter has a wooden dowel that is 72 cm long. She wants to cut it into two pieces so that one piece is 5 times as long as the other. What are the lengths of the two pieces?

29. The coordinate of \(M\) is 2.5, and \(MN = 4\). What are the possible coordinates for \(N\)?

30. Draw three collinear points where \(E\) is between \(D\) and \(F\). Then write an equation using these points and the Segment Addition Postulate.

Suppose \(S\) is between \(R\) and \(T\). Use the Segment Addition Postulate to solve for each variable.

31. \(RS = 7y - 4\)

   \(ST = y + 5\)

   \(RT = 28\)

32. \(RS = 3x + 1\)

   \(ST = \frac{1}{2}x + 3\)

   \(RT = 18\)

33. \(RS = 2z + 6\)

   \(ST = 4z - 3\)

   \(RT = 5z + 12\)

34. **Write About It** In the diagram, \(B\) is not between \(A\) and \(C\). Explain.

35. **Construction** Use a compass and straightedge to construct a segment whose length is \(AB + CD\).
36. Q is between P and R. S is between Q and R, and R is between Q and T. PT = 34, QR = 8, and PQ = SQ = SR. What is the length of RT?

\[ \begin{align*}
\text{A} & \quad 9 \\
\text{B} & \quad 10 \\
\text{C} & \quad 18 \\
\text{D} & \quad 22
\end{align*} \]

37. C is the midpoint of AD. B is the midpoint of AC. BC = 12. What is the length of AD?

\[ \begin{align*}
\text{F} & \quad 12 \\
\text{G} & \quad 24 \\
\text{H} & \quad 36 \\
\text{J} & \quad 48
\end{align*} \]

38. Which expression correctly states that \( XY \) is congruent to \( VW \)?

\[ \begin{align*}
\text{A} & \quad XY \cong VW \\
\text{B} & \quad \overline{XY} \cong \overline{VW} \\
\text{C} & \quad \overline{XY} = \overline{VW} \\
\text{D} & \quad XY = VW
\end{align*} \]

39. A, B, C, D, and E are collinear points. \( AE = 34, BD = 16 \), and \( AB = BC = CD \). What is the length of \( CE \)?

\[ \begin{align*}
\text{F} & \quad 10 \\
\text{G} & \quad 16 \\
\text{H} & \quad 18 \\
\text{J} & \quad 24
\end{align*} \]

**CHALLENGE AND EXTEND**

40. \( HJ \) is twice \( JK \). J is between \( H \) and \( K \). If \( HJ = 4x \) and \( HK = 78 \), find \( JK \).

41. A, D, N, and X are collinear points. D is between N and A. \( NA + AX = NX \). Draw a diagram that represents this information.

**Sports** Use the following information for Exercises 42 and 43.

The table shows regulation distances between hurdles in women's and men's races. In both the women's and men's events, the race consists of a straight track with 10 equally spaced hurdles.

<table>
<thead>
<tr>
<th>Event</th>
<th>Distance of Race</th>
<th>Distance from Start to First Hurdle</th>
<th>Distance Between Hurdles</th>
<th>Distance from Last Hurdle to Finish</th>
</tr>
</thead>
<tbody>
<tr>
<td>Women's</td>
<td>100 m</td>
<td>13.00 m</td>
<td>8.50 m</td>
<td></td>
</tr>
<tr>
<td>Men's</td>
<td>110 m</td>
<td>13.72 m</td>
<td>9.14 m</td>
<td></td>
</tr>
</tbody>
</table>

42. Find the distance from the last hurdle to the finish line for the women's race.

43. Find the distance from the last hurdle to the finish line for the men's race.

44. **Critical Thinking** Given that J, K, and L are collinear and that K is between J and L, is it possible that \( JK = JL \)? If so, draw an example. If not, explain.
Surveyors use angles to help them measure and map the earth’s surface. (See Exercise 27.)

A transit is a tool for measuring angles. It consists of a telescope that swivels horizontally and vertically. Using a transit, a surveyor can measure the angle formed by his or her location and two distant points.

An angle is a figure formed by two rays, or sides, with a common endpoint called the vertex (plural: vertices). You can name an angle several ways: by its vertex, by a point on each ray and the vertex, or by a number.

The set of all points between the sides of the angle is the interior of an angle. The exterior of an angle is the set of all points outside the angle.

Angle Name
∠R, ∠SRT, ∠TRS, or ∠1

You cannot name an angle just by its vertex if the point is the vertex of more than one angle. In this case, you must use all three points to name the angle, and the middle point is always the vertex.

EXAMPLE 1  Naming Angles
A surveyor recorded the angles formed by a transit (point T) and three distant points, Q, R, and S. Name three of the angles.
∠QTR, ∠QTS, and ∠RTS

1. Write the different ways you can name the angles in the diagram.

The measure of an angle is usually given in degrees. Since there are 360° in a circle, one degree is \( \frac{1}{360} \) of a circle. When you use a protractor to measure angles, you are applying the following postulate.

**Postulate 4-3-1 Protractor Postulate**
Given \( \overline{AB} \) and a point \( O \) on \( \overline{AB} \), all rays that can be drawn from \( O \) can be put into a one-to-one correspondence with the real numbers from 0 to 180.
You can use the Protractor Postulate to help you classify angles by their measure. The measure of an angle is the absolute value of the difference of the real numbers that the rays correspond with on a protractor. If $\overrightarrow{OC}$ corresponds with $c$ and $\overrightarrow{OD}$ corresponds with $d$, $m\angle DOC = |d - c|$ or $|c - d|$.

**Types of Angles**

<table>
<thead>
<tr>
<th>Acute Angle</th>
<th>Right Angle</th>
<th>Obtuse Angle</th>
<th>Straight Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measures greater than 0° and less than 90°</td>
<td>Measures 90°</td>
<td>Measures greater than 90° and less than 180°</td>
<td>Formed by two opposite rays and measures 180°</td>
</tr>
</tbody>
</table>

**Example 2**

**Measuring and Classifying Angles**

Find the measure of each angle. Then classify each as acute, right, or obtuse.

A $\angle AOD$
$m\angle AOD = 165°$
$\angle AOD$ is obtuse.

B $\angle COD$
$m\angle COD = |165 - 75| = 90°$
$\angle COD$ is a right angle.

Use the diagram to find the measure of each angle. Then classify each as acute, right, or obtuse.

2a. $\angle BOA$
2b. $\angle DOB$
2c. $\angle EOC$
Congruent angles are angles that have the same measure. In the diagram, \( \angle ABC = \angle DEF \), so you can write \( \angle ABC \cong \angle DEF \). This is read as “angle \( ABC \) is congruent to angle \( DEF \).” Arc marks are used to show that the two angles are congruent.

### Construction: Congruent Angle

Construct an angle congruent to \( \angle A \).

1. Use a straightedge to draw a ray with endpoint \( D \).
2. Place the compass point at \( A \) and draw an arc that intersects both sides of \( \angle A \). Label the intersection points \( B \) and \( C \).
3. Using the same compass setting, place the compass point at \( D \) and draw an arc that intersects the ray. Label the intersection \( E \).
4. Place the compass point at \( B \) and open it to the distance \( BC \). Place the point of the compass at \( E \) and draw an arc. Label its intersection with the first arc \( F \).
5. Use a straightedge to draw \( \overline{DF} \).

\( \angle D \cong \angle A \)

The Angle Addition Postulate is very similar to the Segment Addition Postulate that you learned in the previous lesson.

#### Postulate 4.3.2 Angle Addition Postulate

If \( S \) is in the interior of \( \angle PQR \), then \( m\angle PQS + m\angle SQR = m\angle PQR \).

\((\angle \text{ Add. Post.)}\)

### Example 3

Using the Angle Addition Postulate

If \( \angle ABD = 37^\circ \) and \( \angle ABC = 84^\circ \). Find \( m\angle DBC \).

\[
m\angle ABC = m\angle ABD + m\angle DBC \\
84^\circ = 37^\circ + m\angle DBC \\
-37 \quad -37 \\
47^\circ = m\angle DBC
\]

### Check It Out!

3. \( m\angle XWZ = 121^\circ \) and \( m\angle XWY = 59^\circ \). Find \( m\angle YWZ \).
An **angle bisector** is a ray that divides an angle into two congruent angles. \( \overrightarrow{JK} \) bisects \( \angle LJM \); thus \( \angle LJK \cong \angle KJM \).

### Construction Angle Bisector

**Construct the bisector of \( \angle A \).**

1. Place the point of the compass at \( A \) and draw an arc. Label its points of intersection with \( \angle A \) as \( B \) and \( C \).

2. Without changing the compass setting, draw intersecting arcs from \( B \) and \( C \). Label the intersection of the arcs as \( D \).

3. Use a straightedge to draw \( \overrightarrow{AD} \). \( \overrightarrow{AD} \) bisects \( \angle A \).

### Example 4

**Finding the Measure of an Angle**

\( \overrightarrow{BD} \) bisects \( \angle ABC \), \( m\angle ABD = (6x + 3)\)°, and \( m\angle DBC = (8x - 7)\)°. Find \( m\angle ABD \).

**Step 1** Find \( x \).

\[
\begin{align*}
m\angle ABD &= m\angle DBC \\
(6x + 3)\text{°} &= (8x - 7)\text{°} \\
6x + 10 &= 8x + 7 \\
-6x &= 2x \\
10 &= 2x \\
5 &= x
\end{align*}
\]

**Step 2** Find \( m\angle ABD \).

\[
\begin{align*}
m\angle ABD &= 6x + 3 \\
&= 6(5) + 3 \\
&= 33°
\end{align*}
\]

Find the measure of each angle.

4a. \( \overrightarrow{QS} \) bisects \( \angle PQR \), \( m\angle PQS = (5y - 1)\)°, and \( m\angle PQR = (8y + 12)\)°. Find \( m\angle PQS \).

4b. \( \overrightarrow{JK} \) bisects \( \angle LJM \), \( m\angle LJK = (-10x + 3)\)°, and \( m\angle KJM = (-x + 21)\)°. Find \( m\angle LJM \).
THINK AND DISCUSS

1. Explain why any two right angles are congruent.
2. \( \overrightarrow{BD} \) bisects \( \angle ABC \). How are \( m\angle ABC \), \( m\angle ABD \), and \( m\angle DBC \) related?

3. GET ORGANIZED Copy and complete the graphic organizer. In the cells, sketch, measure, and name an example of each angle type.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Measure</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acute Angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Right Angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obtuse Angle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight Angle</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. \( \angle A \) is an acute angle. \( \angle O \) is an obtuse angle. \( \angle R \) is a right angle. Put \( \angle A \), \( \angle O \), and \( \angle R \) in order from least to greatest by measure.

2. Which point is the vertex of \( \angle BCD \)? Which rays form the sides of \( \angle BCD \)?

3. Music Musicians use a metronome to keep time as they play. The metronome’s needle swings back and forth in a fixed amount of time. Name all of the angles in the diagram.

Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.

4. \( \angle VXW \)
5. \( \angle TXW \)
6. \( \angle RXU \)

\( \overrightarrow{BD} \) bisects \( \angle ABC \). Find each of the following.

7. \( m\angle JKM \) if \( m\angle JKL = 42^\circ \) and \( m\angle LKM = 28^\circ \)
8. \( m\angle LKM \) if \( m\angle JKL = 56.4^\circ \) and \( m\angle JKM = 82.5^\circ \)

\( L \) is in the interior of \( \angle JKM \). Find each of the following.

9. \( m\angle ABD \) if \( m\angle ABD = (6x + 4)^\circ \) and \( m\angle DBC = (8x - 4)^\circ \)
10. \( m\angle ABC \) if \( m\angle ABD = (5y - 3)^\circ \) and \( m\angle DBC = (3y + 15)^\circ \)
11. **Physics** Pendulum clocks have been used since 1656 to keep time. The pendulum swings back and forth once or twice per second. Name all of the angles in the diagram.

Use the protractor to find the measure of each angle. Then classify each as acute, right, or obtuse.

12. \( \angle CGE \)
13. \( \angle BGD \)
14. \( \angle AGB \)

15. If \( m\angle RST = 38^\circ \) and \( m\angle TSU = 28.6^\circ \), find \( m\angle RSU \).
16. If \( m\angle TSU = 46.7^\circ \) and \( m\angle RSU = 83.5^\circ \), find \( m\angle RST \).

17. \( \overrightarrow{SP} \) bisects \( \angle RST \). If \( m\angle RSP = (3x - 2)^\circ \) and \( m\angle PST = (9x - 26)^\circ \), find \( x \).
18. If \( m\angle RSP = \frac{3y}{2}^\circ \) and \( m\angle PST = (y + 5)^\circ \), find \( y \).

**Estimation** Use the following information for Exercises 19–22.

Assume the corner of a sheet of paper is a right angle. Use the corner to estimate the measure and classify each angle in the diagram.

19. \( \angle BOA \)
20. \( \angle COA \)
21. \( \angle EOD \)
22. \( \angle EOB \)

Use a protractor to draw an angle with each of the following measures.

23. \( 33^\circ \)
24. \( 142^\circ \)
25. \( 90^\circ \)
26. \( 168^\circ \)

27. **Surveying** A surveyor at point \( S \) discovers that the angle between peaks \( A \) and \( B \) is 3 times as large as the angle between peaks \( B \) and \( C \). The surveyor knows that \( \angle ASC \) is a right angle. Find \( m\angle ASB \) and \( m\angle BSC \).

28. **Math History** As far back as the 5th century B.C., mathematicians have been fascinated by the problem of trisecting an angle. It is possible to construct an angle with \( \frac{1}{4} \) the measure of a given angle. Explain how to do this.

Find the value of \( x \).

29. \( m\angle AOC = 7x - 2, \ m\angle DOC = 2x + 8, \ m\angle EOD = 27 \)
30. \( m\angle AOB = 4x - 2, \ m\angle BOC = 5x + 10, \ m\angle COD = 3x - 8 \)
31. \( m\angle AOB = 6x + 5, \ m\angle BOC = 4x - 2, \ m\angle AOC = 8x + 21 \)
32. **Multi-Step** \( Q \) is in the interior of right \( \angle PRS \). If \( m\angle PRQ \) is 4 times as large as \( m\angle QRS \), what is \( m\angle PRQ \)?
Data Analysis  Use the circle graph for Exercises 34–36.

34. Find \( m\angle AOB \), \( m\angle BOC \), \( m\angle COD \), and \( m\angle DOA \). Classify each angle as acute, right, or obtuse.

35. What if…? Next year, the music store will use some of the shelves currently holding jazz music to double the space for rap. What will \( m\angle COD \) and \( m\angle BOC \) be next year?

36. Suppose a fifth type of music, salsa, is added. If the space is divided equally among the five types, what will be the angle measure for each type of music in the circle graph?

37. Critical Thinking  Can an obtuse angle be congruent to an acute angle? Why or why not?

38. The measure of an obtuse angle is \((5x + 45)^\circ\). What is the largest value for \(x\)?

39. Write About It  \( \overline{FH} \) bisects \( \angle EFG \). Use the Angle Addition Postulate to explain why \( m\angle EFH = \frac{1}{2}m\angle EFG \).

40. Multi-Step  Use a protractor to draw a 70° angle. Then use a compass and straightedge to bisect the angle. What do you think will be the measure of each angle formed? Use a protractor to support your answer.

41. \( m\angle UOW = 50^\circ \), and \( \overline{OV} \) bisects \( \angle UOW \). What is \( m\angle VOY \)?

   A. 25°  
   B. 65°  
   C. 130°  
   D. 155°

42. What is \( m\angle UOX \)?

   F. 50°  
   G. 115°  
   H. 140°  
   J. 165°

43. \( \overline{BD} \) bisects \( \angle ABC \), \( m\angle ABC = (4x + 5)^\circ \), and \( m\angle ABD = (3x - 1)^\circ \). What is the value of \(x\)?

   A. 2.2  
   B. 3  
   C. 3.5  
   D. 7

44. If an angle is bisected and then 30° is added to the measure of the bisected angle, the result is the measure of a right angle. What is the measure of the original angle?

   F. 30°  
   G. 60°  
   H. 75°  
   J. 120°

45. Short Response  If an obtuse angle is bisected, are the resulting angles acute or obtuse? Explain.
**CHALLENGE AND EXTEND**

46. Find the measure of the angle formed by the hands of a clock when it is 7:00.

47. \( \overline{QS} \) bisects \( \angle PQR \), \( m\angle PQR = (x^2)^\circ \), and \( m\angle PQS = (2x + 6)^\circ \). Find all the possible measures for \( \angle PQR \).

48. For more precise measurements, a degree can be divided into 60 minutes, and each minute can be divided into 60 seconds. An angle measure of 42 degrees, 30 minutes, and 10 seconds is written as 42°30′10″. Subtract this angle measure from the measure 81°24′15″.

49. If 1 degree equals 60 minutes and 1 minute equals 60 seconds, how many seconds are in 2.25 degrees?

50. \( \angle ABC \cong \angle DBC \). \( m\angle ABC = \left( \frac{3x}{2} + 4 \right)^\circ \) and \( m\angle DBC = \left( 2x - 27\frac{1}{4} \right)^\circ \). Is \( \angle ABD \) a straight angle? Explain.

---

**Using Technology  Segment and Angle Bisectors**

1. Construct the bisector of \( \overline{MN} \).

   - Draw \( \overline{MN} \) and construct the midpoint \( B \).
   - Construct a point \( A \) not on the segment.
   - Constructbisector \( \overline{AB} \) and measure \( MB \) and \( NB \).
   - Drag \( M \) and \( N \) and observe \( MB \) and \( NB \).

2. Construct the bisector of \( \angle BAC \).

   - Draw \( \angle BAC \).
   - Construct the angle bisector \( \overrightarrow{AD} \) and measure \( \angle DAC \) and \( \angle DAB \).
   - Drag the angle and observe \( m\angle DAB \) and \( m\angle DAC \).
**Midpoint Formula**

The midpoint $M$ of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is found by

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

**Example 1** Finding the Coordinates of a Midpoint

Find the coordinates of the midpoint of $\overline{CD}$ with endpoints $C(-2, -1)$ and $D(4, 2)$.

$$M\left(\frac{-2 + 4}{2}, \frac{-1 + 2}{2}\right) = \left(\frac{2}{2}, \frac{1}{2}\right) = \left(1, \frac{1}{2}\right)$$

1. Find the coordinates of the midpoint of $\overline{EF}$ with endpoints $E(-2, 3)$ and $F(5, -3)$. 

**Why learn this?**

You can use a coordinate plane to help you calculate distances. (See Example 5.)

Major League baseball fields are laid out according to strict guidelines. Once you know the dimensions of a field, you can use a coordinate plane to find the distance between two of the bases.

A **coordinate plane** is a plane that is divided into four regions by a horizontal line ($x$-axis) and a vertical line ($y$-axis). The location, or coordinates, of a point are given by an ordered pair $(x, y)$.

You can find the midpoint of a segment by using the coordinates of its endpoints. Calculate the average of the $x$-coordinates and the average of the $y$-coordinates of the endpoints.
**Finding the Coordinates of an Endpoint**

**Example 2**

*M* is the midpoint of *AB*. *A* has coordinates (2, 2), and *M* has coordinates (4, –3). Find the coordinates of *B*.

**Step 1** Let the coordinates of *B* equal (*x*, *y*).

**Step 2** Use the Midpoint Formula: \((4, -3) = \left(\frac{2 + x}{2}, \frac{2 + y}{2}\right)\).

**Step 3** Find the *x*-coordinate. Find the *y*-coordinate.

\[
4 = \frac{2 + x}{2} \quad \text{Set the coordinates equal.} \\
2(4) = 2\left(\frac{2 + x}{2}\right) \quad \text{Multiply both sides by 2.} \\
8 = 2 + x \quad \text{Simplify.} \\
-2 = 2 + x \quad \text{Subtract 2 from both sides.} \\
6 = x \quad \text{Simplify.}
\]

\[
-3 = \frac{2 + y}{2} \\
2(-3) = 2\left(\frac{2 + y}{2}\right) \\
-6 = 2 + y \\
-8 = y
\]

The coordinates of *B* are (6, –8).

**Check It Out!**

2. *S* is the midpoint of *RT*. *R* has coordinates (–6, –1), and *S* has coordinates (–1, 1). Find the coordinates of *T*.

The Ruler Postulate can be used to find the distance between two points on a number line. The Distance Formula is used to calculate the distance between two points in a coordinate plane.

**Distance Formula**

In a coordinate plane, the distance *d* between two points \((x_1, y_1)\) and \((x_2, y_2)\) is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]

**Example 3**

Find *AB* and *CD*. Then determine if \(\overline{AB} \cong \overline{CD}\).

**Step 1** Find the coordinates of each point.

*A*(0, 3), *B*(5, 1), *C*(–1, 1), and *D*(–3, –4)

**Step 2** Use the Distance Formula.

\[
a = \sqrt{(5 - 0)^2 + (1 - 3)^2} \\
AB = \sqrt{5^2 + (-2)^2} \\
= \sqrt{25 + 4} \\
= \sqrt{29}
\]

\[
c = \sqrt{[-3 - (-1)]^2 + (-4 - 1)^2} \\
CD = \sqrt{(-2)^2 + (-5)^2} \\
= \sqrt{4 + 25} \\
= \sqrt{29}
\]

Since \(AB = CD\), \(\overline{AB} \cong \overline{CD}\).

**Check It Out!**

3. Find *EF* and *GH*. Then determine if \(\overline{EF} \cong \overline{GH}\).
You can also use the Pythagorean Theorem to find the distance between two points in a coordinate plane. You will learn more about the Pythagorean Theorem later in this course.

In a right triangle, the two sides that form the right angle are the legs. The side across from the right angle that stretches from one leg to the other is the hypotenuse. In the diagram, \( a \) and \( b \) are the lengths of the shorter sides, or legs, of the right triangle. The longest side is called the hypotenuse and has length \( c \).

**Example 4** Finding Distances in the Coordinate Plane

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from \( A \) to \( B \).

**Method 1**

Use the Distance Formula. Substitute the values for the coordinates of \( A \) and \( B \) into the Distance Formula.

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(2 - (-2))^2 + (2 - 3)^2}
\]

\[
= \sqrt{4^2 + (-1)^2}
\]

\[
= \sqrt{16 + 25}
\]

\[
= \sqrt{41}
\]

\[
\approx 6.4
\]

**Method 2**

Use the Pythagorean Theorem. Count the units for sides \( a \) and \( b \).

\[
a = 4 \quad \text{and} \quad b = 5.
\]

\[
c^2 = a^2 + b^2
\]

\[
= 4^2 + 5^2
\]

\[
= 16 + 25
\]

\[
= 41
\]

\[
c \approx 6.4
\]

**Check it Out!**

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, from \( R \) to \( S \).

4a. \( R(3, 2) \) and \( S(-3, -1) \)

4b. \( R(-4, 5) \) and \( S(2, -1) \)
EXAMPLE 5  

**Sports Application**

The four bases on a baseball field form a square with 90 ft sides. When a player throws the ball from home plate to second base, what is the distance of the throw, to the nearest tenth?

Set up the field on a coordinate plane so that home plate $H$ is at the origin, first base $F$ has coordinates $(90, 0)$, second base $S$ has coordinates $(90, 90)$, and third base $T$ has coordinates $(0, 90)$.

The distance $HS$ from home plate to second base is the length of the hypotenuse of a right triangle.

\[
HS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(90 - 0)^2 + (90 - 0)^2} = \sqrt{90^2 + 90^2} = \sqrt{8100 + 8100} = \sqrt{16200} \approx 127.3 \text{ ft}
\]

5. The center of the pitching mound has coordinates $(42.8, 42.8)$. When a pitcher throws the ball from the center of the mound to home plate, what is the distance of the throw, to the nearest tenth?

**THINK AND DISCUSS**

1. Can you exchange the coordinates $(x_1, y_1)$ and $(x_2, y_2)$ in the Midpoint Formula and still find the correct midpoint? Explain.

2. A right triangle has sides lengths of $r$, $s$, and $t$. Given that $s^2 + t^2 = r^2$, which variables represent the lengths of the legs and which variable represents the length of the hypotenuse?

3. Do you always get the same result using the Distance Formula to find distance as you do when using the Pythagorean Theorem? Explain your answer.

4. Why do you think that most cities are laid out in a rectangular grid instead of a triangular or circular grid?

5. GET ORGANIZED  Copy and complete the graphic organizer below. In each box, write a formula. Then make a sketch that will illustrate the formula.
**GUIDED PRACTICE**

1. **Vocabulary** The __ is the side of a right triangle that is directly across from the right angle. (**hypotenuse** or **leg**)

**SEE EXAMPLE 1**

Find the coordinates of the midpoint of each segment.

2. \(\overline{AB}\) with endpoints \(A(4, -6)\) and \(B(-4, 2)\)

3. \(\overline{CD}\) with endpoints \(C(0, -8)\) and \(D(3, 0)\)

**SEE EXAMPLE 2**

4. \(M\) is the midpoint of \(\overline{LN}\). \(L\) has coordinates \((-3, -1)\), and \(M\) has coordinates \((0, 1)\).

Find the coordinates of \(N\).

5. \(B\) is the midpoint of \(\overline{AC}\). \(A\) has coordinates \((-3, 4)\), and \(B\) has coordinates \((-1\frac{1}{2}, 1)\).

Find the coordinates of \(C\).

**SEE EXAMPLE 3**

**Multi-Step** Find the length of the given segments and determine if they are congruent.

6. \(JK\) and \(FG\)

7. \(JK\) and \(RS\)

**SEE EXAMPLE 4**

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

8. \(A(1, -2)\) and \(B(-4, -4)\)

9. \(X(-2, 7)\) and \(Y(-2, -8)\)

10. \(V(2, -1)\) and \(W(-4, 8)\)

**SEE EXAMPLE 5**

**Architecture** The plan for a rectangular living room shows electrical wiring will be run in a straight line from the entrance \(E\) to a light \(L\) at the opposite corner of the room. What is the length of the wire to the nearest tenth?

**PRACTICE AND PROBLEM SOLVING**

Find the coordinates of the midpoint of each segment.

12. \(XY\) with endpoints \(X(-3, -7)\) and \(Y(-1, 1)\)

13. \(MN\) with endpoints \(M(12, -7)\) and \(N(-5, -2)\)

14. \(M\) is the midpoint of \(QR\). \(Q\) has coordinates \((-3, 5)\), and \(M\) has coordinates \((7, -9)\).

Find the coordinates of \(R\).

15. \(D\) is the midpoint of \(\overline{CE}\). \(E\) has coordinates \((-3, -2)\), and \(D\) has coordinates \((2\frac{1}{2}, 1)\).

Find the coordinates of \(C\).

**Multi-Step** Find the length of the given segments and determine if they are congruent.

16. \(DE\) and \(FG\)

17. \(DE\) and \(RS\)
Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

18. \(U(0, 1)\) and \(V(-3, -9)\)  
19. \(M(10, -1)\) and \(N(2, -5)\)  
20. \(P(-10, 1)\) and \(Q(5, 5)\)

21. **Consumer Application**  Televsions and computer screens are usually advertised based on the length of their diagonals. If the height of a computer screen is 11 in. and the width is 14 in., what is the length of the diagonal? Round to the nearest inch.

22. **Multi-Step**  Use the Distance Formula to order \(AB, CD,\) and \(EF\) from shortest to longest.

23. Use the Pythagorean Theorem to find the distance from \(A\) to \(E\). Round to the nearest hundredth.

24. \(X\) has coordinates \((a, 3a)\), and \(Y\) has coordinates \((-5a, 0)\). Find the coordinates of the midpoint of \(XY\).

25. Describe a shortcut for finding the midpoint of a segment when one of its endpoints has coordinates \((a, b)\) and the other endpoint is the origin.

On the map, each square of the grid represents 1 square mile. Find each distance to the nearest tenth of a mile.

26. Find the distance along Highway 201 from Cedar City to Milltown.

27. A car breaks down on Route 1, at the midpoint between Jefferson and Milltown. A tow truck is sent out from Jefferson. How far does the truck travel to reach the car?

28. **History**  The Forbidden City in Beijing, China, is the world’s largest palace complex. Surrounded by a wall and a moat, the rectangular complex is 960 m long and 750 m wide. Find the distance, to the nearest meter, from one corner of the complex to the opposite corner.

29. **Critical Thinking**  Give an example of a line segment with midpoint \((0, 0)\).

The coordinates of the vertices of \(\triangle ABC\) are \(A(1, 4)\), \(B(-2, -1)\), and \(C(-3, -2)\).

30. Find the perimeter of \(\triangle ABC\) to the nearest tenth.

31. The height \(h\) to side \(BC\) is \(\sqrt{2}\), and \(b\) is the length of \(BC\). What is the area of \(\triangle ABC\)?

32. **Write About It**  Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or a vertical line.

33. Tania uses a coordinate plane to map out plans for landscaping a rectangular patio area. On the plan, one square represents 2 feet. She plans to plant a tree at the midpoint of \(AC\). How far from each corner of the patio does she plant the tree? Round to the nearest tenth.
34. Which segment has a length closest to 4 units?
   
   A. \( EF \)  
   B. \( GH \)  
   C. \( JK \)  
   D. \( LM \)
   
35. Find the distance, to the nearest tenth, between the midpoints of \( LM \) and \( JK \).
   
   F. 1.8  
   G. 3.6  
   H. 4.0  
   J. 5.3

36. What are the coordinates of the midpoint of a line segment that connects the points \((7, -3)\) and \((-5, 6)\)?
   
   A. \((6, -4\frac{1}{2})\)  
   B. \((2, 3)\)  
   C. \((2, \frac{1}{2})\)  
   D. \((1, 1\frac{1}{2})\)

37. A coordinate plane is placed over the map of a town. A library is located at \((-5, 1)\), and a museum is located at \((3, 5)\). What is the distance, to the nearest tenth, from the library to the museum?
   
   F. 4.5  
   G. 5.7  
   H. 6.3  
   J. 8.9

**CHALLENGE AND EXTEND**

38. Use the diagram to find the following.
   
   a. \( P \) is the midpoint of \( AB \), and \( R \) is the midpoint of \( BC \). Find the coordinates of \( Q \).
   
   b. Find the area of rectangle \( PBRQ \).
   
   c. Find \( DB \). Round to the nearest tenth.

39. The coordinates of \( X \) are \((a - 5, -2a)\). The coordinates of \( Y \) are \((a + 1, 2a)\). If the distance between \( X \) and \( Y \) is 10, find the value of \( a \).

40. Find two points on the \( y \)-axis that are a distance of 5 units from \((4, 2)\).

41. Given \( \angle ACB \) is a right angle of \( \triangle ABC \), \( AC = x \), and \( BC = y \), find \( AB \) in terms of \( x \) and \( y \).
Quadratic Equations

A quadratic equation is an equation that can be written in the form \( ax^2 + bx + c = 0 \).

Example

Given: \( \triangle ABC \) is isosceles with \( \overline{AB} \cong \overline{AC} \). Solve for \( x \).

\( \textbf{Step 1} \) Set \( x^2 - 5x \) equal to 6 to get \( x^2 - 5x = 6 \).

\( \textbf{Step 2} \) Rewrite the quadratic equation by subtracting 6 from each side to get \( x^2 - 5x - 6 = 0 \).

\( \textbf{Step 3} \) Solve for \( x \).

\( \textbf{Method 1: Factoring} \)

\[
(x - 6)(x + 1) = 0 \quad \text{Factor.}
\]

\( x - 6 = 0 \) or \( x + 1 = 0 \quad \text{Set each factor equal to 0.}

\( x = 6 \quad \text{or} \quad x = -1 \quad \text{Solve.}
\]

\( \textbf{Method 2: Quadratic Formula} \)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{5 \pm \sqrt{49}}{2} \quad \text{Substitute 1 for} \quad a, -5 \text{ for} \quad b, \quad \text{and} \quad -6 \text{ for} \quad c.
\]

\( x = \frac{5 + 7}{2} \quad \text{Simplify.}
\]

\( x = \frac{12}{2} \quad \text{or} \quad x = -2 \quad \text{Find the square root.}
\]

\( x = 6 \text{ or} \quad x = -1 \quad \text{Simplify.}
\]

\( \textbf{Step 4} \) Check each solution in the original equation.

\[
\begin{array}{c|c|c}
\hline
x^2 - 5x & = 6 & x^2 - 5x = 6 \\
(6)^2 - 5(6) & 6 & (-1)^2 - 5(-1) \\
36 - 30 & 6 & 1 + 5 \\
6 & 6 & 6 \\
\hline
\end{array}
\]

Try This

Solve for \( x \) in each isosceles triangle.

1. Given: \( \overline{FE} \cong \overline{FG} \)

\[
x^2 - 3x
\]

2. Given: \( \overline{JK} \cong \overline{JL} \)

\[
x^2 + 4x
\]

3. Given: \( \overline{YZ} \cong \overline{YZ} \)

\[
x^2 - 4x
\]

4. Given: \( \overline{QP} \cong \overline{QR} \)

\[
x^2 + 2x
\]
**Objectives**
- Position figures in the coordinate plane for use in coordinate proofs.
- Prove geometric concepts by using coordinate proof.

**Vocabulary**
- coordinate proof

**Who uses this?**
The Bushmen in South Africa use the Global Positioning System to transmit data about endangered animals to conservationists. (See Exercise 24.)

You have used coordinate geometry to find the midpoint of a line segment and to find the distance between two points. Coordinate geometry can also be used to prove conjectures.

A **coordinate proof** is a style of proof that uses coordinate geometry and algebra. The first step of a coordinate proof is to position the given figure in the plane. You can use any position, but some strategies can make the steps of the proof simpler.

**Strategies for Positioning Figures in the Coordinate Plane**
- Use the origin as a vertex, keeping the figure in Quadrant I.
- Center the figure at the origin.
- Center a side of the figure at the origin.
- Use one or both axes as sides of the figure.

**Example 1**
**Positioning a Figure in the Coordinate Plane**

Position a rectangle with a length of 8 units and a width of 3 units in the coordinate plane.

**Method 1** You can center the longer side of the rectangle at the origin.

**Method 2** You can use the origin as a vertex of the rectangle.

Depending on what you are using the figure to prove, one solution may be better than the other. For example, if you need to find the midpoint of the longer side, use the first solution.

**Check It Out!**
1. Position a right triangle with leg lengths of 2 and 4 units in the coordinate plane. (Hint: Use the origin as the vertex of the right angle.)
Once the figure is placed in the coordinate plane, you can use slope, the coordinates of the vertices, the Distance Formula, or the Midpoint Formula to prove statements about the figure.

**Example 2**

**Writing a Proof Using Coordinate Geometry**

Write a coordinate proof.

**Given:** Right $\triangle ABC$ has vertices $A(0, 6)$, $B(0, 0)$, and $C(4, 0)$. $D$ is the midpoint of $AC$.

**Prove:** The area of $\triangle DBC$ is one half the area of $\triangle ABC$.

**Proof:** $\triangle ABC$ is a right triangle with height $AB$ and base $BC$.

area of $\triangle ABC = \frac{1}{2}bh$

$$= \frac{1}{2}(4)(6) = 12 \text{ square units}$$

By the Midpoint Formula, the coordinates of $D = \left(\frac{0 + 4}{2}, \frac{6 + 0}{2}\right) = (2, 3)$. The $y$-coordinate of $D$ is the height of $\triangle DBC$, and the base is 4 units.

area of $\triangle DBC = \frac{1}{2}bh$

$$= \frac{1}{2}(4)(3) = 6 \text{ square units}$$

Since $6 = \frac{1}{2}(12)$, the area of $\triangle DBC$ is one half the area of $\triangle ABC$.

**Check It Out!**

2. Use the information in Example 2 to write a coordinate proof showing that the area of $\triangle ADB$ is one half the area of $\triangle ABC$.

A coordinate proof can also be used to prove that a certain relationship is always true. You can prove that a statement is true for all right triangles without knowing the side lengths. To do this, assign variables as the coordinates of the vertices.

**Example 3**

**Assigning Coordinates to Vertices**

Position each figure in the coordinate plane and give the coordinates of each vertex.

**A** a right triangle with leg lengths $a$ and $b$

<table>
<thead>
<tr>
<th>$(0, 0)$</th>
<th>$(b, 0)$</th>
</tr>
</thead>
</table>

| $(0, a)$ |

**B** a rectangle with length $c$ and width $d$

<table>
<thead>
<tr>
<th>$(0, 0)$</th>
<th>$(c, 0)$</th>
</tr>
</thead>
</table>

| $(0, d)$ |

| $(c, d)$ |

3. Position a square with side length $4p$ in the coordinate plane and give the coordinates of each vertex.

If a coordinate proof requires calculations with fractions, choose coordinates that make the calculations simpler. For example, use multiples of 2 when you are to find coordinates of a midpoint. Once you have assigned the coordinates of the vertices, the procedure for the proof is the same, except that your calculations will involve variables.
Example 4

Writing a Coordinate Proof

Given: \( \angle B \) is a right angle in \( \triangle ABC \). \( D \) is the midpoint of \( \overline{AC} \).

Prove: The area of \( \triangle DBC \) is one half the area of \( \triangle ABC \).

Step 1 Assign coordinates to each vertex.

Since you will use the Midpoint Formula to find the coordinates of \( D \), use multiples of 2 for the leg lengths.

The coordinates of \( A \) are \((0, 2j)\),

the coordinates of \( B \) are \((0, 0)\),

and the coordinates of \( C \) are \((2n, 0)\).

Step 2 Position the figure in the coordinate plane.

Step 3 Write a coordinate proof.

Proof: \( \triangle ABC \) is a right triangle with height \( 2j \) and base \( 2n \).

\[
\text{area of } \triangle ABC = \frac{1}{2}bh \\
= \frac{1}{2}(2n)(2j) \\
= 2nj \text{ square units}
\]

By the Midpoint Formula, the coordinates of \( D = \left( \frac{0 + 2n}{2}, \frac{2j + 0}{2} \right) = \left( n, j \right) \).

The height of \( \triangle DBC \) is \( j \) units, and the base is \( 2n \) units.

\[
\text{area of } \triangle DBC = \frac{1}{2}bh \\
= \frac{1}{2}(2n)(j) \\
= nj \text{ square units}
\]

Since \( nj = \frac{1}{2}(2nj) \), the area of \( \triangle DBC \) is one half the area of \( \triangle ABC \).

4. Use the information in Example 4 to write a coordinate proof showing that the area of \( \triangle ADB \) is one half the area of \( \triangle ABC \).

Remember!

Because the \( x \)- and \( y \)-axes intersect at right angles, they can be used to form the sides of a right triangle.

Think and Discuss

1. When writing a coordinate proof why are variables used instead of numbers as coordinates for the vertices of a figure?
2. How does the way you position a figure in the coordinate plane affect your calculations in a coordinate proof?
3. Explain why it might be useful to assign \( 2p \) as a coordinate instead of just \( p \).
4. Get Organized Copy and complete the graphic organizer.

In each row, draw an example of each strategy that might be used when positioning a figure for a coordinate proof.

<table>
<thead>
<tr>
<th>Positioning Strategy</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use origin as a vertex.</td>
<td></td>
</tr>
<tr>
<td>Center figure at origin.</td>
<td></td>
</tr>
<tr>
<td>Center side of figure at origin.</td>
<td></td>
</tr>
<tr>
<td>Use axes as sides of figure.</td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary** What is the relationship between coordinate geometry, coordinate plane, and coordinate proof?

Position each figure in the coordinate plane.
2. a rectangle with a length of 4 units and width of 1 unit
3. a right triangle with leg lengths of 1 unit and 3 units

Write a proof using coordinate geometry.
4. Given: Right \( \triangle PQR \) has coordinates \( P(0, 6) \), \( Q(8, 0) \), and \( R(0, 0) \). \( A \) is the midpoint of \( PR \).
   \( B \) is the midpoint of \( QR \).
   Prove: \( AB = \frac{1}{2} PQ \)

Position each figure in the coordinate plane and give the coordinates of each vertex.
5. a right triangle with leg lengths \( m \) and \( n \)
6. a rectangle with length \( a \) and width \( b \)

Multi-Step Assign coordinates to each vertex and write a coordinate proof.
7. Given: \( \angle R \) is a right angle in \( \triangle PQR \). \( A \) is the midpoint of \( PR \).
   \( B \) is the midpoint of \( QR \).
   Prove: \( AB = \frac{1}{2} PQ \)

PRACTICE AND PROBLEM SOLVING

Position each figure in the coordinate plane.
8. a square with side lengths of 2 units
9. a right triangle with leg lengths of 1 unit and 5 units

Write a proof using coordinate geometry.
10. Given: Rectangle \( ABCD \) has coordinates \( A(0, 0) \), \( B(0, 10) \), \( C(6, 10) \), and \( D(6, 0) \). \( E \) is the midpoint of \( AB \), and \( F \) is the midpoint of \( CD \).
    Prove: \( EF = BC \)

Position each figure in the coordinate plane and give the coordinates of each vertex.
11. a square with side length \( 2m \)
12. a rectangle with dimensions \( x \) and \( 3x \)

Multi-Step Assign coordinates to each vertex and write a coordinate proof.
13. Given: \( E \) is the midpoint of \( AB \) in rectangle \( ABCD \). \( F \) is the midpoint of \( CD \).
    Prove: \( EF = AD \)
14. Critical Thinking Use variables to write the general form of the endpoints of a segment whose midpoint is \( (0, 0) \).
15. **Recreation** A hiking trail begins at $E(0, 0)$. Bryan hikes from the start of the trail to a waterfall at $W(3, 3)$ and then makes a $90^\circ$ turn to a campsite at $C(6, 0)$.
   a. Draw Bryan’s route in the coordinate plane.
   b. If one grid unit represents 1 mile, what is the total distance Bryan hiked? Round to the nearest tenth.

Find the perimeter and area of each figure.

16. a right triangle with leg lengths of $a$ and $2a$ units

17. a rectangle with dimensions $s$ and $t$ units

Find the missing coordinates for each figure.

18. $\begin{pmatrix}0, n \end{pmatrix}, \begin{pmatrix}m, m\end{pmatrix}$

19. $\begin{pmatrix}y, y\end{pmatrix}, \begin{pmatrix}p, q\end{pmatrix}$

20. **Conservation** The Bushmen have sighted animals at the following coordinates: $(-25, 31.5)$, $(-23.2, 31.4)$, and $(-24, 31.1)$. Prove that the distance between two of these locations is approximately twice the distance between two other.

21. **Navigation** Two ships depart from a port at $P(20, 10)$. The first ship travels to a location at $A(-30, 50)$, and the second ship travels to a location at $B(70, -30)$. Each unit represents one nautical mile. Find the distance to the nearest nautical mile between the two ships. Verify that the port is at the midpoint between the two.

Write a coordinate proof.

22. **Given:** Rectangle $PQRS$ has coordinates $P(0, 2)$, $Q(3, 2)$, $R(3, 0)$, and $S(0, 0)$. $PR$ and $QS$ intersect at $T(1.5, 1)$.
   **Prove:** The area of $\triangle RST$ is $\frac{1}{4}$ of the area of the rectangle.

23. **Given:** $A(x_1, y_1), B(x_2, y_2)$, with midpoint $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
   **Prove:** $AM = \frac{1}{2}AB$

24. Plot the points on a coordinate plane and connect them to form $\triangle KLM$ and $\triangle MPK$. Write a coordinate proof.
   **Given:** $K(-2, 1), L(-2, 3), M(1, 3), P(1, 1)$
   **Prove:** $\triangle KLM \cong \triangle MPK$

25. **Write About It** When you place two sides of a figure on the coordinate axes, what are you assuming about the figure?

26. Paul designed a doghouse to fit against the side of his house. His plan consisted of a right triangle on top of a rectangle.
   a. Find $BD$ and $CE$.
   b. Before building the doghouse, Paul sketched his plan on a coordinate plane. He placed $A$ at the origin and $AB$ on the $x$-axis. Find the coordinates of $B, C, D,$ and $E$, assuming that each unit of the coordinate plane represents one inch.
27. The coordinates of the vertices of a right triangle are (0, 0), (4, 0), and (0, 2). Which is a true statement?
   - A. The vertex of the right angle is at (4, 2).
   - B. The midpoints of the two legs are at (2, 0) and (0, 1).
   - C. The hypotenuse of the triangle is $\sqrt{6}$ units.
   - D. The shortest side of the triangle is positioned on the x-axis.

28. A rectangle has dimensions of 2g and 2f units. If one vertex is at the origin, which coordinates could NOT represent another vertex?
   - F. (2f, g)
   - G. (2f, 0)
   - H. (2g, 2f)
   - I. (−2f, 2g)

29. The coordinates of the vertices of a rectangle are (0, 0), (a, 0), (a, b), and (0, b). What is the perimeter of the rectangle?
   - A. $a + b$
   - B. $ab$
   - C. $\frac{1}{2}ab$
   - D. $2a + 2b$

30. A coordinate grid is placed over a map. City A is located at (−1, 2) and city C is located at (3, 5). If city C is at the midpoint between city A and city B, what are the coordinates of city B?
   - F. (1, 3.5)
   - G. (−5, −1)
   - H. (7, 8)
   - I. (2, 7)

**CHALLENGE AND EXTEND**

Find the missing coordinates for each figure.

31. The vertices of a right triangle are at (−2s, 2s), (0, 2s), and (0, 0). What coordinates could be used so that a coordinate proof would be easier to complete?

32. Rectangle $ABCD$ has dimensions of 2f and 2g units. The equation of the line containing $BD$ is $y = \frac{g}{f}x$, and the equation of the line containing $AC$ is $y = -\frac{g}{f}x + 2g$. Use algebra to show that the coordinates of $E$ are $(f, g)$.
Who uses this?

Card architects use playing cards to build structures that contain parallel and perpendicular planes.

In 1992, Bryan Berg broke the Guinness World Record for card structures by building a tower 14 feet 6 inches tall. Since then, he has built structures more than 25 feet tall.

Parallel, Perpendicular, and Skew Lines

**Parallel lines** (ǁ) are coplanar and do not intersect. In the figure, \( \overline{AB} \parallel \overline{EF} \), and \( \overline{EG} \parallel \overline{FH} \).

**Perpendicular lines** (⊥) intersect at 90° angles. In the figure, \( \overline{AB} \perp \overline{AE} \), and \( \overline{EG} \perp \overline{GH} \).

**Skew lines** are not coplanar. Skew lines are not parallel and do not intersect. In the figure, \( \overline{AB} \) and \( \overline{EG} \) are skew.

**Parallel planes** are planes that do not intersect. In the figure, plane \( \overline{ABE} \parallel \overline{CDG} \).

Identifying Types of Lines and Planes

**Example 1**

Identify each of the following.

**A** a pair of parallel segments

\( \overline{KN} \parallel \overline{PS} \)

**B** a pair of skew segments

\( \overline{LM} \) and \( \overline{RS} \) are skew.

**C** a pair of perpendicular segments

\( \overline{MR} \perp \overline{RS} \)

**D** a pair of parallel planes

plane \( \overline{KPS} \parallel \overline{LQR} \)

**Check It Out!**

Identify each of the following.

1a. a pair of parallel segments

1b. a pair of skew segments

1c. a pair of perpendicular segments

1d. a pair of parallel planes
**Example 2**

Classifying Pairs of Angles

Give an example of each angle pair.

A. **corresponding angles**  
∠4 and ∠8

B. **alternate interior angles**  
∠4 and ∠6

C. **alternate exterior angles**  
∠2 and ∠8

D. **same-side interior angles**  
∠4 and ∠5

**Check It Out!**

Give an example of each angle pair.

2a. corresponding angles
2b. alternate interior angles
2c. alternate exterior angles
2d. same-side interior angles

**Example 3**

Identifying Angle Pairs and Transversals

Identify the transversal and classify each angle pair.

A. ∠1 and ∠5  
transversal: n; alternate interior angles

B. ∠3 and ∠6  
transversal: m; corresponding angles

C. ∠1 and ∠4  
transversal: ℓ; alternate exterior angles

**Check It Out!**

3. Identify the transversal and classify the angle pair ∠2 and ∠5 in the diagram above.
1. **Vocabulary**  Two angles are located on opposite sides of a transversal, between the two lines that intersect the transversal. (corresponding angles, alternate interior angles, alternate exterior angles, or same-side interior angles)

Identify each of the following.

2. one pair of perpendicular segments
3. one pair of skew segments
4. one pair of parallel segments
5. one pair of parallel planes

Give an example of each angle pair.

6. alternate interior angles
7. alternate exterior angles
8. corresponding angles
9. same-side interior angles

Identify the transversal and classify each angle pair.

10. \(\angle 1\) and \(\angle 2\)
11. \(\angle 2\) and \(\angle 3\)
12. \(\angle 2\) and \(\angle 4\)
13. \(\angle 4\) and \(\angle 5\)
Identify each of the following.

14. one pair of parallel segments
15. one pair of skew segments
16. one pair of perpendicular segments
17. one pair of parallel planes

Give an example of each angle pair.
18. same-side interior angles
19. alternate exterior angles
20. corresponding angles
21. alternate interior angles

Identify the transversal and classify each angle pair.
22. \(\angle 2\) and \(\angle 3\)
23. \(\angle 4\) and \(\angle 5\)
24. \(\angle 2\) and \(\angle 4\)
25. \(\angle 1\) and \(\angle 2\)

26. **Sports** A football player runs across the 30-yard line at an angle. He continues in a straight line and crosses the goal line at the same angle. Describe two parallel lines and a transversal in the diagram.

Name the type of angle pair shown in each letter.
27. F
28. Z
29. C

**Entertainment** Use the following information for Exercises 30–32.
In an Ames room, the floor is tilted and the back wall is closer to the front wall on one side.

30. Name a pair of parallel segments in the diagram.
31. Name a pair of skew segments in the diagram.
32. Name a pair of perpendicular segments in the diagram.
33. **Buildings that are tilted like the one shown are sometimes called mystery spots.**
   - **a.** Name a plane parallel to plane \( KLP \), a plane parallel to plane \( KNP \), and a plane parallel to \( KLM \).
   - **b.** In the diagram, \( QR \) is a transversal to \( PQ \) and \( RS \). What type of angle pair is \( \angle PQR \) and \( \angle QRS \)?

34. **Critical Thinking** Line \( \ell \) is contained in plane \( P \) and line \( m \) is contained in plane \( Q \). If \( P \) and \( Q \) are parallel, what are the possible classifications of \( \ell \) and \( m \)? Include diagrams to support your answer.

Use the diagram for Exercises 35–40.

35. Name a pair of alternate interior angles with transversal \( n \).

36. Name a pair of same-side interior angles with transversal \( \ell \).

37. Name a pair of corresponding angles with transversal \( m \).

38. Identify the transversal and classify the angle pair for \( \angle 3 \) and \( \angle 7 \).

39. Identify the transversal and classify the angle pair for \( \angle 5 \) and \( \angle 8 \).

40. Identify the transversal and classify the angle pair for \( \angle 1 \) and \( \angle 6 \).

41. **Aviation** Describe the type of lines formed by two planes when flight 1449 is flying from San Francisco to Atlanta at 32,000 feet and flight 2390 is flying from Dallas to Chicago at 28,000 feet.

42. **Multi-Step** Draw line \( p \), then draw two lines \( m \) and \( n \) that are both perpendicular to \( p \). Make a conjecture about the relationship between lines \( m \) and \( n \).

43. **Write About It** Discuss a real-world example of skew lines. Include a sketch.

44. Which pair of angles in the diagram are alternate interior angles?
   - **A** \( \angle 1 \) and \( \angle 5 \)
   - **B** \( \angle 2 \) and \( \angle 6 \)
   - **C** \( \angle 7 \) and \( \angle 5 \)
   - **D** \( \angle 2 \) and \( \angle 3 \)

45. How many pairs of corresponding angles are in the diagram?
   - **F** 2
   - **G** 4
   - **H** 8
   - **I** 16
46. Which type of lines are NOT represented in the diagram?  
- A) Parallel lines  
- B) Intersection lines  
- C) Skew lines  
- D) Perpendicular lines

47. For two lines and a transversal, \( \angle 1 \) and \( \angle 8 \) are alternate exterior angles, and \( \angle 1 \) and \( \angle 5 \) are corresponding angles. Classify the angle pair \( \angle 5 \) and \( \angle 8 \).  
- F) Vertical angles  
- G) Alternate interior angles  
- H) Adjacent angles  
- I) Same-side interior angles

48. Which angles in the diagram are NOT corresponding angles?  
- A) \( \angle 1 \) and \( \angle 5 \)  
- B) \( \angle 2 \) and \( \angle 6 \)  
- C) \( \angle 4 \) and \( \angle 8 \)  
- D) \( \angle 2 \) and \( \angle 7 \)

**CHALLENGE AND EXTEND**

Name all the angle pairs of each type in the diagram. Identify the transversal for each pair.

49. Corresponding  
50. Alternate interior  
51. Alternate exterior  
52. Same-side interior

53. **Multi-Step** Draw two lines and a transversal such that \( \angle 1 \) and \( \angle 3 \) are corresponding angles, \( \angle 1 \) and \( \angle 2 \) are alternate interior angles, and \( \angle 3 \) and \( \angle 4 \) are alternate exterior angles. What type of angle pair is \( \angle 2 \) and \( \angle 4 \)?

54. If the figure shown is folded to form a cube, which faces of the cube will be parallel?
Explore Parallel Lines and Transversals

Geometry software can help you explore angles that are formed when a transversal intersects a pair of parallel lines.

Use with Angles Formed by Parallel Lines and Transversals

Activity

1. Construct a line and label two points on the line $A$ and $B$.

2. Create point $C$ not on $\overrightarrow{AB}$. Construct a line parallel to $\overrightarrow{AB}$ through point $C$. Create another point on this line and label it $D$.

3. Create two points outside the two parallel lines and label them $E$ and $F$. Construct transversal $\overrightarrow{EF}$. Label the points of intersection $G$ and $H$.

4. Measure the angles formed by the parallel lines and the transversal. Write the angle measures in a chart like the one below. Drag point $E$ or $F$ and chart with the new angle measures. What relationships do you notice about the angle measures? What conjectures can you make?

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\angle AGE$</th>
<th>$\angle BGE$</th>
<th>$\angle AGH$</th>
<th>$\angle BGH$</th>
<th>$\angle CHG$</th>
<th>$\angle DHG$</th>
<th>$\angle CHF$</th>
<th>$\angle DHF$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measure</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Try This

1. Identify the pairs of corresponding angles in the diagram. What conjecture can you make about their angle measures? Drag a point in the figure to confirm your conjecture.

2. Repeat steps in the previous problem for alternate interior angles, alternate exterior angles, and same-side interior angles.

3. Try dragging point $C$ to change the distance between the parallel lines. What happens to the angle measures in the figure? Why do you think this happens?
**Objective**
Prove and use theorems about the angles formed by parallel lines and a transversal.

**Who uses this?**
Piano makers use parallel strings for the higher notes. The longer strings used to produce the lower notes can be viewed as transversals. (See Example 3.)

When parallel lines are cut by a transversal, the angle pairs formed are either congruent or supplementary.

### Postulate 4-7-1 Corresponding Angles Postulate

<table>
<thead>
<tr>
<th>POSTULATE</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
</table>
| If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent. | ![Diagram of parallel lines and transversal](image) | \( \angle 1 \cong \angle 3 \)  
\( \angle 2 \cong \angle 4 \)  
\( \angle 5 \cong \angle 7 \)  
\( \angle 6 \cong \angle 8 \) |

### Example 1
**Using the Corresponding Angles Postulate**
Find each angle measure.

**A**
\[
\text{m} \angle ABC = 80^\circ \\
\text{m} \angle ABC = x = 80 \\
\text{Corr. \angle Post.}
\]

**B**
\[
\begin{align*}
(2x - 45)^\circ &= (x + 30)^\circ \\
2x - 45 &= x + 30 \\
x &= 75 \\
\text{Corr. \angle Post.} \\
\text{Subtract } x \text{ from both sides.} \\
\text{Add 45 to both sides.} \\
\text{Substitute 75 for } x.
\end{align*}
\]

\[
\text{m} \angle DEF = x + 30 \\
= 75 + 30 \\
= 105^\circ
\]

**1.** Find \( \text{m} \angle QRS \).

Remember that postulates are statements that are accepted without proof. Since the Corresponding Angles Postulate is given as a postulate, it can be used to prove the next three theorems.
Alternate Interior Angles Theorem

Given: \( \ell \parallel m \)

Prove: \( \angle 2 \cong \angle 3 \)

Proof:

1. Given: \( \ell \parallel m \)
2. \( \angle 1 \cong \angle 3 \) (Corr. \& Post.)
3. \( \angle 2 \cong \angle 1 \) (Trans. Prop. of \( \cong \))
4. \( \angle 2 \cong \angle 3 \) (Vert. \( \cong \) Thm.)

You will prove Theorems 4-7-3 and 4-7-4 in Exercises 25 and 26.

Example 2

Finding Angle Measures

Find each angle measure.

A. \( m\angle EDF \)

\[ x = 125 \]

\[ m\angle EDF = 125^\circ \] (Alt. Ext. \( \angle \) Thm.)

B. \( m\angle TUS \)

\[ 13x^\circ + 23x^\circ = 180^\circ \]

\[ 36x = 180 \]

\[ x = 5 \]

\[ m\angle TUS = 23(5) = 115^\circ \] (Substitute 5 for \( x \)).

2. Find \( m\angle ABD \).
**EXAMPLE 3**

**Music Application**

The treble strings of a grand piano are parallel. Viewed from above, the bass strings form transversals to the treble strings. Find \( x \) and \( y \) in the diagram.

By the Alternate Exterior Angles Theorem, \((25x + 5y)^\circ = 125^\circ\).

By the Corresponding Angles Postulate, \((25x + 4y)^\circ = 120^\circ\).

\[
25x + 5y = 125 \\
- (25x + 4y = 120) \\
y = 5 \\
25x + 5(5) = 125 \\
x = 4, y = 5
\]

Subtract the second equation from the first equation.
Substitute 5 for \( y \) in \( 25x + 5y = 125 \). Simplify and solve for \( x \).

3. Find the measures of the acute angles in the diagram.

**THINK AND DISCUSS**

1. Explain why a transversal that is perpendicular to two parallel lines forms eight congruent angles.

2. **GET ORGANIZED** Copy the diagram and graphic organizer. Complete the graphic organizer by explaining why each of the three theorems is true.
GUIDED PRACTICE

SEE EXAMPLE 1

Find each angle measure.

1. \( m\angle JKL \)

\[ \angle JKL = 127^\circ \]

2. \( m\angle BEF \)

\[ \angle BEF = (7x - 14)^\circ \]

\[ \angle BEF = (4x + 19)^\circ \]

SEE EXAMPLE 2

3. \( m\angle 1 \)

SEE EXAMPLE 3

4. \( m\angle CBY \)

5. **Safety** The railing of a wheelchair ramp is parallel to the ramp. Find \( x \) and \( y \) in the diagram.

\[ \angle CBY = (3x + 9)^\circ \]

\[ (4x + 6y)^\circ \]

86\(^\circ\)

94\(^\circ\)

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises See Example

6–7 1
8–11 2
12 3

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.

6. \( m\angle KLM \)

\[ \angle KLM = 115^\circ \]

7. \( m\angle VYX \)

\[ \angle VYX = (2a + 50)^\circ \]

8. \( m\angle ABC \)

\[ \angle ABC = x^\circ \]

\[ \angle A = 116^\circ \]

9. \( m\angle EFG \)

\[ \angle EFG = 13x^\circ \]

\[ 17x^\circ \]

10. \( m\angle PQR \)

\[ \angle PQR = (3n - 45)^\circ \]

\[ \angle Q = (2n + 15)^\circ \]

11. \( m\angle STU \)

\[ \angle STU = (4x - 14)^\circ \]

\[ \angle T = (3x + 12)^\circ \]
12. **Parking** In the parking lot shown, the lines that mark the width of each space are parallel.

\[ m\angle 1 = (2x - 3y)^\circ \]
\[ m\angle 2 = (x + 3y)^\circ \]

Find \( x \) and \( y \).

Find each angle measure. Justify each answer with a postulate or theorem.

13. \( m\angle 1 \)
14. \( m\angle 2 \)
15. \( m\angle 3 \)
16. \( m\angle 4 \)
17. \( m\angle 5 \)
18. \( m\angle 6 \)
19. \( m\angle 7 \)

**Algebra** State the theorem or postulate that is related to the measures of the angles in each pair. Then find the angle measures.

20. \( m\angle 1 = (7x + 15)^\circ \), \( m\angle 2 = (10x - 9)^\circ \)
21. \( m\angle 3 = (23x + 11)^\circ \), \( m\angle 4 = (14x + 21)^\circ \)
22. \( m\angle 4 = (37x - 15)^\circ \), \( m\angle 5 = (44x - 29)^\circ \)
23. \( m\angle 1 = (6x + 24)^\circ \), \( m\angle 4 = (17x - 9)^\circ \)

24. **Architecture** The Luxor Hotel in Las Vegas, Nevada, is a 30-story pyramid. The hotel uses an elevator called an inclinator to take people up the side of the pyramid. The inclinator travels at a 39\(^\circ\) angle. Which theorem or postulate best illustrates the angles formed by the path of the inclinator and each parallel floor? (Hint: Draw a picture.)

25. Complete the two-column proof of the Alternate Exterior Angles Theorem.

Given: \( \ell \parallel m \)
Prove: \( \angle 1 \cong \angle 2 \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \ell \parallel m )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. a. ____? ____</td>
<td>2. Vert. ( \angle ) Thm.</td>
</tr>
<tr>
<td>3. ( \angle 3 \cong \angle 2 )</td>
<td>3. b. ____? ____</td>
</tr>
<tr>
<td>4. c. ____? ____</td>
<td>4. d. ____? ____</td>
</tr>
</tbody>
</table>

26. Write a paragraph proof of the Same-Side Interior Angles Theorem.

Given: \( r \parallel s \)
Prove: \( m\angle 1 + m\angle 2 = 180^\circ \)

Draw the given situation or tell why it is impossible.

27. Two parallel lines are intersected by a transversal so that the corresponding angles are supplementary.

28. Two parallel lines are intersected by a transversal so that the same-side interior angles are complementary.
29. In the diagram, which represents the side view of a mystery spot, $m\angle SRT = 25^\circ$. $\overrightarrow{RT}$ is a transversal to $\overrightarrow{PS}$ and $\overrightarrow{QR}$.
   a. What type of angle pair is $\angle QRT$ and $\angle STR$?
   b. Find $m\angle STR$. Use a theorem or postulate to justify your answer.

30. Land Development A piece of property lies between two parallel streets as shown. $m\angle 1 = (2x + 6)^\circ$, and $m\angle 2 = (3x + 9)^\circ$.
   What is the relationship between the angles? What are their measures?

31. ERROR ANALYSIS In the figure, $m\angle ABC = (15x + 5)^\circ$, and $m\angle BCD = (10x + 25)^\circ$.
   Which value of $m\angle BCD$ is incorrect? Explain.

32. Critical Thinking In the diagram, $\ell \parallel m$.
   Explain why $\frac{x}{y} = 1$.

33. Write About It Suppose that lines $\ell$ and $m$ are intersected by transversal $p$. One of the angles formed by $\ell$ and $p$ is congruent to every angle formed by $m$ and $p$. Draw a diagram showing lines $\ell$, $m$, and $p$, mark any congruent angles that are formed, and explain what you know is true.

34. $m\angle RST = (x + 50)^\circ$, and $m\angle STU = (3x + 20)^\circ$.
   Find $m\angle RVT$. 
   A $15^\circ$   B $27.5^\circ$   C $65^\circ$   D $77.5^\circ$

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35. For two parallel lines and a transversal, $m\angle 1 = 83^\circ$. For which pair of angle measures is the sum the least?
   - F $\angle 1$ and a corresponding angle
   - G $\angle 1$ and a same-side interior angle
   - H $\angle 1$ and its supplement
   - I $\angle 1$ and its complement

36. **Short Response** Given $a \parallel b$ with transversal $t$, explain why $\angle 1$ and $\angle 3$ are supplementary.

---

**CHALLENGE AND EXTEND**

**Multi-Step** Find $m\angle 1$ in each diagram. (*Hint:* Draw a line parallel to the given parallel lines.)

37.

38.

39. Find $x$ and $y$ in the diagram. Justify your answer.

40. Two lines are parallel. The measures of two corresponding angles are $a^\circ$ and $2b^\circ$, and the measures of two same-side interior angles are $a^\circ$ and $b^\circ$. Find the value of $a$. 

---

4-7 Angles Formed by Parallel Lines and Transversals 189
Recall that the converse of a theorem is found by exchanging the hypothesis and conclusion. The converse of a theorem is not automatically true. If it is true, it must be stated as a postulate or proved as a separate theorem.

**Example 1**

Use the Converse of the Corresponding Angles Postulate and the given information to show that \( \ell \parallel m \).

\[ \angle 1 \equiv \angle 5 \]
\[ \angle 1 \equiv \angle 5 \]
\[ \ell \parallel m \]

**A**

\[ \angle 1 \equiv \angle 5 \]
\[ \angle 1 \equiv \angle 5 \]
\[ \ell \parallel m \]

Conv. of Corr. \( \angle \)s Post.

**B**

\[ m \angle 4 = (2x + 10)^\circ, \ m \angle 8 = (3x - 55)^\circ, \ x = 65 \]
\[ m \angle 4 = 2(65) + 10 = 140 \]
\[ m \angle 8 = 3(65) - 55 = 140 \]
\[ m \angle 4 = m \angle 8 \]
\[ \angle 4 \equiv \angle 8 \]
\[ \ell \parallel m \]

Substitute 65 for \( x \).

Trans. Prop. of Equality

Def. of \( \equiv \) \( \angle \)

Conv. of Corr. \( \angle \) Post.

**Check It Out!**

Use the Converse of the Corresponding Angles Postulate and the given information to show that \( \ell \parallel m \).

1a. \( m \angle 1 = m \angle 3 \)

1b. \( m \angle 7 = (4x + 25)^\circ \),
\[ m \angle 5 = (5x + 12)^\circ, \ x = 13 \]
The Converse of the Corresponding Angles Postulate is used to construct parallel lines. The Parallel Postulate guarantees that for any line \( \ell \), you can always construct a parallel line through a point that is not on \( \ell \).

Construction Parallel Lines

1. Draw a line \( \ell \) and a point \( P \) that is not on \( \ell \).
2. Draw a line \( m \) through \( P \) that intersects \( \ell \). Label the angle \( 1 \).
3. Construct an angle congruent to \( \angle 1 \) at \( P \). By the converse of the Corresponding Angles Postulate, \( \ell \parallel n \).

Theorems Proving Lines Parallel

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-8-3</td>
<td>Converse of the Alternate Interior Angles Theorem</td>
<td>If two coplanar lines are cut by a transversal so that a pair of alternate interior angles are congruent, then the two lines are parallel.</td>
</tr>
<tr>
<td>4-8-4</td>
<td>Converse of the Alternate Exterior Angles Theorem</td>
<td>If two coplanar lines are cut by a transversal so that a pair of alternate exterior angles are congruent, then the two lines are parallel.</td>
</tr>
<tr>
<td>4-8-5</td>
<td>Converse of the Same-Side Interior Angles Theorem</td>
<td>If two coplanar lines are cut by a transversal so that a pair of same-side interior angles are supplementary, then the two lines are parallel.</td>
</tr>
</tbody>
</table>

You will prove Theorems 4-8-3 and 4-8-5 in Exercises 38–39.
Converse of the Alternate Exterior Angles Theorem

Given: \( \angle 1 \cong \angle 2 \)
Prove: \( \ell \parallel m \)

Proof: It is given that \( \angle 1 \cong \angle 2 \). Vertical angles are congruent, so \( \angle 1 \cong \angle 3 \). By the Transitive Property of Congruence, \( \angle 2 \cong \angle 3 \). So \( \ell \parallel m \) by the Converse of the Corresponding Angles Postulate.

Example 2: Determining Whether Lines are Parallel

Use the given information and the theorems you have learned to show that \( r \parallel s \).

A. \( \angle 2 \cong \angle 6 \)

\( \angle 2 \cong \angle 6 \) and \( \angle 6 \) are alternate interior angles.

\( \ell \parallel m \) \hspace{1cm} \text{Conv. of Alt. Int. \( \triangle \) Thm.}

B. \( m\angle 6 = (6x + 18)^\circ, m\angle 7 = (9x + 12)^\circ, x = 10 \)

\( m\angle 6 = 6x + 18 \)
\( m\angle 7 = 9x + 12 \)
\( m\angle 6 + m\angle 7 = 78^\circ + 102^\circ = 180^\circ \) \hspace{1cm} \text{\( \angle 6 \) and \( \angle 7 \) are same-side interior angles.}

\( r \parallel s \) \hspace{1cm} \text{Conv. of Same-Side Int. \( \triangle \) Thm.}

Check It Out! Refer to the diagram above. Use the given information and the theorems you have learned to show that \( r \parallel s \).

2a. \( m\angle 4 = m\angle 8 \) \hspace{1cm} 2b. \( m\angle 3 = 2x^\circ, m\angle 7 = (x + 50)^\circ, x = 50 \)

Example 3: Proving Lines Parallel

Given: \( \ell \parallel m, \angle 1 \cong \angle 3 \)
Prove: \( \ell \parallel p \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \ell \parallel m )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \cong \angle 2 )</td>
<td>2. Corr. ( \triangle ) Post.</td>
</tr>
<tr>
<td>3. ( \angle 1 \cong \angle 3 )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \angle 2 \cong \angle 3 )</td>
<td>4. Trans. Prop. of ( \cong )</td>
</tr>
<tr>
<td>5. ( \ell \parallel p )</td>
<td>5. Conv. of Alt. Ext. ( \triangle ) Thm.</td>
</tr>
</tbody>
</table>

Check It Out! 3. Given: \( \angle 1 \cong \angle 4, \angle 3 \) and \( \angle 4 \) are supplementary.

Prove: \( \ell \parallel m \)
EXAMPLE 4

*Sports Application*

During a race, all members of a rowing team should keep the oars parallel on each side. If \( m\angle 1 = (3x + 13)^\circ \), \( m\angle 2 = (5x - 5)^\circ \), and \( x = 9 \), show that the oars are parallel.

A line through the center of the boat forms a transversal to the two oars on each side of the boat.

\( \angle 1 \) and \( \angle 2 \) are corresponding angles. If \( \angle 1 \cong \angle 2 \), then the oars are parallel.

Substitute 9 for \( x \) in each expression:

\[
\begin{align*}
m\angle 1 &= 3x + 13 \\
&= 3(9) + 13 = 40^\circ \\
m\angle 2 &= 5x - 5 \\
&= 5(9) - 5 = 40^\circ \\
\end{align*}
\]

Substitute 9 for \( x \) in each expression. \( m\angle 1 = m\angle 2 \), so \( \angle 1 \cong \angle 2 \).

The corresponding angles are congruent, so the oars are parallel by the Converse of the Corresponding Angles Postulate.

4. **What if...?** Suppose the corresponding angles on the opposite side of the boat measure \((4y - 2)^\circ\) and \((3y + 6)^\circ\), where \( y = 8 \). Show that the oars are parallel.

**THINK AND DISCUSS**

1. Explain three ways of proving that two lines are parallel.

2. If you know \( m\angle 1 \), how could you use the measures of \( \angle 5, \angle 6, \angle 7, \) or \( \angle 8 \) to prove \( m \parallel n \)?

3. **GET ORGANIZED** Copy and complete the graphic organizer. Use it to compare the Corresponding Angles Postulate with the Converse of the Corresponding Angles Postulate.
GUIDED PRACTICE

**SEE EXAMPLE 1**

Use the Converse of the Corresponding Angles Postulate and the given information to show that \( p \parallel q \).

1. \( \angle 4 \cong \angle 5 \)
2. \( m\angle 1 = (4x + 16)^\circ, \ m\angle 8 = (5x - 12)^\circ, \ x = 28 \)
3. \( m\angle 4 = (6x - 19)^\circ, \ m\angle 5 = (3x + 14)^\circ, \ x = 11 \)

**SEE EXAMPLE 2**

Use the theorems and given information to show that \( r \parallel s \).

4. \( \angle 1 \cong \angle 5 \)
5. \( m\angle 3 + m\angle 4 = 180^\circ \)
6. \( \angle 3 \cong \angle 7 \)
7. \( m\angle 4 = (13x - 4)^\circ, \ m\angle 8 = (9x + 16)^\circ, \ x = 5 \)
8. \( m\angle 8 = (17x + 37)^\circ, \ m\angle 7 = (9x - 13)^\circ, \ x = 6 \)
9. \( m\angle 2 = (25x + 7)^\circ, \ m\angle 6 = (24x + 12)^\circ, \ x = 5 \)

**SEE EXAMPLE 3**

Complete the following two-column proof.

Given: \( \angle 1 \cong \angle 2, \ \angle 3 \cong \angle 1 \)
Prove: \( XY \parallel WV \)
Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 \cong \angle 2, \ \angle 3 \cong \angle 1 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 2 \cong \angle 3 )</td>
<td>2. a. ?</td>
</tr>
<tr>
<td>3. b. ?</td>
<td>3. c. ?</td>
</tr>
</tbody>
</table>

**SEE EXAMPLE 4**

11. **Architecture** In the fire escape, \( m\angle 1 = (17x + 9)^\circ, \ m\angle 2 = (14x + 18)^\circ \), and \( x = 3 \). Show that the two landings are parallel.

PRACTICE AND PROBLEM SOLVING

Use the Converse of the Corresponding Angles Postulate and the given information to show that \( \ell \parallel m \).

12. \( \angle 3 \cong 7 \)
13. \( m\angle 4 = 54^\circ, \ m\angle 8 = (7x + 5)^\circ, \ x = 7 \)
14. \( m\angle 2 = (8x + 4)^\circ, \ m\angle 6 = (11x - 41)^\circ, \ x = 15 \)
15. \( m\angle 1 = (3x + 19)^\circ, \ m\angle 5 = (4x + 7)^\circ, \ x = 12 \)
Use the theorems and given information to show that \( n \parallel p \).

16. \( \angle 3 \cong \angle 6 \)
17. \( \angle 2 \cong \angle 7 \)
18. \( m\angle 4 + m\angle 6 = 180^\circ \)
19. \( m\angle 1 = (8x - 7)^\circ, m\angle 8 = (6x + 21)^\circ, x = 14 \)
20. \( m\angle 4 = (4x + 3)^\circ, m\angle 5 = (5x - 22)^\circ, x = 25 \)
21. \( m\angle 3 = (2x + 15)^\circ, m\angle 5 = (3x + 15)^\circ, x = 30 \)
22. Complete the following two-column proof.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AB} \parallel \overline{CD} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \cong \angle 3 )</td>
<td>2. a. ____?</td>
</tr>
<tr>
<td>3. ( \angle 1 \cong \angle 2, \angle 3 \cong \angle 4 )</td>
<td>3. b. ____?</td>
</tr>
<tr>
<td>4. ( \angle 2 \cong \angle 4 )</td>
<td>4. c. ____?</td>
</tr>
<tr>
<td>5. d. ____?</td>
<td>5. e. ____?</td>
</tr>
</tbody>
</table>

23. Art Edmund Dulac used perspective when drawing the floor titles in an illustration for *The Wind's Tale* by Hans Christian Andersen. Show that \( \overline{DJ} \parallel \overline{EK} \) if \( m\angle 1 = (3x + 2)^\circ \), \( m\angle 2 = (5x - 10)^\circ \), and \( x = 6 \).

Name the postulate or theorem that proves that \( \ell \parallel m \).

24. \( \angle 8 \cong \angle 6 \)
25. \( \angle 8 \cong \angle 4 \)
26. \( \angle 2 \cong \angle 6 \)
27. \( \angle 7 \cong \angle 5 \)
28. \( \angle 3 \cong \angle 7 \)
29. \( m\angle 2 + m\angle 3 = 180^\circ \)

For the given information, tell which pair of lines must be parallel. Name the postulate or theorem that supports your answer.

30. \( m\angle 2 = m\angle 10 \)
31. \( m\angle 8 + m\angle 9 = 180^\circ \)
32. \( \angle 1 \cong \angle 7 \)
33. \( m\angle 10 = m\angle 6 \)
34. \( \angle 11 \cong \angle 5 \)
35. \( m\angle 2 + m\angle 5 = 180^\circ \)

36. Multi-Step Two lines are intersected by a transversal so that \( \angle 1 \) and \( \angle 2 \) are corresponding angles, \( \angle 1 \) and \( \angle 3 \) are alternate exterior angles, and \( \angle 3 \) and \( \angle 4 \) are corresponding angles. If \( \angle 2 \cong \angle 4 \), what theorem or postulate can be used to prove the lines parallel?
37. In the diagram, which represents the side view of a mystery spot, \( \angle SRT = 25^\circ \), and \( \angle SUR = 65^\circ \).
   a. Name a same-side interior angle of \( \angle SUR \) for lines \( SU \) and \( RT \) with transversal \( RU \). What is its measure? Explain your reasoning.
   b. Prove that \( SU \) and \( RT \) are parallel.

38. Complete the flowchart proof of the Converse of the Alternate Interior Angles Theorem.
   Given: \( \angle 2 \cong \angle 3 \)
   Prove: \( \ell \parallel m \)
   Proof:
   \[ \angle 2 \cong \angle 3 \]
   [Given]
   \[ \angle 1 \cong \angle 3 \]
   [Vert. \( \triangle \) Thm.] (a. ?)
   \[ \angle ? \]
   [b. ?]
   \[ \angle ? \]
   [c. ?]
   \[ \angle ? \]
   [d. ?]

39. Use the diagram to write a paragraph proof of the Converse of the Same-Side Interior Angles Theorem.
   Given: \( \angle 1 \) and \( \angle 2 \) are supplementary.
   Prove: \( \ell \parallel m \)

40. **Carpentry** A **plumb bob** is a weight hung at the end of a string, called a **plumb line**. The weight pulls the string down so that the plumb line is perfectly vertical. Suppose that the angle formed by the wall and the roof is \( 123^\circ \) and the angle formed by the plumb line and the roof is \( 123^\circ \). How does this show that the wall is perfectly vertical?

41. **Critical Thinking** Are the Reflexive, Symmetric, and Transitive Properties true for parallel lines? Explain why or why not.
   Reflexive: \( \ell \parallel \ell \)
   Symmetric: If \( \ell \parallel m \), then \( m \parallel \ell \).
   Transitive: If \( \ell \parallel m \) and \( m \parallel n \), then \( \ell \parallel n \).

42. **Write About It** Does the information given in the diagram allow you to conclude that \( a \parallel b \)? Explain.

43. Which postulate or theorem can be used to prove \( \ell \parallel m \)?
   - **A** Converse of the Corresponding Angles Postulate
   - **B** Converse of the Alternate Interior Angles Theorem
   - **C** Converse of the Alternate Exterior Angles Theorem
   - **D** Converse of the Same-Side Interior Angles Theorem
44. Two coplanar lines are cut by a transversal. Which condition does NOT guarantee that the two lines are parallel?
   A) A pair of alternate interior angles are congruent.
   B) A pair of same-side interior angles are supplementary.
   C) A pair of corresponding angles are congruent.
   D) A pair of alternate exterior angles are complementary.

45. **Gridded Response** Find the value of $x$ so that $\ell \parallel m$.

46. **Challenge and Extend**
Determine which lines, if any, can be proven parallel using the given information. Justify your answers.

46. $\angle 1 \cong \angle 15$
47. $\angle 8 \cong \angle 14$
48. $\angle 3 \cong \angle 7$
49. $\angle 8 \cong \angle 10$
50. $\angle 6 \cong \angle 8$
51. $\angle 13 \cong \angle 11$
52. $m\angle 12 + m\angle 15 = 180^\circ$
53. $m\angle 5 + m\angle 8 = 180^\circ$
54. Write a paragraph proof that $AE \parallel BD$.

55. **Given:** $m\angle 2 + m\angle 3 = 180^\circ$
    **Prove:** $\ell \parallel m$
56. **Given:** $m\angle 2 + m\angle 5 = 180^\circ$
    **Prove:** $\ell \parallel n$
Construct Parallel Lines

You have learned one method of constructing parallel lines using a compass and straightedge. Another method, called the rhombus method, uses a property of a figure called a rhombus. The rhombus method is shown below.

**Activity 1**

1. Draw a line ℓ and a point \( P \) not on the line.

2. Choose a point \( Q \) on the line. Place your compass point at \( Q \) and draw an arc through \( P \) that intersects \( ℓ \). Label the intersection \( R \).

3. Using the same compass setting as the first arc, draw two more arcs: one from \( P \), the other from \( R \). Label the intersection of the two arcs \( S \).

4. Draw \( PS \parallel ℓ \).

**Try This**

1. Repeat Activity 1 using a different point not on the line. Are your results the same?

2. Using the lines you constructed in Problem 1, draw transversal \( PQ \). Verify that the lines are parallel by using a protractor to measure alternate interior angles.

3. What postulate ensures that this construction is always possible?

4. A rhombus is a quadrilateral with four congruent sides. Explain why this method is called the rhombus method.
**Activity 2**

1. Draw a line \( \ell \) and point \( P \) on a piece of patty paper.

2. Fold the paper through \( P \) so that both sides of line \( \ell \) match up.

3. Crease the paper to form line \( m \). \( P \) should be on line \( m \).

4. Fold the paper again through \( P \) so that both sides of line \( m \) match up.

5. Crease the paper to form line \( n \). Line \( n \) is parallel to line \( \ell \) through \( P \).

**Try This**

5. Repeat Activity 2 using a point in a different place not on the line. Are your results the same?

6. Use a protractor to measure corresponding angles. How can you tell that the lines are parallel?

7. Draw a triangle and construct a line parallel to one side through the vertex that is not on that side.

8. Line \( m \) is perpendicular to both \( \ell \) and \( n \). Use this statement to complete the following conjecture: If two lines in a plane are perpendicular to the same line, then ______? ______.
Why learn this?
Rip currents are strong currents that flow away from the shoreline and are perpendicular to it. A swimmer who gets caught in a rip current can get swept far out to sea. (See Example 3.)

The perpendicular bisector of a segment is a line perpendicular to a segment at the segment’s midpoint. A construction of a perpendicular bisector is shown below.

Construction Perpendicular Bisector of a Segment

1. Draw \( \overline{AB} \). Open the compass wider than half of \( AB \) and draw an arc centered at \( A \).
2. Using the same compass setting, draw an arc centered at \( B \) that intersects the first arc at \( C \) and \( D \).
3. Draw \( \overrightarrow{CD} \). \( \overrightarrow{CD} \) is the perpendicular bisector of \( \overline{AB} \).

The shortest segment from a point to a line is perpendicular to the line. This fact is used to define the **distance from a point to a line** as the length of the perpendicular segment from the point to the line.

**Example 1** Distance From a Point to a Line

A. Name the shortest segment from \( P \) to \( \overrightarrow{AC} \).
   The shortest distance from a point to a line is the length of the perpendicular segment, so \( PB \) is the shortest segment from \( P \) to \( \overrightarrow{AC} \).

   \[ PA > PB \]
   \[ x + 3 > 5 \]
   \[ -3 \quad -3 \]
   \[ x > 2 \]

B. Write and solve an inequality for \( x \).
   - \( PA > PB \)  \( PB \) is the shortest segment.
   - \( x + 3 > 5 \)  Substitute \( x + 3 \) for \( PA \) and 5 for \( PB \).
   - \( -3 \quad -3 \)
   - \( x > 2 \)

**Check It Out!**

1a. Name the shortest segment from \( A \) to \( \overrightarrow{BC} \).
1b. Write and solve an inequality for \( x \).
Theorem 4-9-1: If two intersecting lines form a linear pair of congruent angles, then the lines are perpendicular. (2 intersecting lines form lin. pair of \( \equiv \angle \) \( \rightarrow \) lines \( \perp \).)

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-9-1</td>
<td>( BC \parallel DE, AB \perp BC )</td>
<td>( \ell \perp m )</td>
</tr>
</tbody>
</table>

**Proof:**

Given: \( BC \parallel DE, AB \perp BC \)

Prove: \( AB \perp DE \)

Proof:

It is given that \( BC \parallel DE \), so \( \angle ABC \equiv \angle BDE \) by the Corresponding Angles Postulate. It is also given that \( AB \perp BC \), so \( \angle ABC = 90^\circ \). By the definition of congruent angles, \( m\angle ABC = m\angle BDE \), so \( m\angle BDE = 90^\circ \) by the Transitive Property of Equality. By the definition of perpendicular lines, \( AB \perp DE \).

**Example 2:**

Proving Properties of Lines

Write a two-column proof.

Given: \( \overrightarrow{AD} \parallel \overrightarrow{BC}, \overrightarrow{AD} \perp \overrightarrow{AB}, \overrightarrow{BC} \perp \overrightarrow{DC} \)

Prove: \( \overrightarrow{AB} \parallel \overrightarrow{DC} \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overrightarrow{AD} \parallel \overrightarrow{BC}, \overrightarrow{BC} \perp \overrightarrow{DC} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overrightarrow{AD} \perp \overrightarrow{AB} )</td>
<td>2. ( \perp ) Transv. Thm.</td>
</tr>
<tr>
<td>3. ( \overrightarrow{AD} \perp \overrightarrow{AB} )</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overrightarrow{AB} \parallel \overrightarrow{DC} )</td>
<td>4. 2 lines ( \perp ) to same line ( \rightarrow ) 2 lines ( \parallel ).</td>
</tr>
</tbody>
</table>

**Check It Out!**

2. Write a two-column proof.

Given: \( \angle EHF \equiv \angle HFG, \overrightarrow{FG} \perp \overrightarrow{GH} \)

Prove: \( \overrightarrow{EH} \perp \overrightarrow{GH} \)
**Oceanography Application**

Rip currents may be caused by a sandbar parallel to the shoreline. Waves cause a buildup of water between the sandbar and the shoreline. When this water breaks through the sandbar, it flows out in a direction perpendicular to the sandbar. Why must the rip current be perpendicular to the shoreline?

The rip current forms a transversal to the shoreline and the sandbar.

The shoreline and the sandbar are parallel, and the rip current is perpendicular to the sandbar. So by the Perpendicular Transversal Theorem, the rip current is perpendicular to the shoreline.

3. A swimmer who gets caught in a rip current should swim in a direction perpendicular to the current. Why should the path of the swimmer be parallel to the shoreline?

**THINK AND DISCUSS**

1. Describe what happens if two intersecting lines form a linear pair of congruent angles.

2. Explain why a transversal that is perpendicular to two parallel lines forms eight congruent angles.

3. **GET ORGANIZED** Copy and complete the graphic organizer. Use the diagram and the theorems from this lesson to complete the table.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>If you are given . . .</th>
<th>Then you can conclude . . .</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>$m\angle 1 = m\angle 2$</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>$m\angle 2 = 90^\circ$ $m\angle 3 = 90^\circ$</td>
<td></td>
</tr>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td>$m\angle 2 = 90^\circ$ $m \parallel n$</td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary** \( \overline{CD} \) is the perpendicular bisector of \( \overline{AB} \). \( \overline{CD} \) intersects \( \overline{AB} \) at \( C \).
   What can you say about \( \overline{AB} \) and \( \overline{CD} \)? What can you say about \( \overline{AC} \) and \( \overline{BC} \)?

2. Name the shortest segment from point \( E \) to \( \overline{AD} \).

3. Write and solve an inequality for \( x \).

4. Complete the two-column proof.
   Given: \( \angle ABC \cong \angle CBE \), \( \overline{DE} \perp \overline{AF} \)
   Prove: \( \overline{CB} \parallel \overline{DE} \)
   Proof:
   
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle ABC \cong \angle CBE )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( \overline{CB} \perp \overline{AF} )</td>
<td>a. ______</td>
</tr>
<tr>
<td>3. b. ______</td>
<td>Given</td>
</tr>
<tr>
<td>4. ( \overline{CB} \parallel \overline{DE} )</td>
<td>c. ______</td>
</tr>
</tbody>
</table>

5. **Sports** The center line in a tennis court is perpendicular to both service lines. Explain why the service lines must be parallel to each other.

PRACTICE AND PROBLEM SOLVING

6. Name the shortest segment from point \( W \) to \( \overline{XZ} \).

7. Write and solve an inequality for \( x \).

8. Complete the two-column proof below.
   Given: \( \overline{AB} \perp \overline{BC} \), \( m\angle 1 + m\angle 2 = 180^\circ \)
   Prove: \( \overline{BC} \perp \overline{CD} \)
   Proof:
   
<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{AB} \perp \overline{BC} )</td>
<td>Given</td>
</tr>
<tr>
<td>2. ( m\angle 1 + m\angle 2 = 180^\circ )</td>
<td>a. ______</td>
</tr>
<tr>
<td>3. ( \angle 1 ) and ( \angle 2 ) are supplementary.</td>
<td>Def. of supplementary</td>
</tr>
<tr>
<td>4. b. ______</td>
<td>Converse of the Same-Side Interior Angles Theorem</td>
</tr>
<tr>
<td>5. ( \overline{BC} \perp \overline{CD} )</td>
<td>c. ______</td>
</tr>
</tbody>
</table>
9. **Music**  The frets on a guitar are all perpendicular to one of the strings. Explain why the frets must be parallel to each other.

For each diagram, write and solve an inequality for $x$.

10. \[2x - 5\]

11. \[9x - 3, 6x + 5\]

**Multi-Step** Solve to find $x$ and $y$ in each diagram.

12. \[\n \begin{align*}
  2x^2 & = \quad (3y - 2x)^{\circ} \\
  (3y - 2x)^{\circ} & = \quad (10x - 4y)^{\circ}
\end{align*}
\]

14. \[\n \begin{align*}
  (2x + y)^{\circ} & = \quad (2x + y)^{\circ} \\
  (10x - 4y)^{\circ} & = \quad (x + y)^{\circ}
\end{align*}
\]

Determine if there is enough information given in the diagram to prove each statement.

16. $\angle 1 \equiv \angle 2$
17. $\angle 1 \equiv \angle 3$
18. $\angle 2 \equiv \angle 3$
19. $\angle 2 \equiv \angle 4$
20. $\angle 3 \equiv \angle 4$
21. $\angle 3 \equiv \angle 5$

22. **Critical Thinking** Are the Reflexive, Symmetric, and Transitive Properties true for perpendicular lines? Explain why or why not.

   **Reflexive:** $\ell \perp \ell$
   **Symmetric:** If $\ell \perp m$, then $m \perp \ell$.
   **Transitive:** If $\ell \perp m$ and $m \perp n$, then $\ell \perp n$.

23. In the diagram, which represents the side view of a mystery spot, $QR \perp PQ$, $PQ \parallel RS$, and $PS \parallel QR$.
   a. Prove $QR \perp RS$ and $PS \perp RS$.
   b. Prove $PQ \perp PS$. 

---

204  Chapter 4  Parallel and Perpendicular Lines
24. **Geography** Felton Avenue, Arlee Avenue, and Viehl Avenue are all parallel. Broadway Street is perpendicular to Felton Avenue. Use the satellite photo and the given information to determine the values of $x$ and $y$.

25. **Estimation** Copy the diagram onto a grid with 1 cm by 1 cm squares. Estimate the distance from point $P$ to line $\ell$.

![Diagram](image)

26. **Critical Thinking** Draw a figure to show that Theorem 4-9-3 is not true if the lines are not in the same plane.

27. Draw a figure in which $AB$ is a perpendicular bisector of $XY$ but $XY$ is not a perpendicular bisector of $AB$.

28. **Write About It** A ladder is formed by rungs that are perpendicular to the sides of the ladder. Explain why the rungs of the ladder are parallel.

**Construction** Construct a segment congruent to each given segment and then construct its perpendicular bisector.

29. 

30. 

31. Which inequality is correct for the given diagram?

- A. $2x + 5 < 3x$
- B. $x > 1$
- C. $2x + 5 > 3x$
- D. $x > 5$

32. In the diagram, $\ell \perp m$. Find $x$ and $y$.

- A. $x = 5$, $y = 7$
- B. $x = 7$, $y = 5$
- C. $x = 90$, $y = 90$
- D. $x = 10$, $y = 5$

33. If $\ell \perp m$, which statement is NOT correct?

- A. $m\angle 2 = 90^\circ$
- B. $m\angle 1 + m\angle 2 = 180^\circ$
- C. $\angle 1 \equiv \angle 2$
- D. $\angle 1 \perp \angle 2$
34. In a plane, both lines \( m \) and \( n \) are perpendicular to both lines \( p \) and \( q \). Which conclusion CANNOT be made?

\[ \text{A} \quad p \parallel q \]
\[ \text{B} \quad m \parallel n \]
\[ \text{C} \quad p \perp q \]
\[ \text{D} \quad \text{All angles formed by lines } m, n, p, \text{ and } q \text{ are congruent.} \]

35. **Extended Response** Lines \( m \) and \( n \) are parallel. Line \( p \) intersects line \( m \) at \( A \) and line \( n \) at \( B \), and is perpendicular to line \( m \).

\( \text{a. What is the relationship between line } n \text{ and line } p? \text{ Draw a diagram to support your answer.} \)

\( \text{b. What is the distance from point } A \text{ to line } n? \text{ What is the distance from point } B \text{ to line } m? \text{ Explain.} \)

\( \text{c. How would you define the distance between two parallel lines in a plane?} \)

**CHALLENGE AND EXTEND**

36. **Multi-Step** Find \( m\angle1 \) in the diagram.

(\( \text{Hint: Draw a line parallel to the given parallel lines.} \))

37. Prove Theorem 4-9-1: If two intersecting lines form a linear pair of congruent angles, then the two lines are perpendicular.

38. Prove Theorem 4-9-3: If two coplanar lines are perpendicular to the same line, then the two lines are parallel to each other.
Construct Perpendicular Lines

You have learned to construct the perpendicular bisector of a segment. This is the basis of the construction of a line perpendicular to a given line through a given point. The steps in the construction are the same whether the point is on or off the line.

Activity

Copy the given line \( \ell \) and point \( P \).

Place the compass point on \( P \) and draw an arc that intersects \( \ell \) at two points. Label the points \( A \) and \( B \).

Construct the perpendicular bisector of \( AB \).

Try This

Copy each diagram and construct a line perpendicular to line \( \ell \) through point \( P \). Use a protractor to verify that the lines are perpendicular.

1. Given a line \( \ell \), draw a point \( P \) not on \( \ell \).  
   
2. Construct line \( m \) perpendicular to \( \ell \) through \( P \).

   
3. Follow the steps below to construct two parallel lines. Explain why \( \ell \parallel n \).

   Step 1 Given a line \( \ell \), draw a point \( P \) not on \( \ell \).  
   
   Step 2 Construct line \( m \) perpendicular to \( \ell \) through \( P \).  
   
   Step 3 Construct line \( n \) perpendicular to \( m \) through \( P \).
Complete the sentences below with vocabulary words from the list above.

1. A(n) ___________ divides an angle into two congruent angles.
2. ___________ are two angles whose measures have a sum of 90°.
3. The length of the longest side of a right triangle is called the ___________.

4-6 Understanding Points, Lines, and Planes

**E X A M P L E S**

- Name the common endpoint of $\overrightarrow{SR}$ and $\overrightarrow{ST}$.

$\overrightarrow{SR}$ and $\overrightarrow{ST}$ are opposite rays with common endpoint $S$.

**E X E R C I S E S**

Name each of the following.

4. four coplanar points
5. line containing $B$ and $C$
6. plane that contains $A$, $G$, and $E$
Draw and label three coplanar lines intersecting in one point.

Draw and label each of the following.
7. line containing P and Q
8. pair of opposite rays both containing C
9. CD intersecting plane P at B

4-2 Measuring and Constructing Segments

**Examples**
- Find the length of XY.
  \[ XY = | -2 - 1 | = | -3 | = 3 \]

- S is between R and T. Find RT.
  \[ RT = RS + ST \]
  \[ 3x + 2 = 5x - 6 + 2x \]
  \[ 3x + 2 = 7x - 6 \]
  \[ x = 2 \]
  \[ RT = 3(2) + 2 = 8 \]

**Exercises**
Find each length.
10. JL
11. HK
12. Y is between X and Z, XY = 13.8, and XZ = 21.4. Find YZ.
13. Q is between P and R. Find PR.
14. U is the midpoint of TV, TU = 3x + 4, and UV = 5x − 2. Find TU, UV, and TV.
15. E is the midpoint of DF, DE = 9x, and EF = 4x + 10. Find DE, EF, and DF.

4-3 Measuring and Constructing Angles

**Examples**
- Classify each angle as acute, right, or obtuse.
  \[ \angle ABC \text{ acute; } \angle CBD \text{ acute; } \angle ABD \text{ obtuse; } \angle DBC \text{ acute; } \angle CBE \text{ obtuse} \]

- KM bisects \( \angle JKL \), \( m \angle JKM = (3x + 4)^\circ \), and \( m \angle MKL = (6x - 5)^\circ \). Find \( m \angle JKL \).
  \[ 3x + 4 = 6x - 5 \]  
  Def. of \( \angle \) bisector
  \[ 3x + 9 = 6x \]  
  Add 5 to both sides.
  \[ 9 = 3x \]  
  Subtract 3x from both sides.
  \[ x = 3 \]  
  Divide both sides by 3.

  \[ m \angle JKL = 3x + 4 + 6x - 5 \]
  \[ = 9x - 1 \]
  \[ = 9(3) - 1 = 26^\circ \]

**Exercises**
16. Classify each angle as acute, right, or obtuse.
17. \( m \angle HJL = 116^\circ \). Find \( m \angle HKJ \).
18. \( NP \) bisects \( \angle MNQ \), \( m \angle MNP = (6x - 12)^\circ \), and \( m \angle PNQ = (4x + 8)^\circ \). Find \( m \angle MNQ \).
4-4 Pairs of Angles

**EXAMPLES**

- Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.
  
  \[ \angle 1 \text{ and } \angle 2 \text{ are only adjacent.} \]
  \[ \angle 2 \text{ and } \angle 4 \text{ are not adjacent.} \]
  \[ \angle 2 \text{ and } \angle 3 \text{ are adjacent and form a linear pair.} \]
  \[ \angle 1 \text{ and } \angle 4 \text{ are adjacent and form a linear pair.} \]

- Find the measure of the complement and supplement of each angle.
  
  \[
  90 - 67.3 = 22.7^\circ \\
  180 - 67.3 = 112.7^\circ \\
  90 - (3x - 8) = (98 - 3x)^\circ \\
  180 - (3x - 8) = (188 - 3x)^\circ 
  \]

4-5 Using Formulas in Geometry

**EXAMPLES**

- Find the perimeter and area of the triangle.
  
  \[
  P = 2x + 3x + 5 + 10 = 5x + 15 \\
  A = \frac{1}{2}(3x + 5)(2x) = 3x^2 + 5x 
  \]

- Find the circumference and area of the circle to the nearest tenth.
  
  \[
  C = 2\pi r = 2\pi (11) = 22\pi \approx 69.1 \text{ cm} \\
  A = \pi r^2 = \pi (11)^2 = 121\pi \approx 380.1 \text{ cm}^2 
  \]

**EXERCISES**

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

19. \( \angle 1 \text{ and } \angle 2 \)
20. \( \angle 3 \text{ and } \angle 4 \)
21. \( \angle 2 \text{ and } \angle 5 \)

Find the measure of the complement and supplement of each angle.

22. \( 74.6^\circ \)
23. \( (2x - 4)^\circ \)
24. An angle measures 5 degrees more than 4 times its complement. Find the measure of the angle.

Find the perimeter and area of each figure.

25. \( 4x - 1 \)
26. \( x + 4 \)
27. \( 12 \text{ and } 8 \)
28. \( 20 \)
29. \( 21 \text{ m} \)
30. \( 14 \text{ ft} \)
31. The area of a triangle is 102 m\(^2\). The base of the triangle is 17 m. What is the height of the triangle?
**4-6 Midpoint and Distance in the Coordinate Plane**

**EXAMPLES**

- X is the midpoint of CD. C has coordinates (−4, 1), and X has coordinates (3, −2). Find the coordinates of D.

\[
(3, -2) = \left( \frac{-4 + x}{2}, \frac{1 + y}{2} \right)
\]

\[
3 = \frac{-4 + x}{2} \quad -2 = \frac{1 + y}{2}
\]

\[
x = 10 \quad y = -5
\]

The coordinates of D are (10, −5).

- Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
c^2 = a^2 + b^2
\]

\[
= \sqrt{3^2 + (-4)^2} \quad = 3^2 + 4^2
\]

\[
= \sqrt{9 + 16} \quad = 9 + 16 = 25
\]

\[
= 5.0
\]

**EXERCISES**

Y is the midpoint of AB. Find the missing coordinates of each point.

32. A(3, 2); B(−1, 4); Y(\_\_\_\_)

33. A(5, 0); B(\_\_\_\_); Y(−2, 3)

34. A(\_\_\_\_); B(−4, 4); Y(−2, 3)

Use the Distance Formula and the Pythagorean Theorem to find the distance, to the nearest tenth, between each pair of points.

35. X(−2, 4) and Y(6, 1)

36. H(0, 3) and K(−2, −4)

37. L(−4, 2) and M(3, −2)

**4-7 Transformations in the Coordinate Plane**

**EXAMPLES**

- Identify the transformation. Then use arrow notation to describe the transformation.

The transformation is a reflection.

\[\triangle ABC \rightarrow \triangle A'B'C'\]

- The coordinates of the vertices of rectangle HJKL are H(2, −1), J(5, −1), K(5, −3), and L(2, −3). Find the coordinates of the image of rectangle HJKL after the translation \((x, y) \rightarrow (x - 4, y + 1)\).

\[H' = (2 - 4, -1 + 1) = H'(-2, 0)\]

\[J' = (5 - 4, -1 + 1) = J'(1, 0)\]

\[K' = (5 - 4, -3 + 1) = K'(1, -2)\]

\[L' = (2 - 4, -3 + 1) = L'(-2, -2)\]

**EXERCISES**

Identify each transformation. Then use arrow notation to describe the transformation.

38. \[\begin{array}{c}
E \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \Quad
1. Draw and label plane \( \mathcal{N} \) containing two lines that intersect at \( B \).

Use the figure to name each of the following.

2. four noncoplanar points
3. line containing \( B \) and \( E \)
4. The coordinate of \( A \) is \(-3\), and the coordinate of \( B \) is \( 0.5 \). Find \( AB \).
5. \( E \), \( F \), and \( G \) represent mile markers along a straight highway. Find \( EF \).
6. \( J \) is the midpoint of \( HK \). Find \( HJ \), \( JK \), and \( HK \).

Classify each angle by its measure.

7. \( \angle LMP = 70^\circ \)
8. \( \angle QMN = 90^\circ \)
9. \( \angle PMN = 125^\circ \)
10. \( \overline{TV} \) bisects \( \angle RTS \). If the \( \angle RTV = (16x - 6)^\circ \) and \( \angle VTS = (13x + 9)^\circ \), what is the \( \angle RTV \)?
11. An angle’s measure is 5 degrees less than 3 times the measure of its supplement. Find the measure of the angle and its supplement.

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

12. \( \angle 2 \) and \( \angle 3 \)
13. \( \angle 4 \) and \( \angle 5 \)
14. \( \angle 1 \) and \( \angle 4 \)
15. Find the perimeter and area of a rectangle with \( b = 8 \) ft and \( h = 4 \) ft.

Find the circumference and area of each circle to the nearest tenth.

16. \( r = 15 \) m
17. \( d = 25 \) ft
18. \( d = 2.8 \) cm
19. Find the midpoint of the segment with endpoints \((-4, 6) \) and \((3, 2) \).
20. \( M \) is the midpoint of \( \overline{LN} \). \( M \) has coordinates \((-5, 1) \), and \( L \) has coordinates \((2, 4) \). Find the coordinates of \( N \).
21. Given \( A(-5, 1) \), \( B(-1, 3) \), \( C(1, 4) \), and \( D(4, 1) \), is \( \overline{AB} \equiv \overline{CD} \)? Explain.

Identify each transformation. Then use arrow notation to describe the transformation.

22. \( Q \rightarrow S' \rightarrow R' \)
23. \( X \rightarrow W' \rightarrow X' \)
24. A designer used the translation \((x, y) \rightarrow (x + 3, y - 3) \) to transform a triangular-shaped pin \( \triangle ABC \). Find the coordinates and draw the image of \( \triangle ABC \).
Is That Your Foot?
Criminologists use measurements, such as the size of footprints, and functions to help them identify criminals.
**Reading Strategy: Read and Interpret Math Symbols**

It is essential that as you read through each lesson of the textbook, you can interpret mathematical symbols.

**Common Math Symbols**

- Less than: $<$
- Less than or equal to: $\leq$
- Greater than: $>$
- Greater than or equal to: $\geq$
- Square root: $\sqrt{}$
- Absolute value of $x$: $|x|$
- Not equal to: $\neq$

You must be able to translate symbols into words . . .

<table>
<thead>
<tr>
<th>Using Symbols</th>
<th>Using Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\left(\frac{x}{12}\right) - 1 = 21$</td>
<td>Three times the quotient of $x$ and 12, minus 1 equals 21.</td>
</tr>
<tr>
<td>$25x + 6 \geq 17$</td>
<td>Twenty-five times $x$ plus 6 is greater than or equal to 17.</td>
</tr>
<tr>
<td>$</td>
<td>x</td>
</tr>
<tr>
<td>$\sqrt{60 + x} \leq 40$</td>
<td>The square root of the sum of 60 and $x$ is less than or equal to 40.</td>
</tr>
</tbody>
</table>

. . . and words into symbols.

<table>
<thead>
<tr>
<th>Using Words</th>
<th>Using Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>The height of the shed is at least 9 feet.</td>
<td>$h \geq 9$ ft</td>
</tr>
<tr>
<td>The distance is at most one tenth of a mile.</td>
<td>$d \leq 0.1$ mi</td>
</tr>
<tr>
<td>The silo contains more than 600 cubic feet of corn.</td>
<td>$c &gt; 600$ ft$^3$</td>
</tr>
</tbody>
</table>

**Try This**

**Translate the symbols into words.**

1. $x \leq \sqrt{10}$
2. $|x| + 2 > 45$
3. $-5 \leq x < 8$
4. $-6 - \frac{1}{5}x = -32$

**Translate the words into symbols.**

5. There are less than 15 seconds remaining.
6. The tax rate is 8.25 percent of the cost.
7. Ann counted over 100 pennies.
8. Joe can spend at least $22 but no more than $30.
Graphing Relationships

Objectives

- Match simple graphs with situations.
- Graph a relationship.

Vocabulary

- continuous graph
- discrete graph

Who uses this?

Cardiologists can use graphs to analyze their patients’ heartbeats. (See Example 2.)

Graphs can be used to illustrate many different situations. For example, trends shown on a cardiograph can help a doctor see how the patient’s heart is functioning.

To relate a graph to a given situation, use key words in the description.

Example 1: Relating Graphs to Situations

The air temperature was constant for several hours at the beginning of the day and then rose steadily for several hours. It stayed the same temperature for most of the day before dropping sharply at sundown. Choose the graph that best represents this situation.

Step 1 Read the graphs from left to right to show time passing.

Step 2 List key words in order and decide which graph shows them.

<table>
<thead>
<tr>
<th>Key Words</th>
<th>Segment Description</th>
<th>Graphs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Was constant</td>
<td>Horizontal</td>
<td>Graphs A and B</td>
</tr>
<tr>
<td>Rose steadily</td>
<td>Slanting upward</td>
<td>Graphs A and B</td>
</tr>
<tr>
<td>Stayed the same</td>
<td>Horizontal</td>
<td>Graph A and B</td>
</tr>
<tr>
<td>Dropped sharply</td>
<td>Slanting downward</td>
<td>Graph B</td>
</tr>
</tbody>
</table>

Step 3 Pick the graph that shows all the key phrases in order.

- horizontal, slanting upward,
- horizontal, slanting downward

The correct graph is B.

1. The air temperature increased steadily for several hours and then remained constant. At the end of the day, the temperature increased slightly again before dropping sharply. Choose the graph above that best represents this situation.
As seen in Example 1, some graphs are connected lines or curves called continuous graphs. Some graphs are only distinct points. These are called discrete graphs.

The graph on theme-park attendance is an example of a discrete graph. It consists of distinct points because each year is distinct and people are counted in whole numbers only. The values between the whole numbers are not included, since they have no meaning for the situation.

**Example 2** Sketching Graphs for Situations

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

**A** Simon is selling candles to raise money for the school dance. For each candle he sells, the school will get $2.50. He has 10 candles that he can sell.

![Simon's Earnings Graph]

The amount earned (y-axis) increases by $2.50 for each candle Simon sells (x-axis).

Since Simon can only sell whole numbers of candles, the graph is 11 distinct points.

The graph is discrete.

**B** Angelique’s heart rate is being monitored while she exercises on a treadmill. While walking, her heart rate remains the same. As she increases her pace, her heart rate rises at a steady rate. When she begins to run, her heart rate increases more rapidly and then remains high while she runs. As she decreases her pace, her heart rate slows down and returns to her normal rate.

As time passes during her workout (moving left to right along the x-axis), her heart rate (y-axis) does the following:

- remains the same,
- rises at a steady rate,
- increases more rapidly (steeper than previous segment),
- remains high,
- slows down,
- and then returns to her normal rate.

The graph is continuous.

**Check it Out!** Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

2a. Jamie is taking an 8-week keyboarding class. At the end of each week, she takes a test to find the number of words she can type per minute. She improves each week.

2b. Henry begins to drain a water tank by opening a valve. Then he opens another valve. Then he closes the first valve. He leaves the second valve open until the tank is empty.
When sketching or interpreting a graph, pay close attention to the labels on each axis. Both graphs below show a relationship about a child going down a slide. **Graph A** represents the child’s distance from the ground over time. **Graph B** represents the child’s speed over time.

### Example 3

#### Writing Situations for Graphs

Write a possible situation for the given graph.

1. **Step 1** Identify labels.  
   \( x \)-axis: time  
   \( y \)-axis: water level

2. **Step 2** Analyze sections.  
   Over time, the water level  
   • increases steadily,  
   • remains unchanged,  
   • and then decreases steadily.

**Possible Situation:** A watering can is filled with water. It sits for a while until some flowers are planted. The water is then emptied on top of the planted flowers.

#### Check It Out!  

3. Write a possible situation for the given graph.

### Think and Discuss

1. Should a graph of age related to height be a continuous graph or a discrete graph? Explain.

2. Give an example of a situation that, when graphed, would include a horizontal segment.

3. **Get Organized** Copy and complete the graphic organizer. Write an example of key words that suggest the given segments on a graph. One example for each segment is given for you.

![Key Words for Graph Segments]

- Increases
- Decreases
- Stays the same
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. A ______ graph is made of connected lines or curves. (continuous or discrete)
2. A ______ graph is made of only distinct points. (continuous or discrete)

Choose the graph that best represents each situation.
3. A person alternates between running and walking.
4. A person gradually speeds up to a constant running pace.
5. A person walks, gradually speeds up to a run, and then slows back down to a walk.

6. Maxine is buying extra pages for her photo album. Each page holds exactly 8 photos. Sketch a graph to show the maximum number of photos she can add to her album if she buys 1, 2, 3, or 4 extra pages. Tell whether the graph is continuous or discrete.

Write a possible situation for each graph.
7.  
8.  
9.  

PRACTICE AND PROBLEM SOLVING

Choose the graph that best represents each situation.
10. A flag is raised up a flagpole quickly at the beginning and then more slowly near the top.
11. A flag is raised up a flagpole in a jerky motion, using a hand-over-hand method.
12. A flag is raised up a flagpole at a constant rate of speed.
13. For six months, a puppy gained weight at a steady rate. Sketch a graph to illustrate the weight of the puppy during that time period. Tell whether the graph is continuous or discrete.

Write a possible situation for each graph.

14. 

15. 

16. 

17. **Data Collection** Use a graphing calculator and motion detector for the following.
   a. On a coordinate plane, draw a graph relating distance from a starting point walking at various speeds and time.
   b. Using the motion detector as the starting point, walk away from the motion detector to make a graph on the graphing calculator that matches the one you drew.
   c. Compare your walking speeds to each change in steepness on the graph.

**Sports** The graph shows the speed of a horse during and after a race. Use it to describe the changing pace of the horse during the race.

19. **Recreation** You hike up a mountain path starting at 10 A.M. You camp overnight and then walk back down the same path at the same pace at 10 A.M. the next morning. On the same set of axes, graph the relationship between distance from the top of the mountain and the time of day for both the hike up and the hike down. What does the point of intersection of the graphs represent?

20. **Critical Thinking** Suppose that you sketched a graph of speed related to time for a brick that fell from the top of a building. Then you sketched a graph for speed related to time for a ball that was rolled down a hill and then came to rest. How would the graphs be the same? How would they be different?

21. **Write About It** Describe a real-life situation that could be represented by a distinct graph. Then describe a real-life situation that could be represented by a continuous graph.

22. A rectangular pool that is 4 feet deep at all places is being filled at a constant rate.
   a. Sketch a graph to show the depth of the water as it increases over time.
   b. The side view of another swimming pool is shown. If the pool is being filled at a constant rate, sketch a graph to show the depth of the water as it increases over time.

On November 1, 1938, the underdog Seabiscuit beat the heavily favored Triple-Crown winner War Admiral in a historic horse race at Pimlico Race Course in Baltimore, Maryland.
23. Which situation would NOT be represented by a discrete graph?
   - A. Amount of money earned based on the number of cereal bars sold
   - B. Number of visitors to a grocery store per day for one week
   - C. The amount of iced tea in a pitcher at a restaurant during the lunch hour
   - D. The total cost of buying 1, 2, or 3 CDs at the music store

24. Which situation is best represented by the graph?
   - F. A snowboarder starts at the bottom of the hill and takes a ski lift to the top.
   - G. A cruise boat travels at a steady pace from the port to its destination.
   - H. An object falls from the top of a building and gains speed at a rapid pace before hitting the ground.
   - J. A marathon runner starts at a steady pace and then runs faster at the end of the race before stopping at the finish line.

25. **Short Response** Marla participates in a triathlon consisting of swimming, biking, and running. Would a graph of Marla’s speed during the triathlon be a continuous graph or a distinct graph? Explain.

**CHALLENGE AND EXTEND**

Pictured are three vases and graphs representing the height of water as it is poured into each of the vases at a constant rate. Match each vase with the correct graph.

- [A]  
- [B]  
- [C]  

26.  
27.  
28.
Objectives
Identify functions.
Find the domain and range of relations and functions.

Vocabulary
relation
domain
range
function

Why learn this?
You can use a relation to show finishing positions and scores in a track meet.

Previously, you saw relationships represented by graphs. Relationships can also be represented by a set of ordered pairs, called a relation.

In the scoring system of some track meets, first place is worth 5 points, second place is worth 3 points, third place is worth 2 points, and fourth place is worth 1 point. This scoring system is a relation, so it can be shown as ordered pairs, \{(1, 5), (2, 3), (3, 2), (4, 1)\}. You can also show relations in other ways, such as tables, graphs, or mapping diagrams.

Example 1: Showing Multiple Representations of Relations

Express the relation for the track meet scoring system, \{(1, 5), (2, 3), (3, 2), (4, 1)\}, as a table, as a graph, and as a mapping diagram.

Table

<table>
<thead>
<tr>
<th>Place</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

Graph

Use the x- and y-values to plot the ordered pairs.

Mapping Diagram

Write all x-values under “Place” and all y-values under “Points.”

1. Express the relation \{(1, 3), (2, 4), (3, 5)\} as a table, as a graph, and as a mapping diagram.

The domain of a relation is the set of first coordinates (or x-values) of the ordered pairs. The range of a relation is the set of second coordinates (or y-values) of the ordered pairs. The domain of the track meet scoring system is \{1, 2, 3, 4\}. The range is \{5, 3, 2, 1\}. 
Finding the Domain and Range of a Relation

Give the domain and range of the relation.

The domain is all x-values from 1 through 3, inclusive.
The range is all y-values from 2 through 4, inclusive.

D: \(1 \leq x \leq 3\)  
R: \(2 \leq y \leq 4\)

Give the domain and range of each relation.

2a.  

2b.  

A function is a special type of relation that pairs each domain value with exactly one range value.

Identifying Functions

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

A Field Trip

<table>
<thead>
<tr>
<th>Students (x)</th>
<th>Buses (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>2</td>
</tr>
<tr>
<td>68</td>
<td>2</td>
</tr>
<tr>
<td>125</td>
<td>3</td>
</tr>
</tbody>
</table>

D: \(\{75, 68, 125\}\)  
R: \(\{2, 3\}\)

Even though 2 appears twice in the table, it is written only once when writing the range.

This relation is a function. Each domain value is paired with exactly one range value.

B

Use the arrows to determine which domain values correspond to each range value.

D: \(\{7, 9, 12, 15\}\)  
R: \(\{-7, -1, 0\}\)

This relation is not a function. Each domain value does not have exactly one range value. The domain value 7 is paired with the range values \(-1\) and 0.
Give the domain and range of each relation. Tell whether the relation is a function. Explain.

**Check It Out!** Give the domain and range of each relation. Tell whether the relation is a function. Explain.

3a. \((8, 2), (-4, 1), (-6, 2), (1, 9)\)  
3b.

![Graph of a circle]

**Draw lines to see the domain and range values.**

- **Domain:** \(-4 \leq x \leq 4\)
- **Range:** \(-4 \leq y \leq 4\)

<table>
<thead>
<tr>
<th>x</th>
<th>4</th>
<th>0</th>
<th>0</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>4</td>
<td>-4</td>
<td>0</td>
</tr>
</tbody>
</table>

This relation is not a function because there are several domain values that have more than one range value. For example, the domain value 0 is paired with both 4 and -4.

**Think and Discuss**

1. Describe how to tell whether a set of ordered pairs is a function.
2. Can the graph of a vertical line segment represent a function? Explain.
3. **Get Organized** Copy and complete the graphic organizer by explaining when a relation is a function and when it is not a function.

---

**Helpful Hint**

To find the domain and range of a graph, it may help to draw lines to see the x- and y-values.
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. Use a mapping diagram to show a relation that is not a function.
2. The set of x-values for a relation is also called the  ?  . (domain or range)

Express each relation as a table, as a graph, and as a mapping diagram.
3. \{1, 1\}, \{1, 2\}
4. \{-1, 1\}, \{-2, \frac{1}{2}\}, \{-3, \frac{1}{3}\}, \{-4, \frac{1}{4}\}
5. \{-1, 1\}, \{-3, 3\}, \{-5, 5\}, \{-7, 7\}
6. \{0, 0\}, \{-2, -4\}, \{-2, -2\}

Give the domain and range of each relation.
7. \{-5, 7\}, \{0, 0\}, \{-2, 2\}, \{-5, -2\}
8. \{1, 2\}, \{2, 4\}, \{3, 6\}, \{4, 8\}, \{5, 10\}

Multi-Step  Give the domain and range of each relation. Tell whether the relation is a function. Explain.
9. \begin{tabular}{|c|cccccccc|}
\hline
x & 3 & 5 & 2 & 8 & 6 \\
\hline
y & 9 & 25 & 4 & 81 & 36 \\
\hline
\end{tabular}

10. \begin{tabular}{|c|cccc|}
\hline
x & -2 & -1 & 0 & 1 & 2 \\
\hline
y & 1 & 1 & 1 & 1 & 1 \\
\hline
\end{tabular}

PRACTICE AND PROBLEM SOLVING

Express each relation as a table, as a graph, and as a mapping diagram.
15. \{-2, -4\}, \{-1, -1\}, \{0, 0\}, \{1, -1\}, \{2, -4\}
16. \{2, 1\}, \{2, \frac{1}{2}\}, \{2, 2\}, \{2, 2 \frac{1}{2}\}

Give the domain and range of each relation.
17. \begin{tabular}{|c|c|}
\hline
x & y \\
\hline
4 & \\
2 & 2 \\
0 & 4 \\
\hline
\end{tabular}
18. \begin{tabular}{|c|c|}
\hline
x & y \\
\hline
4 & 4 \\
5 & 5 \\
6 & 6 \\
7 & 7 \\
8 & 8 \\
\hline
\end{tabular}
**Multi-Step** Give the domain and range of each relation. Tell whether the relation is a function. Explain.

19. [Graph showing a triangle with vertices at (-2, 2), (0, 0), and (2, 2).]

20. [Graph showing points with coordinates (-1, 3), (0, 2), (1, 1), and (2, 0).]

21. **Consumer Application** An electrician charges a base fee of $75 plus $50 for each hour of work. Create a table that shows the amount the electrician charges for 1, 2, 3, and 4 hours of work. Let $x$ represent the number of hours and $y$ represent the amount charged for $x$ hours. Is this relation a function? Explain.

22. **Geometry** Write a relation as a set of ordered pairs in which the $x$-value represents the side length of a square and the $y$-value represents the area of that square. Use a domain of 2, 4, 6, 9, and 11.

23. **Multi-Step** Create a mapping diagram to display the numbers of days in 1, 2, 3, and 4 weeks. Is this relation a function? Explain.

24. **Nutrition** The illustrations list the number of grams of fat and the number of Calories from fat for selected foods.
   a. Create a graph for the relation between grams of fat and Calories from fat.
   b. Is this relation a function? Explain.

   - **Hamburger** Fat (g): 14, Fat (Cal): 126
   - **Cheeseburger** Fat (g): 18, Fat (Cal): 162
   - **Grilled chicken filet** Fat (g): 3.5, Fat (Cal): 31.5
   - **Breaded chicken filet** Fat (g): 11, Fat (Cal): 99
   - **Taco salad** Fat (g): 19, Fat (Cal): 171

25. **Recreation** A shop rents canoes for a $7 equipment fee plus $2 per hour, with a maximum cost of $15 per day. Express the number of hours $x$ and the cost $y$ as a relation in table form, and find the cost to rent a canoe for 1, 2, 3, 4, and 5 hours. Is this relation a function? Explain.

26. **Health** You can burn about 6 Calories per minute bicycling. Let $x$ represent the number of minutes bicycled, and let $y$ represent the number of Calories burned.
   a. Write ordered pairs to show the number of Calories burned by bicycling for 60, 120, 180, 240, or 300 minutes. Graph the ordered pairs.
   b. Find the domain and range of the relation.
   c. Does this graph represent a function? Explain.

27. **Critical Thinking** For a function, can the number of elements in the range be greater than the number of elements in the domain? Explain.

28. **Critical Thinking** Tell whether each statement is true or false. If false, explain why.
   a. All relations are functions.
   b. All functions are relations.
29. a. The graph shows the amount of water being pumped into a pool over a 5-hour time period. Find the domain and range.
   b. Does the graph represent a function? Explain.
   c. Give the time and volume as ordered pairs at 2 hours and at 3 hours 30 minutes.

30. **ERROR ANALYSIS** When asked whether the relation \{(-4, 16), (-2, 4), (0, 0), (2, 4)\} is a function, a student stated that the relation is not a function because 4 appears twice. What error did the student make? How would you explain to the student why this relation is a function?

31. **Write About It** Describe a real-world situation for a relation that is NOT a function. Create a mapping diagram to show why the relation is not a function.

32. Which of the following relations is NOT a function?

   A) \{(6, 2), (-1, 2), (-3, 2), (-5, 2)\}
   B) \{-5, 5, 10, 15, 16, 26, 36\}
   C) \[
   \begin{array}{ccc}
   x & 3 & 5 & 7 \\
   y & 1 & 15 & 30 \\
   \end{array}
   \]
   D) A mapping diagram with points at (2, y), (0, y), (-2, y)

33. Which is NOT a correct way to describe the function \{(-3, 2), (1, 8), (-1, 5), (3, 11)\}?

   F) A mapping diagram with points at (-3, 11), (-1, 5), (1, 8), (3, 2)
   G) A grid with points at (2, 2), (0, 4), (1, 8), (3, 11)
   H) Domain: \{-3, 1, -1, 3\}
   Range: \{2, 8, 5, 11\}
   J) A table with points at (-3, 2), (-1, 5), (1, 8), (3, 11)
34. Which graph represents a function?

[Images of four graphs labeled A, B, C, and D]

35. Extended Response Use the table for the following.

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
</tr>
</tbody>
</table>

a. Express the relation as ordered pairs.
b. Give the domain and range of the relation.
c. Does the relation represent a function? Explain your answer.

---

**CHALLENGE AND EXTEND**

36. What values of $a$ make the relation $\{(a, 1), (2, 3), (4, 5)\}$ a function? Explain.
37. What values of $b$ make the relation $\{(5, 6), (7, 8), (9, b)\}$ a function? Explain.
38. The inverse of a relation is created by interchanging the $x$- and $y$-coordinates of each ordered pair in the relation.

a. Find the inverse of the following relation: $\{(-2, 5), (0, 4), (3, -8), (7, 5)\}$.
b. Is the original relation a function? Why or why not? Is the inverse of the relation a function? Why or why not?
c. The statement “If a relation is a function, then the inverse of the relation is also a function” is sometimes true. Give an example of a relation and its inverse that are both functions. Then give an example of a relation and its inverse that are both not functions.
Activity

1. Look at the values in Table 1. Is every \( x \)-value paired with exactly one \( y \)-value? If not, what \( x \)-value(s) are paired with more than one \( y \)-value?

2. Is the relation a function? Explain.

3. Graph the points from the Table 1. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

4. Look at the values in Table 2. Is every \( x \)-value paired with exactly one \( y \)-value? If not, what \( x \)-value(s) are paired with more than one \( y \)-value?

5. Is the relation a function? Explain.

6. Graph the points from the Table 2. Draw a vertical line through each point of the graph. Does any vertical line touch more than one point?

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

7. What is the \( x \)-value of the two points that are on the same vertical line? Is that \( x \)-value paired with more than one \( y \)-value?

8. Write a statement describing how to use a vertical line to tell if a relation is a function. This is called the vertical-line test.

9. Why does the vertical-line test work?

Try This

Use the vertical-line test to determine whether each relation is a function. If a relation is not a function, list two ordered pairs that show the same \( x \)-value with two different \( y \)-values.

1. [Circle graph]

2. [Graph with vertical line through points]

3. [Graph with vertical line through points]
Similar to the way that numbers are classified into sets based on common characteristics, functions can be classified into families of functions. The parent function is the simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent function.

<table>
<thead>
<tr>
<th>Family</th>
<th>Constant</th>
<th>Linear</th>
<th>Quadratic</th>
<th>Cubic</th>
<th>Square root</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule</td>
<td>( f(x) = c )</td>
<td>( f(x) = x )</td>
<td>( f(x) = x^2 )</td>
<td>( f(x) = x^3 )</td>
<td>( f(x) = \sqrt{x} )</td>
</tr>
<tr>
<td>Graph</td>
<td></td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Domain</td>
<td>( \mathbb{R} )</td>
<td>( \mathbb{R} )</td>
<td>( \mathbb{R} )</td>
<td>( \mathbb{R} )</td>
<td>( x \geq 0 )</td>
</tr>
<tr>
<td>Range</td>
<td>( y = c )</td>
<td>( \mathbb{R} )</td>
<td>( y \geq 0 )</td>
<td>( \mathbb{R} )</td>
<td>( y \geq 0 )</td>
</tr>
<tr>
<td>Intersects y-axis</td>
<td>( (0, c) )</td>
<td>( (0, 0) )</td>
<td>( (0, 0) )</td>
<td>( (0, 0) )</td>
<td>( (0, 0) )</td>
</tr>
</tbody>
</table>

**Example 1**

**Identifying Transformations of Parent Functions**

Identify the parent function for \( g \) from its function rule. Then graph \( g \) on your calculator and describe what transformation of the parent function it represents.

**A**

\[ \begin{align*}
\text{g}(x) &= x + 5 \\
\text{g}(x) &= x + 5 \text{ is linear.} \\
\text{x has a power of 1.}
\end{align*} \]

The linear parent function \( f(x) = x \) intersects the \( y \)-axis at the point \((0, 0)\).

Graph \( Y_1 = X + 5 \) on a graphing calculator. The function \( g(x) = x + 5 \) intersects the \( y \)-axis at the point \((0, 5)\).

So \( g(x) = x + 5 \) represents a vertical translation of the linear parent function 5 units up.
Identify the parent function for \( g \) from its function rule. Then graph \( g \) on your calculator and describe what transformation of the parent function it represents.

**B**

\[ g(x) = (x - 3)^2 \]

1. \( g(x) = (x - 3)^2 \) is quadratic. \( x - 3 \) has a power of 2.

The quadratic parent function \( f(x) = x^2 \) intersects the \( x \)-axis at the point \((0, 0)\).

Graph \( y_1 = (x - 3)^2 \) on a graphing calculator. The function \( g(x) = (x - 3)^2 \) intersects the \( x \)-axis at the point \((3, 0)\).

So \( g(x) = (x - 3)^2 \) represents a horizontal translation of the quadratic parent function 3 units right.

---

**CHECK IT OUT!**

Identify the parent function for \( g \) from its function rule. Then graph \( g \) on your calculator and describe what transformation of the parent function it represents.

1a. \( g(x) = x^3 + 2 \)

1b. \( g(x) = (-x)^2 \)

It is often necessary to work with a set of data points like the ones represented by the table at right.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

With only the information in the table, it is impossible to know the exact behavior of the data between and beyond the given points. However, a working knowledge of the parent functions can allow you to sketch a curve to approximate those values not found in the table.

---

**EXAMPLE 2**

Identifying Parent Functions to Model Data Sets

Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

The graph of the data points resembles the shape of the quadratic parent function \( f(x) = x^2 \).

The quadratic parent function passes through the points \((2, 4)\) and \((4, 16)\). The data set contains the points \((2, 2) = (2, \frac{1}{2}(4))\) and \((4, 8) = (2, \frac{1}{2}(16))\).

The data set seems to represent a vertical compression of the quadratic parent function by a factor of \( \frac{1}{2} \).

---

2. Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-12</td>
<td>-6</td>
<td>0</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>
Consider the two data points \((0, 0)\) and \((1, 1)\). If you plot them on a coordinate plane you might very well think that they are part of a linear function. In fact they belong to each of the parent functions below.

### Ocean Waves

<table>
<thead>
<tr>
<th>Wave Height (ft)</th>
<th>Wind Speed (knots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8.8</td>
</tr>
<tr>
<td>4</td>
<td>12.4</td>
</tr>
<tr>
<td>6</td>
<td>15.2</td>
</tr>
<tr>
<td>8</td>
<td>17.5</td>
</tr>
<tr>
<td>10</td>
<td>19.6</td>
</tr>
</tbody>
</table>

Remember that any parent function you use to approximate a set of data should never be considered exact. However, these function approximations are often useful for estimating unknown values.

### Example 3

**Oceanography Application**

An oceanographer wants to determine a model that can be used to estimate wind speed based upon wave height. Graph the relationship from wave height to wind speed and identify which parent function best describes it. Then use the graph to estimate the wave height when the wind speed is 10 knots.

**Step 1** Graph the relation.

Graph the points given in the table. Draw a smooth curve through them to help you see the shape.

**Step 2** Identify the parent function.

The graph of the data set resembles the shape of the square-root parent function \(f(x) = \sqrt{x}\).

**Step 3** Estimate the wave height when the wind speed is 10 knots.

The curve indicates that a wind speed of 10 knots would create a wave that is approximately 2.5 feet high.

### Check It Out!

3. The cost of playing an online video game depends on the number of months for which the online service is used. Graph the relationship from number of months to cost, and identify which parent function best describes the data. Then use the graph to estimate the cost for 5 months of online service.

<table>
<thead>
<tr>
<th>Time (mo)</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>40</td>
<td>56</td>
<td>80</td>
<td>104</td>
<td>128</td>
</tr>
</tbody>
</table>
1. **Vocabulary** Explain how transformations, families of functions, and *parent functions* are related.

2. Identify the parent function for $g$ from its function rule. Then graph $g$ on your calculator and describe what transformation of the parent function it represents.

   2. $g(x) = (x - 1)^3$
   3. $g(x) = (x + 1)^2$
   4. $g(x) = -x$
   5. $g(x) = \sqrt{x + 3}$
   6. $g(x) = x^2 + 4$
   7. $g(x) = x - \sqrt{2}$

3. Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

   8. $x$ | $-3$ | $-1$ | $0$ | $1$ | $3$
   $y$  | $-15$ | $-5$ | $0$ | $5$ | $15$

   9. $x$ | $-3$ | $-1$ | $0$ | $1$ | $3$
   $y$  | $-1$ | $\frac{1}{27}$ | $0$ | $\frac{1}{27}$ | $1$

4. **Physics** The time it takes a pendulum to make one complete swing back and forth depends on its string length.
   a. Graph the relationship from string length to time.
   b. Identify which parent function best describes the data.
   c. Use your graph to estimate the string length of a pendulum that takes 4.5 seconds to make one complete swing.
   d. Use your graph to estimate the time it takes to make a complete swing for a string of length 14 meters.

<table>
<thead>
<tr>
<th>String Length (m)</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.8</td>
</tr>
<tr>
<td>4</td>
<td>4.0</td>
</tr>
<tr>
<td>6</td>
<td>4.9</td>
</tr>
<tr>
<td>8</td>
<td>5.7</td>
</tr>
<tr>
<td>10</td>
<td>6.3</td>
</tr>
</tbody>
</table>
**PRACTICE AND PROBLEM SOLVING**

Identify the parent function for \( g \) from its function rule. Then graph \( g \) on your calculator and describe what transformation of the parent function it represents.

11. \( g(x) = x^2 - 1 \)  
12. \( g(x) = \sqrt{x} - 2 \)  
13. \( g(x) = x^3 + 3 \)

Graph the data from the table. Describe the parent function and the transformation that best approximates the data set.

14. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 3 )</th>
<th>( -1 )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( 3 )</td>
<td>( 1 )</td>
<td>( 3 )</td>
<td>( 0 )</td>
<td>( 1 )</td>
</tr>
</tbody>
</table>

15. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( 4 )</th>
<th>( 9 )</th>
<th>( 16 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( 0 )</td>
<td>( 2 )</td>
<td>( 4 )</td>
<td>( 6 )</td>
<td>( 8 )</td>
</tr>
</tbody>
</table>

**16. Geometry** The number of segments required to connect a given number of points is shown in the table.

- a. Graph the relationship from the number of points to the number of segments.
- b. Identify which parent function best describes the data.
- c. Use your graph to estimate the number of points if there are 45 segments.
- d. Use your graph to estimate the number of segments if there are 7 points.

<table>
<thead>
<tr>
<th>Connecting Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Points</td>
</tr>
<tr>
<td>Number of Segments</td>
</tr>
</tbody>
</table>

**Graphing Calculator** Graph each function with a graphing calculator. Identify the domain and range of the function, and describe the transformation from its parent function.

17. \( g(x) = 3\sqrt{x} \)  
18. \( g(x) = \frac{2}{3}x \)  
19. \( g(x) = -\sqrt{x} \)

20. \( g(x) = -(x - 2)^2 \)  
21. \( g(x) = -x^2 + 1 \)  
22. \( g(x) = -\frac{1}{2}x^3 \)

**23. Sports** Based on the information in the table, what is the total cost of 15 tickets to the hockey game? Explain how you determined your answer.

<table>
<thead>
<tr>
<th>Hockey Tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tickets</td>
</tr>
<tr>
<td>Total Cost ($)</td>
</tr>
</tbody>
</table>

Graph each function. Identify the parent function that best describes the set of points, and describe the transformation from the parent function.

24. \( \{(-2, 8), (-1, 1), (0, 0), (1, -1), (2, -8)\} \)  
25. \( \{(5, 4), (7, 0), (9, 4), (10, 9), (11, 16)\} \)  
26. \( \{(0, 0), (-1, 1), (-4, 2), (-9, 3), (-16, 4)\} \)

27. \( \{(-4, 3), (-2, 1), (0, -1), (2, 3), (4, -5)\} \)

28. a. One function used in the Multi-Step Test Prep in the lesson Exploring Transformations was \( f(x) = 20 + 1.25x \). What is its parent function?

b. The graph for a given function has a U shape. What could be the parent function?

c. Plot the data set \( \{(0, 0), (1, 2), (4, 4), (9, 6), (16, 8), (25, 10)\} \). Which parent function best models the data set?
Photography When resizing a digital photo, it is often important to preserve its aspect ratio, the ratio of its width to its height. Use the table for Exercises 29–31.

29. Graph the relationship from width to height and identify which parent function best describes the data. Use the graph to estimate the width of a photo with a height of 1000 pixels.

30. Graph the relationship from height to width and identify which parent function best describes the data. Use the graph to estimate the height of a photo with a width of 500 pixels.

31. Resizing a photo changes the file size. Graph the relationship from width to file size and identify which parent function best describes the data. Use the graph to estimate the width of a photo with a file size of 1000 KB.

Sketch a graph for each situation and identify the related parent function. Then explain what the reasonable domain and range for the function is and compare it with the domain and range of the parent function.

32. distance traveled after \( h \) hours at a speed of 55 mi/h

33. volume of a cube with side length \( \ell \)

34. area of a room with width \( w \) and a length of 15 feet

35. cost to wash \( n \) loads of laundry at $1.00 per load

36. cost of an item with original price \( p \) after a 15% discount

37. side length of a square with area \( A \)

38. Chemistry The table shows properties of aerogel. Graph the relationship from mass to volume, and then estimate the volume of 1 gram of aerogel.

Aerogel has been called the world's lowest density solid. It is 99.8% air and is an excellent heat insulator. As shown above, a layer of aerogel can prevent a flame from melting crayons.

39. What if…? Use the set of points \( \{(−1, −1), (0, 0), (1, 1)\} \) to answer each question.
   a. What parent function best describes the set of points?
   b. If the points \((-2, 8)\) and \((2, 8)\) were added, what parent function would best describe the set?
   c. If the point \((1, 1)\) were replaced with \((1, −1)\), what parent function would best describe the set?
   d. If the point \((-1, −1)\) were replaced with \((4, 2)\), what parent function would best describe the set?
   e. Multi-Step If the \(x\)-coordinate of each point were doubled and 3 were added to each \(y\)-coordinate, what parent function would best describe the set? What transformation of the parent function would the set represent?

40. Critical Thinking Explain any relationship you have noticed between the quadratic parent function and a function rule that represents a horizontal translation, a vertical translation, or a reflection across the \(x\)-axis.
41. **Write About It** Order the parent functions covered in this lesson from least to greatest by the rate at which \( f(x) \) increases as \( x \) increases for \( x > 1 \). Explain your answer.

42. Which situation could be represented by the graph?
   - A) The area of a circle based on its radius
   - B) The volume of a sphere based on its radius
   - C) The surface area of a sphere based on its radius
   - D) The circumference of a circle based on its radius

43. Which graph best represents the function \( f(x) = 2x^2 - 2 \)?

44. Which equation describes a relationship in which every nonzero real number \( x \) corresponds to a negative real number \( y \)?
   - A) \( y = -x^3 \)
   - B) \( y = -x^2 \)
   - C) \( y = (-x)^2 \)
   - D) \( y = -x \)

45. For which function is \(-1\) NOT an element of the range?
   - F) \( y = -1 \)
   - G) \( y = (-x)^2 \)
   - H) \( y = -x \)
   - J) \( y = x^3 \)

46. What type of function can be used to determine the side length of a square if the independent variable is the square's area?
   - A) Cubic
   - B) Linear
   - C) Quadratic
   - D) Square root

**CHALLENGE AND EXTEND**

Identify the parent function for each function.

47. \( g(x) = 3(x - 1)^2 - 6 \)

48. \( h(x) = (4x^3)^0 + 2 \)

49. \( g(x) = 5(3x - 2) - 11x \)

50. Another parent function is an exponential function of the form \( f(x) = a^x \).
   a. Graph \( f(x) = 2^x \).
   b. Find the domain and range of the function.
   c. Identify the point where the function crosses the \( y \)-axis.
   d. Predict where \( f(x) = 3^x \) crosses the \( y \)-axis and explain your answer.
5-4 Linear, Quadratic, and Exponential Models

Objectives
Compare linear, quadratic, and exponential models.
Given a set of data, decide which type of function models the data and write an equation to describe the function.

Why learn this?
Different situations in sports can be described by linear, quadratic, or exponential models.

Look at the tables and graphs below. The data show three ways you have learned that variable quantities can be related. The relationships shown are linear, quadratic, and exponential.

In the real world, people often gather data and then must decide what kind of relationship (if any) they think best describes their data.

**Example 1**
**Graphing Data to Choose a Model**
Graph each data set. Which kind of model best describes the data?

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacteria</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

Plot the data points and connect them.
The data appear to be exponential.
Graph each data set. Which kind of model best describes the data?

### Celsius to Fahrenheit

<table>
<thead>
<tr>
<th>°C</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>°F</td>
<td>32</td>
<td>41</td>
<td>50</td>
<td>59</td>
<td>68</td>
</tr>
</tbody>
</table>

Plot the data points and connect them.
The data appear to be linear.

Graph each data set. Which kind of model best describes the data?

1a. \([-3, 0.30), (-2, 0.44), (0, 1), (1, 1.5), (2, 2.25), (3, 3.38)\]

1b. \([-3, -14), (-2, -9), (-1, -6), (0, -5), (1, -6), (2, -9), (3, -14)\]

Another way to decide which kind of relationship (if any) best describes a data set is to use patterns.

**Example 2**

**Using Patterns to Choose a Model**

Look for a pattern in each data set to determine which kind of model best describes the data.

#### A. Height of Bridge Suspension Cables

<table>
<thead>
<tr>
<th>Cable’s Distance from Tower (ft)</th>
<th>Cable’s Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 100</td>
<td>0</td>
</tr>
<tr>
<td>+ 100</td>
<td>100</td>
</tr>
<tr>
<td>+ 100</td>
<td>200</td>
</tr>
<tr>
<td>+ 100</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>256</td>
</tr>
<tr>
<td></td>
<td>144</td>
</tr>
<tr>
<td></td>
<td>64</td>
</tr>
</tbody>
</table>

For every constant change in distance of +100 feet, there is a constant second difference of +32.
The data appear to be quadratic.

#### B. Value of a Car

<table>
<thead>
<tr>
<th>Car’s Age (yr)</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ 1 yr</td>
<td>0</td>
</tr>
<tr>
<td>+ 1 yr</td>
<td>1</td>
</tr>
<tr>
<td>+ 1 yr</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value ($)</th>
<th>20,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value ($)</td>
<td>17,000</td>
</tr>
<tr>
<td>Value ($)</td>
<td>14,450</td>
</tr>
<tr>
<td>Value ($)</td>
<td>12,282.50</td>
</tr>
</tbody>
</table>

For every constant change in age of +1 year, there is a constant ratio of 0.85.
The data appear to be exponential.

2. Look for a pattern in the data set \([-2, 10), (-1, 1), (0, -2), (1, 1), (2, 10)\) to determine which kind of model best describes the data.
After deciding which model best fits the data, you can write a function. Recall the general forms of linear, quadratic, and exponential functions.

<table>
<thead>
<tr>
<th>LINEAR</th>
<th>QUADRATIC</th>
<th>EXPONENTIAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = mx + b )</td>
<td>( y = ax^2 + bx + c )</td>
<td>( y = ab^x )</td>
</tr>
</tbody>
</table>

**Example 3**

**Problem-Solving Application**

Use the data in the table to describe how the ladybug population is changing. Then write a function that models the data. Use your function to predict the ladybug population after one year.

<table>
<thead>
<tr>
<th>Ladybug Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (mo)</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

**1. Understand the Problem**

The answer will have three parts—a description, a function, and a prediction.

**2. Make a Plan**

Determine whether the data is linear, quadratic, or exponential. Use the general form to write a function. Then use the function to find the population after one year.

**3. Solve**

**Step 1** Describe the situation in words.

Ladybug Population

<table>
<thead>
<tr>
<th>Time (mo)</th>
<th>Ladybugs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>270</td>
</tr>
</tbody>
</table>

Each month, the ladybug population is multiplied by 3. In other words, the population triples each month.

**Step 2** Write the function.

There is a constant ratio of 3. The data appear to be exponential.

\[
\begin{align*}
  y &= ab^x \\  y &= a(3)^x \\ 10 &= a(3)^0 \\ 10 &= a(1) \\ 10 &= a \\
  y &= 10(3)^x
\end{align*}
\]

Write the general form of an exponential function.

Substitute the constant ratio, 3, for \( b \).

Choose an ordered pair from the table, such as \((0, 10)\).

Substitute for \( x \) and \( y \).

Simplify. \( 3^0 = 1 \)

The value of \( a \) is 10.

Substitute 10 for \( a \) in \( y = a(3)^x \).
Step 3  Predict the ladybug population after one year.

\[ y = 10(3)^x \quad \text{Write the function.} \]
\[ = 10(3)^{12} \quad \text{Substitute 12 for } x \text{ (1 year = 12 mo).} \]
\[ = 5,314,410 \quad \text{Use a calculator.} \]

There will be 5,314,410 ladybugs after one year.

Look Back

You chose the ordered pair (0, 10) to write the function. Check that every other ordered pair in the table satisfies your function.

\[
\begin{array}{c|c}
\text{ } & y = 10(3)^x \\
\hline
30 & 10(3)^3 \\
30 & 10(3) \\
30 & 30 \checkmark \\
\end{array}
\quad
\begin{array}{c|c}
\text{ } & y = 10(3)^x \\
\hline
90 & 10(3)^2 \\
90 & 10(9) \\
90 & 90 \checkmark \\
\end{array}
\quad
\begin{array}{c|c}
\text{ } & y = 10(3)^x \\
\hline
270 & 10(3)^3 \\
270 & 10(27) \\
270 & 270 \checkmark \\
\end{array}
\]

3. Use the data in the table to describe how the oven temperature is changing. Then write a function that models the data. Use your function to predict the temperature after 1 hour.

<table>
<thead>
<tr>
<th>Oven Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (min)</td>
</tr>
<tr>
<td>Temperature (°F)</td>
</tr>
</tbody>
</table>

Student to Student  Checking Units

I used to get a lot of answers wrong because of the units. If a question asked for the value of something after 1 year, I would always just substitute 1 into the function.

I finally figured out that you have to check what x is. If x represents months and you’re trying to find the value after 1 year, then you have to substitute 12, not 1, because there are 12 months in a year.

THINK AND DISCUSS

1. Do you think that every data set will be able to be modeled by a linear, quadratic, or exponential function? Why or why not?

2. In Example 3, is it certain that there will be 5,314,410 ladybugs after one year? Explain.

3. GET ORGANIZED  Copy and complete the graphic organizer. In each box, list some characteristics and sketch a graph of each type of model.
**GUIDED PRACTICE**

Graph each data set. Which kind of model best describes the data?

1. \( \{(−1, 4), (−2, 0.8), (0, 20), (1, 100), (−3, 0.16)\} \)
2. \( \{(0, 3), (1, 9), (2, 11), (3, 9), (4, 3)\} \)
3. \( \{(2, −7), (−2, −9), (0, −8), (4, −6), (6, −5)\} \)

Look for a pattern in each data set to determine which kind of model best describes the data.

4. \( \{(−2, 1), (−1, 2.5), (0, 3), (1, 2.5), (2, 1)\} \)
5. \( \{(−2, 0.75), (−1, 1.5), (0, 3), (1, 6), (2, 12)\} \)
6. \( \{(−2, 2), (−1, 4), (0, 6), (1, 8), (2, 10)\} \)

7. **Consumer Economics** Use the data in the table to describe the cost of grapes. Then write a function that models the data. Use your function to predict the cost of 6 pounds of grapes.

<table>
<thead>
<tr>
<th>Total Cost of Grapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount (lb)</td>
</tr>
<tr>
<td>Cost ($)</td>
</tr>
</tbody>
</table>

**PRACTICE AND PROBLEM SOLVING**

Graph each data set. Which kind of model best describes the data?

8. \( \{(−3, −5), (−2, −8), (−1, −9), (0, −8), (1, −5), (2, 0), (3, 7)\} \)
9. \( \{(−3, −1), (−2, 0), (−1, 1), (0, 2), (1, 3), (2, 4), (3, 5)\} \)
10. \( \{(0, 0.1), (2, 0.9), (3, 2.7), (4, 8.1)\} \)

Look for a pattern in each data set to determine which kind of model best describes the data.

11. \( \{(−2, 5), (−1, 4), (0, 3), (1, 2), (2, 1)\} \)
12. \( \{(−2, 12), (−1, 15), (0, 16), (1, 15), (2, 12)\} \)
13. \( \{(−2, 8), (−1, 4), (0, 2), (1, 1), (2, 0.5)\} \)

14. **Business** Use the data in the table to describe how the company’s sales are changing. Then write a function that models the data. Use your function to predict the amount of sales after 10 years.

<table>
<thead>
<tr>
<th>Company Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>Sales ($)</td>
</tr>
</tbody>
</table>

15. **Multi-Step** Jay’s hair grows about 6 inches each year. Write a function that describes the length \( ℓ \) in inches that Jay’s hair will grow for each year \( k \). Which kind of model best describes the function?
Tell which kind of model best describes each situation.

16. The height of a plant at weekly intervals over the last 6 weeks was 1 inches, 1.5 inches, 2 inches, 2.5 inches, 3 inches, and 3.5 inches.

17. The number of games a baseball player played in the last four years was 162, 162, 162, and 162.

18. The height of a ball in a certain time interval was recorded as 30.64 feet, 30.96 feet, 31 feet, 30.96 feet, and 30.64 feet.

Write a function to model each set of data.

19. \[
\begin{array}{c|c}
  x & -1 & 0 & 1 & 2 & 4 \\
  y & 0.05 & 0.2 & 0.8 & 3.2 & 51.2 \\
\end{array}
\]

20. \[
\begin{array}{c|c}
  x & -2 & 0 & 2 & 4 & 8 \\
  y & 5 & 4 & 3 & 2 & 0 \\
\end{array}
\]

Tell which kind of model best describes each graph.

21. ![Graph of Cost of a Telephone Call](image)

22. ![Graph of Height of Basketball](image)

23. **Write About It** Write a set of data that you could model with an exponential function. Explain why the exponential model would work.

24. **ERROR ANALYSIS** A student concluded that the data set would best be modeled by a quadratic function. Explain the student’s error.

25. **Critical Thinking** Sometimes the graphs of quadratic data and exponential data can look very similar. Describe how you can tell them apart.

26. a. Examine the two models that represent annual tuition for two colleges. Describe each model as linear, quadratic, or exponential.

   b. Write a function rule for each model.

   c. Both models have the same values for 2004. What does this mean?

   d. Why do both models have the same value for year 1?

<table>
<thead>
<tr>
<th>Years After 2004</th>
<th>Tuition at College 1 ($)</th>
<th>Tuition at College 2 ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000.00</td>
<td>2000.00</td>
</tr>
<tr>
<td>1</td>
<td>2200.00</td>
<td>2200.00</td>
</tr>
<tr>
<td>2</td>
<td>2400.00</td>
<td>2420.00</td>
</tr>
<tr>
<td>3</td>
<td>2600.00</td>
<td>2662.00</td>
</tr>
<tr>
<td>4</td>
<td>2800.00</td>
<td>2928.20</td>
</tr>
</tbody>
</table>
27. Which function best models the data: \{(-4, -2), (-2, -1), (0, 0), (2, 1), (4, 2)\}?

A) \( y = \left( \frac{1}{2} \right)^x \)  
B) \( y = \frac{1}{2}x^2 \)  
C) \( y = \frac{1}{2}x \)  
D) \( y = \left( \frac{1}{2}x \right)^2 \)

28. A city’s population is increasing at a rate of 2% per year. Which type of model describes this situation?

F) Exponential  
G) Quadratic  
H) Linear  
J) None of these

29. Which data set is best modeled by a linear function?

A) \{(-2, 0), (-1, 2), (0, -4), (1, -1), (2, 2)\}  
B) \{(-2, 2), (-1, 4), (0, 6), (1, 16), (2, 32)\}  
C) \{(-2, 2), (-1, 4), (0, 6), (1, 8), (2, 10)\}  
D) \{(-2, 0), (-1, 5), (0, 7), (1, 5), (2, 0)\}

CHALLENGE AND EXTEND

30. Finance An accountant estimates that a certain new automobile worth $18,000 will lose value at a rate of 16% per year.

a. Make a table that shows the worth of the car for years 0, 1, 2, 3, and 4. What is the real-world meaning of year 0?

b. Which type of model best represents the data in your table? Explain.

c. Write a function for your data.

d. What is the value of the car after 5 \( \frac{1}{2} \) years?

e. What is the value of the car after 8 years?

31. Pet Care The table shows general guidelines for the weight of a Great Dane at various ages.

\begin{tabular}{|c|c|}
\hline
Age (mo) & Weight (kg) \\
\hline
2 & 12 \\
4 & 23 \\
6 & 33 \\
8 & 40 \\
10 & 45 \\
\hline
\end{tabular}

a. None of the three models in this lesson—linear, quadratic, or exponential—fits this data exactly. Which of these is the best model for the data? Explain your choice.

b. What would you predict for the weight of a Great Dane who is 1 year old?

c. Do you think you could use your model to find the weight of a Great Dane at any age? Why or why not?
Model Variable Relationships

You can use models to represent an algebraic relationship. Using these models, you can write an algebraic expression to help describe and extend patterns.

Use with Writing Functions

The diagrams below represent the side views of tables. Each has a tabletop and a base. Copy and complete the chart using the pattern shown in the diagrams.

Tabletop ———— Base

<table>
<thead>
<tr>
<th>TERM NUMBER</th>
<th>FIGURE</th>
<th>DESCRIPTION OF FIGURE</th>
<th>EXPRESSION FOR NUMBER OF BLOCKS</th>
<th>VALUE OF TERM (NUMBER OF BLOCKS)</th>
<th>ORDERED PAIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>length of tabletop = 4</td>
<td>4 + (2)1</td>
<td>6</td>
<td>(1, 6)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>height of base = 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>length of tabletop = 4</td>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>height of base = 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>length of tabletop = 4</td>
<td></td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>height of base = 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Try This

1. Explain why you must multiply the height of the base by 2.
2. What does the ordered pair (1, 6) mean?
3. Does the ordered pair (10, 24) belong in this pattern? Why or why not?
4. Which expression from the table describes how you would find the total number of blocks for any term number \( n \)?
5. Use your rule to find the 25th term in this pattern.
Writing Functions

Objectives
Identify independent and dependent variables.
Write an equation in function notation and evaluate a function for given input values.

Vocabulary
independent variable
dependent variable
function rule
function notation

Why learn this?
You can use a function rule to calculate how much money you will earn for working specific amounts of time.

Suppose Tasha baby-sits and charges $5 per hour.

<table>
<thead>
<tr>
<th>Time Worked (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount Earned ($)</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

The amount of money Tasha earns is $5 times the number of hours she works. Write an equation using two different variables to show this relationship.

Amount earned is $5 times the number of hours worked.

\[ y = 5 \cdot x \]

Tasha can use this equation to find how much money she will earn for any number of hours she works.

Example 1
Using a Table to Write an Equation

Determine a relationship between the \( x \)- and \( y \)-values. Write an equation.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Step 1 List possible relationships between the first \( x \)- and \( y \)-values.

\( 1 - 3 = -2 \) or \( 1(-2) = -2 \)

Step 2 Determine if one relationship works for the remaining values.

\( 2 - 3 = -1 \) \( \checkmark \) \( 2(-2) \neq -1 x \)
\( 3 - 3 = 0 \) \( \checkmark \) \( 3(-2) \neq 0 x \)
\( 4 - 3 = 1 \) \( \checkmark \) \( 4(-2) \neq 1 x \)

The first relationship works. The value of \( y \) is 3 less than \( x \).

Step 3 Write an equation.

\[ y = x - 3 \quad \text{The value of } y \text{ is 3 less than } x. \]

Check it out!

1. Determine a relationship between the \( x \)- and \( y \)-values in the relation \( \{ (1, 3), (2, 6), (3, 9), (4, 12) \} \). Write an equation.

The equation in Example 1 describes a function because for each \( x \)-value (input), there is only one \( y \)-value (output).
The input of a function is the independent variable. The output of a function is the dependent variable. The value of the dependent variable depends on, or is a function of, the value of the independent variable. For Tasha, the amount she earns depends on, or is a function of, the amount of time she works.

### Example 2: Identifying Independent and Dependent Variables

Identify the independent and dependent variables in each situation.

**A** In the winter, more electricity is used when the temperature goes down, and less is used when the temperature rises.

The amount of electricity used depends on the temperature.

- Dependent: amount of electricity
- Independent: temperature

**B** The cost of shipping a package is based on its weight.

The cost of shipping a package depends on its weight.

- Dependent: cost
- Independent: weight

**C** The faster Ron walks, the quicker he gets home.

The time it takes Ron to get home depends on the speed he walks.

- Dependent: time
- Independent: speed

### Check It Out!

Identify the independent and dependent variables in each situation.

2a. A company charges $10 per hour to rent a jackhammer.

2b. Apples cost $0.99 per pound.

An algebraic expression that defines a function is a function rule. $5 \cdot x$ in the equation about Tasha's earnings is a function rule.

If $x$ is the independent variable and $y$ is the dependent variable, then function notation for $y$ is $f(x)$, read “$f$ of $x$,” where $f$ names the function. When an equation in two variables describes a function, you can use function notation to write it.

The dependent variable is a function of the independent variable.

\[
\begin{align*}
y & \quad \text{is a function of} \quad x. \\
y & = f(x)
\end{align*}
\]

Since $y = f(x)$, Tasha's earnings, $y = 5x$, can be rewritten in function notation by substituting $f(x)$ for $y$: $f(x) = 5x$. Sometimes functions are written using $y$, and sometimes functions are written using $f(x)$.

### Example 3: Writing Functions

Identify the independent and dependent variables. Write an equation in function notation for each situation.

**A** A lawyer's fee is $200 per hour for her services.

The fee for the lawyer depends on how many hours she works.

- Dependent: fee
- Independent: hours

Let $h$ represent the number of hours the lawyer works.

The function for the lawyer's fee is $f(h) = 200h$. 

---

**Helpful Hint**

There are several different ways to describe the variables of a function.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$-values</td>
<td>$y$-values</td>
</tr>
<tr>
<td>Domain</td>
<td>Range</td>
</tr>
<tr>
<td>Input</td>
<td>Output</td>
</tr>
<tr>
<td>$x$</td>
<td>$f(x)$</td>
</tr>
</tbody>
</table>
identify the independent and dependent variables. write an equation in function notation for each situation.

b. the admission fee to a local carnival is $8. each ride costs $1.50.
the total cost depends on the number of rides ridden, plus $8.
dependent: total cost independent: number of rides
let r represent the number of rides ridden.
the function for the total cost of the carnival is f(r) = 1.50r + 8.

identify the independent and dependent variables. write an equation in function notation for each situation.

3a. steven buys lettuce that costs $1.69/lb.
3b. an amusement park charges a $6.00 parking fee plus $29.99 per person.

you can think of a function as an input-output machine. for tasha’s earnings, f(x) = 5x, if you input a value x, the output is 5x.

if tasha wanted to know how much money she would earn by working 6 hours, she could input 6 for x and find the output. this is called evaluating the function.

example 4

evaluate each function for the given input values.

a. for f(x) = 5x, find f(x) when x = 6 and when x = 7.5.

f(x) = 5x
f(6) = 5(6) substitute 6 for x.
= 30 simplify.

f(7.5) = 5(7.5) substitute 7.5 for x.
= 37.5 simplify.

b. for g(t) = 2.30t + 10, find g(t) when t = 2 and when t = -5.

g(t) = 2.30t + 10

\[ g(2) = 2.30(2) + 10 \]
\[ = 4.6 + 10 \]
\[ = 14.6 \]

\[ g(-5) = 2.30(-5) + 10 \]
\[ = -11.5 + 10 \]
\[ = -1.5 \]

\[ g(-8) = \frac{1}{2}(-8) - 3 \]
\[ = -4 - 3 \]
\[ = -7 \]

\[ g(-8) = \frac{1}{2}(-8) - 3 \]
\[ = -4 - 3 \]
\[ = -7 \]

check it out!

evaluate each function for the given input values.

4a. for h(c) = 2c - 1, find h(c) when c = 1 and c = -3.

4b. for g(t) = \frac{1}{4}t + 1, find g(t) when t = -24 and t = 400.
When a function describes a real-world situation, every real number is not always reasonable for the domain and range. For example, a number representing the length of an object cannot be negative, and only whole numbers can represent a number of people.

**Example 5** Finding the Reasonable Domain and Range of a Function

Manuel has already sold $20 worth of tickets to the school play. He has 4 tickets left to sell at $2.50 per ticket. Write a function to describe how much money Manuel can collect from selling tickets. Find the reasonable domain and range for the function.

Money collected from ticket sales is $2.50 per ticket plus the $20 already sold.

If he sells $x$ more tickets, he will have collected $f(x) = 2.50x + 20$ dollars.

Manuel has only 4 tickets left to sell, so he could sell 0, 1, 2, 3, or 4 tickets. A reasonable domain is \{0, 1, 2, 3, 4\}.

Substitute these values into the function rule to find the range values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2.50(0) + 20</td>
<td>2.50(1) + 20</td>
<td>2.50(2) + 20</td>
<td>2.50(3) + 20</td>
<td>2.50(4) + 20</td>
</tr>
<tr>
<td></td>
<td>= 20</td>
<td>= 22.50</td>
<td>= 25</td>
<td>= 27.50</td>
<td>= 30</td>
</tr>
</tbody>
</table>

The reasonable range for this situation is \{$20, 22.50, 25, 27.50, 30\}$.

**Check It Out!** The settings on a space heater are the whole numbers from 0 to 3. The total number of watts used for each setting is 500 times the setting number. Write a function to describe the number of watts used for each setting. Find the reasonable domain and range for the function.

**Think and Discuss**

1. When you input water into an ice machine, the output is ice cubes. Name another real-world object that has an input and an output.

2. How do you identify the independent and dependent variables in a situation?

3. Explain how to find reasonable domain values for a function.

4. GET ORGANIZED Copy and complete the graphic organizer. Use the function $y = x + 3$ and the domain \{-2, -1, 0, 1, 2\}.

![Graphic Organizer](image-url)
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.
1. The output of a function is the ___?____ variable. (independent or dependent)
2. An algebraic expression that defines a function is a ____?____. (function rule or function notation)

Determine a relationship between the x- and y-values. Write an equation.

3.  
   \[
   \begin{array}{c|cccc}
   x & 1 & 2 & 3 & 4 \\
   \hline
   y & -1 & 0 & 1 & 2 \\
   \end{array}
   \]

4. \{(1, 4), (2, 7), (3, 10), (4, 13)\}

Identify the independent and dependent variables in each situation.
5. A small-size bottle of water costs $1.99 and a large-size bottle of water costs $3.49.
6. An employee receives 2 vacation days for every month worked.

Identify the independent and dependent variables. Write an equation in function notation for each situation.
7. An air-conditioning technician charges customers $75 per hour.
8. An ice rink charges $3.50 for skates and $1.25 per hour.

Evaluate each function for the given input values.
9. For \(f(x) = 7x + 2\), find \(f(x)\) when \(x = 0\) and when \(x = 1\).
10. For \(g(x) = 4x - 9\), find \(g(x)\) when \(x = 3\) and when \(x = 5\).
11. For \(h(t) = \frac{1}{3}t - 10\), find \(h(t)\) when \(t = 27\) and when \(t = -15\).

A construction company uses beams that are 2, 3, or 4 meters long. The measure of each beam must be converted to centimeters. Write a function to describe the situation. Find the reasonable domain and range for the function. (Hint: 1 m = 100 cm)

PRACTICE AND PROBLEM SOLVING

Determine a relationship between the x- and y-values. Write an equation.

13.  
   \[
   \begin{array}{c|cccc}
   x & 1 & 2 & 3 & 4 \\
   \hline
   y & -2 & -4 & -6 & -8 \\
   \end{array}
   \]

14. \{(1, -1), (2, -2), (3, -3), (4, -4)\}

Identify the independent and dependent variables in each situation.
15. Gardeners buy fertilizer according to the size of a lawn.
16. The cost to gift wrap an order is $3 plus $1 per item wrapped.

Identify the independent and dependent variables. Write an equation in function notation for each situation.
17. To rent a DVD, a customer must pay $3.99 plus $0.99 for every day that it is late.
18. Stephen charges $25 for each lawn he mows.
19. A car can travel 28 miles per gallon of gas.
Evaluate each function for the given input values.

20. For \( f(x) = x^2 - 5 \), find \( f(x) \) when \( x = 0 \) and when \( x = 3 \).

21. For \( g(x) = x^2 + 6 \), find \( g(x) \) when \( x = 1 \) and when \( x = 2 \).

22. For \( f(x) = \frac{2}{3}x + 3 \), find \( f(x) \) when \( x = 9 \) and when \( x = -3 \).

23. A mail-order company charges $5 per order plus $2 per item in the order, up to a maximum of 4 items. Write a function to describe the situation. Find the reasonable domain and range for the function.

24. **Transportation** Air Force One can travel 630 miles per hour. Let \( h \) be the number of hours traveled. The function \( d = 630h \) gives the distance \( d \) in miles that Air Force One travels in \( h \) hours.
   
   a. Identify the independent and dependent variables. Write \( d = 630h \) using function notation.
   
   b. What are reasonable values for the domain and range in the situation described?
   
   c. How far can Air Force One travel in 12 hours?

25. Complete the table for \( g(z) = 2z - 5 \).

26. Complete the table for \( h(x) = x^2 + x \).

<table>
<thead>
<tr>
<th>( z )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(z) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

27. **Estimation** For \( f(x) = 3x + 5 \), estimate the output when \( x = -6.89 \), \( x = 1.01 \), and \( x = 4.67 \).

28. **Transportation** A car can travel 30 miles on a gallon of gas and has a 20-gallon gas tank. Let \( g \) be the number of gallons of gas the car has in its tank. The function \( d = 30g \) gives the distance \( d \) in miles that the car travels on \( g \) gallons.
   
   a. What are reasonable values for the domain and range in the situation described?
   
   b. How far can the car travel on 12 gallons of gas?

29. **Critical Thinking** Give an example of a real-life situation for which the reasonable domain consists of 1, 2, 3, and 4 and the reasonable range consists of 2, 4, 6, and 8.

30. **ERROR ANALYSIS** Rashid saves $150 each month. He wants to know how much he will have saved in 2 years. He writes the rule \( s = m + 150 \) to help him figure out how much he will save, where \( s \) is the amount saved and \( m \) is the number of months he saves. Explain why his rule is incorrect.

31. **Write About It** Give a real-life situation that can be described by a function. Identify the independent variable and the dependent variable.

32. The table shows the volume \( v \) of water pumped into a pool after \( t \) hours.
   
   a. Determine a relationship between the time and the volume of water and write an equation.
   
   b. Identify the independent and dependent variables.
   
   c. If the pool holds 10,000 gallons, how long will it take to fill?

<table>
<thead>
<tr>
<th>Amount of Water in Pool</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>
33. Marsha buys $x$ pens at $0.70$ per pen and one pencil for $0.10$. Which function gives the total amount Marsha spends?

- $A$ \( c(x) = 0.70x + 0.10 \)
- $B$ \( c(x) = 0.70x + 1 \)
- $C$ \( c(x) = (0.70 + 0.10)x \)
- $D$ \( c(x) = 0.70x + 0.10 \)

34. Belle is buying pizzas for her daughter’s birthday party, using the prices in the table. Which equation best describes the relationship between the total cost $c$ and the number of pizzas $p$?

- $F$ \( c = 26.25p \)
- $H$ \( c = p + 26.25 \)
- $G$ \( c = 5.25p \)
- $I$ \( c = 6p - 3.75 \)

35. **Gridded Response** What is the value of $f(x) = 5 - \frac{1}{2}x$ when $x = 3$?

### CHALLENGE AND EXTEND

36. The formula to convert a temperature that is in degrees Celsius $x$ to degrees Fahrenheit $f(x)$ is $f(x) = \frac{9}{5}x + 32$. What are reasonable values for the domain and range when you convert to Fahrenheit the temperature of water as it rises from $0^\circ$ to $100^\circ$ Celsius?

37. **Math History** In his studies of the motion of free-falling objects, Galileo Galilei found that regardless of its mass, an object will fall a distance $d$ that is related to the square of its travel time $t$ in seconds. The modern formula that describes free-fall motion is $d = \frac{1}{2}gt^2$, where $g$ is the acceleration due to gravity and $t$ is the length of time in seconds the object falls. Find the distance an object falls in 3 seconds. *(Hint: Research to find acceleration due to gravity in meters per second squared.)*
### Objectives

- Graph functions given a limited domain.
- Graph functions given a domain of all real numbers.

### Who uses this?

Scientists can use a function to make conclusions about rising sea level.

Sea level is rising at an approximate rate of 2.5 millimeters per year. If this rate continues, the function \( y = 2.5x \) can describe how many millimeters \( y \) sea level will rise in the next \( x \) years.

One way to understand functions such as the one above is to graph them. You can graph a function by finding ordered pairs that satisfy the function.

### Example 1

#### Graphing Solutions Given a Domain

Graph each function for the given domain.

**A** \(-x + 2y = 6; D: \{-4, -2, 0, 2\}\)

**Step 1** Solve for \( y \) since you are given values of the domain, or \( x \).

\[
\begin{align*}
-2y &= x + 6 \\
2y &= -x - 6 \\
\frac{2y}{2} &= \frac{-x - 6}{2} \\
y &= \frac{-1}{2}x - 3
\end{align*}
\]

**Step 2** Substitute the given values of the domain for \( x \) and find values of \( y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = \frac{1}{2}x + 3 )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>( y = \frac{1}{2}(-4) + 3 = 1 )</td>
<td>(-4, 1)</td>
</tr>
<tr>
<td>-2</td>
<td>( y = \frac{1}{2}(-2) + 3 = 2 )</td>
<td>(-2, 2)</td>
</tr>
<tr>
<td>0</td>
<td>( y = \frac{1}{2}(0) + 3 = 3 )</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>2</td>
<td>( y = \frac{1}{2}(2) + 3 = 4 )</td>
<td>(2, 4)</td>
</tr>
</tbody>
</table>

**Step 3** Graph the ordered pairs.

![Graph of ordered pairs](image)
Graph each function for the given domain.

**B** \( f(x) = |x|; \) D: \([-2, -1, 0, 1, 2]\)

**Step 1** Use the given values of the domain to find values of \( f(x) \).

| \( x \) | \( f(x) = |x| \) | \((x, f(x))\) |
|---|---|---|
| -2 | \( f(x) = |-2| = 2 \) | \((-2, 2)\) |
| -1 | \( f(x) = |-1| = 1 \) | \((-1, 1)\) |
| 0 | \( f(x) = |0| = 0 \) | \((0, 0)\) |
| 1 | \( f(x) = |1| = 1 \) | \((1, 1)\) |
| 2 | \( f(x) = |2| = 2 \) | \((2, 2)\) |

**Step 2** Graph the ordered pairs.

Graph each function for the given domain.

1a. \(-2x + y = 3; \) D: \([-5, -3, 1, 4]\)
1b. \( f(x) = x^2 + 2; \) D: \([-3, -1, 0, 1, 3]\)

If the domain of a function is all real numbers, any number can be used as an input value. This process will produce an infinite number of ordered pairs that satisfy the function. Therefore, arrowheads are drawn at both “ends” of a smooth line or curve to represent the infinite number of ordered pairs. If a domain is not given, assume that the domain is all real numbers.

### Graphing Functions Using a Domain of All Real Numbers

| Step 1 | Use the function to generate ordered pairs by choosing several values for \( x \). |
| Step 2 | Plot enough points to see a pattern for the graph. |
| Step 3 | Connect the points with a line or smooth curve. |

### Example 2

Graph each function.

**A** \( 2x + 1 = y \)

**Step 1** Choose several values of \( x \) and generate ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2x + 1 = y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( 2(-3) + 1 = -5 )</td>
<td>((-3, -5))</td>
</tr>
<tr>
<td>-2</td>
<td>( 2(-2) + 1 = -3 )</td>
<td>((-2, -3))</td>
</tr>
<tr>
<td>-1</td>
<td>( 2(-1) + 1 = -1 )</td>
<td>((-1, -1))</td>
</tr>
<tr>
<td>0</td>
<td>( 2(0) + 1 = 1 )</td>
<td>((0, 1))</td>
</tr>
<tr>
<td>1</td>
<td>( 2(1) + 1 = 3 )</td>
<td>((1, 3))</td>
</tr>
<tr>
<td>2</td>
<td>( 2(2) + 1 = 5 )</td>
<td>((2, 5))</td>
</tr>
<tr>
<td>3</td>
<td>( 2(3) + 1 = 7 )</td>
<td>((3, 7))</td>
</tr>
</tbody>
</table>

**Step 2** Plot enough points to see a pattern.

**Step 3** The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both “ends” of the line.
Graph each function.

**B.** $y = x^2$

**Step 1** Choose several values of $x$ and generate ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = x^2$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$(-3)^2 = 9$</td>
<td>$(-3, 9)$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$(-2)^2 = 4$</td>
<td>$(-2, 4)$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$(-1)^2 = 1$</td>
<td>$(-1, 1)$</td>
</tr>
<tr>
<td>$0$</td>
<td>$(0)^2 = 0$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>$1$</td>
<td>$(1)^2 = 1$</td>
<td>$(1, 1)$</td>
</tr>
<tr>
<td>$2$</td>
<td>$(2)^2 = 4$</td>
<td>$(2, 4)$</td>
</tr>
</tbody>
</table>

**Step 2** Plot enough points to see a pattern.

**Step 3** The ordered pairs appear to form an almost U-shaped graph. Draw a smooth curve through the points to show all the ordered pairs that satisfy the function. Draw arrowheads on the “ends” of the curve.

**Check** If the graph is correct, any point on it will satisfy the function. Choose an ordered pair on the graph that was not in your table, such as $(3, 9)$. Check whether it satisfies $y = x^2$.

$$y = x^2$$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$x^2$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9$</td>
<td>$3^2$</td>
<td>$9$</td>
</tr>
<tr>
<td>$9$</td>
<td>$9$</td>
<td>✓</td>
</tr>
</tbody>
</table>

The ordered pair $(3, 9)$ satisfies the function.

---

**Graph each function.**

2a. $f(x) = 3x - 2$

2b. $y = |x - 1|$

---

**EXAMPLE 3**

**Finding Values Using Graphs**

Use a graph of the function $f(x) = \frac{1}{3}x + 2$ to find the value of $f(x)$ when $x = 6$. Check your answer.

Locate 6 on the $x$-axis. Move up to the graph of the function. Then move left to the $y$-axis to find the corresponding value of $y$.

$f(x) = 4$

**Check** Use substitution.

$$f(x) = \frac{1}{3}x + 2$$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{1}{3}x + 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$\frac{1}{3}(6) + 2$</td>
</tr>
<tr>
<td>4</td>
<td>$2 + 2$</td>
</tr>
<tr>
<td>4</td>
<td>$4$ ✓</td>
</tr>
</tbody>
</table>

The ordered pair $(4, 6)$ satisfies the function.

---

3. Use the graph above to find the value of $x$ when $f(x) = 3$. Check your answer.
Recall that in real-world situations you may have to limit the domain to make answers reasonable. For example, quantities such as time, distance, and number of people can be represented using only nonnegative values. When both the domain and the range are limited to nonnegative values, the function is graphed only in Quadrant I.

**Problem-Solving Application**

The function $y = 2.5x$ describes how many millimeters sea level $y$ rises in $x$ years. Graph the function. Use the graph to estimate how many millimeters sea level will rise in 3.5 years.

1. **Understand the Problem**
   - The answer is a graph that can be used to find the value of $y$ when $x$ is 3.5.
   - List the important information:
     - The function $y = 2.5x$ describes how many millimeters sea level rises.

2. **Make a Plan**
   - Think: What values should I use to graph this function? Both, the number of years sea level has risen and the distance sea level rises, cannot be negative. Use only nonnegative values for both the domain and the range. The function will be graphed in Quadrant I.

3. **Solve**
   - Choose several nonnegative values of $x$ to find values of $y$.
   - Then graph the ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 2.5x$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$y = 2.5(0) = 0$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>1</td>
<td>$y = 2.5(1) = 2.5$</td>
<td>$(1, 2.5)$</td>
</tr>
<tr>
<td>2</td>
<td>$y = 2.5(2) = 5$</td>
<td>$(2, 5)$</td>
</tr>
<tr>
<td>3</td>
<td>$y = 2.5(3) = 7.5$</td>
<td>$(3, 7.5)$</td>
</tr>
<tr>
<td>4</td>
<td>$y = 2.5(4) = 10$</td>
<td>$(4, 10)$</td>
</tr>
</tbody>
</table>

   - Use the graph to estimate the $y$-value when $x$ is 3.5.
   - Sea level will rise about 8.75 millimeters in 3.5 years.

4. **Look Back**
   - As the number of years increases, sea level also increases, so the graph is reasonable. When $x$ is between 3 and 4, $y$ is between 7.5 and 10. Since 3.5 is between 3 and 4, it is reasonable to estimate $y$ to be 8.75 when $x$ is 3.5.

4. The fastest recorded Hawaiian lava flow moved at an average speed of 6 miles per hour. The function $y = 6x$ describes the distance $y$ the lava moved on average in $x$ hours. Graph the function. Use the graph to estimate how many miles the lava moved after 5.5 hours.
Guided Practice

Graph each function for the given domain.

1. \(3x - y = 1; \ D: \{-3, -1, 0, 4\}\)
2. \(f(x) = -|x|; \ D: \{-5, -3, 0, 3, 5\}\)
3. \(f(x) = x + 4; \ D: \{-5, -3, 0, 4\}\)
4. \(y = x^2 - 1; \ D: \{-3, -1, 0, 1, 3\}\)

Graph each function.

5. \(f(x) = 6x + 4\)
6. \(y = \frac{1}{2} x + 4\)
7. \(x + y = 0\)
8. \(y = |x| - 4\)
9. \(f(x) = 2x^2 - 7\)
10. \(y = -x^2 + 5\)

11. Use a graph of the function \(f(x) = \frac{1}{2} x - 2\) to find the value of \(y\) when \(x = 2\). Check your answer.

12. Oceanography The floor of the Atlantic Ocean is spreading at an average rate of 1 inch per year. The function \(y = x\) describes the number of inches \(y\) the ocean floor spreads in \(x\) years. Graph the function. Use the graph to estimate the number of inches the ocean floor will spread in 10 \(\frac{1}{2}\) years.

Practice and Problem Solving

Graph each function for the given domain.

13. \(2x + y = 4; \ D: \{-3, -1, 4, 7\}\)
14. \(y = |x| - 1; \ D: \{-4, -2, 0, 2, 4\}\)
15. \(f(x) = -7x; \ D: \{-2, -1, 0, 1\}\)
16. \(y = (x + 1)^2; \ D: \{-2, -1, 0, 1, 2\}\)

Graph each function.

17. \(y = -3x + 5\)
18. \(f(x) = 3x\)
19. \(x + y = 8\)
20. \(f(x) = 2x + 2\)
21. \(y = -|x| + 10\)
22. \(f(x) = -5 + x^2\)
23. \(y = |x + 1| + 1\)
24. \(y = (x - 2)^2 - 1\)
25. Use a graph of the function \(f(x) = -2x - 3\) to find the value of \(y\) when \(x = -4\). Check your answer.
26. Use a graph of the function \(f(x) = \frac{1}{3} x + 1\) to find the value of \(y\) when \(x = 6\). Check your answer.
27. **Transportation** An electric motor scooter can travel at 0.25 miles per minute. The function \( y = 0.25x \) describes the number of miles \( y \) the scooter can travel in \( x \) minutes. Graph the function. Use the graph to estimate the number of miles an electric motor scooter travels in 15 minutes.

Graph each function.

28. \( f(x) = x - 1 \) 
29. \( 12 - x - 2y = 0 \) 
30. \( 3x - y = 13 \) 
31. \( y = x^2 - 2 \) 
32. \( x^2 - y = -4 \) 
33. \( 2x^2 = f(x) \) 
34. \( f(x) = |2x| - 2 \) 
35. \( y = |-x| \) 
36. \( -|2x + 1| = y \)

37. Find the value of \( x \) so that \((x, 12)\) satisfies \( y = 4x + 8 \).
38. Find the value of \( x \) so that \((x, 6)\) satisfies \( y = -x - 4 \).
39. Find the value of \( y \) so that \((-2, y)\) satisfies \( y = -2x^2 \).

For each function, determine whether the given points are on the graph.

40. \( y = 7x - 2; (1, 5) \) and \((2, 10)\) 
41. \( y = |x| + 2; (3, 5) \) and \((-1, 3)\) 
42. \( y = x^2; (1, 1) \) and \((-3, -9)\) 
43. \( y = \frac{1}{4}x - 2; \left(1, -\frac{3}{4}\right) \) and \((4, -1)\) 

44. **ERROR ANALYSIS** Student A says that \((3, 2)\) is on the graph of \( y = 4x - 5 \), but student B says that it is not. Who is incorrect? Explain the error.

Determine whether \((0, -7), \left(-6, -\frac{5}{3}\right), \) and \((-2, -3)\) lie on the graph of each function.

45. \( x + 3y = -11 \) 
46. \( y + |x| = -1 \) 
47. \( x^2 - y = 7 \)

For each function, find three ordered pairs that lie on the graph of the function.

48. \(-6 = 3x + 2y\) 
49. \(y = 1.1x + 2\)
50. \(y = \frac{4}{3}x\) 
51. \(y = 3x - 1\)
52. \(y = |x| + 6\) 
53. \(y = x^2 - 5\)

54. **Critical Thinking** Graph the functions \( y = |x| \) and \( y = -|x| \). Describe how they are alike. How are they different?

55. A pool containing 10,000 gallons of water is being drained. Every hour, the volume of the water in the pool decreases by 1500 gallons.
   a. Write an equation to describe the volume \( v \) of water in the pool after \( h \) hours.
   b. How much water is in the pool after 1 hour?
   c. Create a table of values showing the volume of the water in gallons in the pool as a function of the time in hours and graph the function.
56. **Estimation** Use the graph to estimate the value of $y$ when $x = 2.117$.

57. **Write About It** Why is a graph a convenient way to show the ordered pairs that satisfy a function?

---

58. **Test Prep**
   Which function is graphed?
   - A  $2y - 3x = 2$
   - B  $5x + y = 1$
   - C  $y = 2x - 1$
   - D  $y = 5x + 8$

59. Which ordered pair is NOT on the graph of $y = 4 - |x|$?
   - F  $(0, 4)$
   - G  $(4, 0)$
   - H  $(-1, 3)$
   - I  $(3, -1)$

60. Which function has $(3, 2)$ on its graph?
   - A  $2x - 3y = 12$
   - B  $-2x - 3y = 12$
   - C  $y = -\frac{2}{3}x + 4$
   - D  $y = -\frac{3}{2}x + 4$

61. Which statement(s) is true about the function $y = x^2 + 1$?
   - I. All points on the graph are above the origin.
   - II. All ordered pairs have positive $x$-values.
   - III. All ordered pairs have positive $y$-values.
   - F  I Only
   - G  II Only
   - H  I and II
   - I  I and III

---

**CHALLENGE AND EXTEND**

62. Graph the function $y = x^3$. Make sure you have enough ordered pairs to see the shape of the graph.

63. The temperature of a liquid that started at 64 °F is increasing by 4 °F per hour. Write a function that describes the temperature of the liquid over time. Graph the function to show the temperatures over the first 10 hours.
Connect Function Rules, Tables, and Graphs

You can use a graphing calculator to understand the connections among function rules, tables, and graphs.

**Activity**

Make a table of values for the function \( f(x) = 4x + 3 \). Then graph the function.

1. Press \( \text{Y=} \) and enter the function rule \( 4x + 3 \).
2. Press \( \text{2nd} \) \( \text{WINDOW} \). Make sure \texttt{Indpnt: Auto} and \texttt{Depend: Auto} are selected.
3. To view the table, press \( \text{2nd} \) \( \text{GRAPH} \). The \( x \)-values and the corresponding \( y \)-values appear in table form. Use the up and down arrow keys to scroll through the table.
4. To view the table with the graph, press \( \text{MODE} \) and select \texttt{G-T view}. Press \( \text{ENTER} \). Be sure to use the standard window.
5. Press \( \text{TRACE} \) to see both the graph and a table of values.
6. Press the left arrow key several times to move the cursor. Notice that the point on the graph and the values in the table correspond.

**Try This**

Make a table of values for each function. Then graph the function.

1. \( f(x) = 2x - 1 \)
2. \( f(x) = 1.5x \)
3. \( f(x) = \frac{1}{2}x + 2 \)
4. Explain the relationship between a function, its table of values, and the graph of the function.
A **cubic function** is a function that can be written in the form 
\[ f(x) = ax^3 + bx^2 + cx + d, \] 
where \( a \neq 0 \). The parent cubic function is \( f(x) = x^3 \).

To graph this function, choose several values of \( x \) and find ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>-2</td>
<td>-8</td>
</tr>
</tbody>
</table>

From the graph of \( f(x) = x^3 \), you can see
- the general shape of a cubic function.
- that the domain and the range are all real numbers.
- that the \( x \)-intercept and the \( y \)-intercept are both 0.

The graph of \( f(x) = 2x^3 + 5x^2 - x + 1 \) illustrates another characteristic of the graphs of cubic functions. Points \( A \) and \( B \) are called **turning points**.

In general, the graph of a cubic function will have two turning points.

**EXEMPLARY 1**

**Graphing Cubic Functions**

Graph \( f(x) = -2x^3 + 3x^2 + x - 4 \). Identify the intercepts and give the domain and range.

Choose positive, negative, and zero values for \( x \), and find ordered pairs.
Plot the ordered pairs and connect them with a smooth curve.

Notice that, in general, this graph falls from left to right. This is because the value of $a$ is negative.

The $x$-intercept is $-1$. The $y$-intercept is $-4$. The domain and range are all real numbers.

Graph each cubic function. Identify the intercepts and give the domain and range.

1a. $f(x) = (x - 1)^3$
1b. $f(x) = 2x^3 - 12x^2 + 18x$

Previously, you saw that every quadratic function has a related quadratic equation. Cubic functions also have related cubic equations. A cubic equation is an equation that can be written in the form $ax^3 + bx^2 + cx + d = 0$, where $a \neq 0$.

One way to solve a cubic equation is by graphing the related function and finding its zeros.

### Example 2

**Solving Cubic Equations by Graphing**

Solve $x^3 - 2x^2 - x = -2$ by graphing. Check your answer.

**Step 1** Rewrite the equation in the form $ax^3 + bx^2 + cx + d = 0$.

$x^3 - 2x^2 - x = -2$

$x^3 - 2x^2 - x + 2 = 0$ Add 2 to both sides of the equation.

**Step 2** Write and graph the related function: $f(x) = x^3 - 2x^2 - x + 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = x^3 - 2x^2 - x + 2$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>$(-1)^3 - 2(-1)^2 - (-1) + 2$</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>$(0)^3 - 2(0)^2 - 0 + 2$</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>$(1)^3 - 2(1)^2 - 1 + 2$</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>$(2)^3 - 2(2)^2 - 2 + 2$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>$(3)^3 - 2(3)^2 - 3 + 2$</td>
<td>8</td>
</tr>
</tbody>
</table>

**Step 3** Find the zeros.

The zeros appear to be $-1$, $1$, and $2$. Check these values in the original equation.

$x^3 - 2x^2 - x = -2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$x^3 - 2x^2 - x = -2$</th>
<th>$x^3 - 2x^2 - x = -2$</th>
<th>$x^3 - 2x^2 - x = -2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-1)^3 - 2(-1)^2 - (-1)$</td>
<td>$-2$</td>
<td>$1^3 - 2(1)^2 - 1$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$-1 - 2 + 1$</td>
<td>$-2$</td>
<td>$1 - 2 - 1$</td>
<td>$-2$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$-2$</td>
<td>$x - 8 - 2$</td>
<td>$-2$</td>
</tr>
</tbody>
</table>

$\boxed{-2}$
Solve each equation by graphing. Check your answer.

2a. \(x^3 - 2x^2 - 25x = -50\) 
2b. \(2x^3 + 12x^2 = 30x + 200\)

Cubic equations can also be solved algebraically. Many of the methods used to solve quadratic equations can be applied to cubic equations as well.

**Example 3**

**Solving Cubic Equations Algebraically**

Solve each equation. Check your answer.

A \((x + 5)^3 = 27\)

\[
\sqrt[3]{(x + 5)^3} = \sqrt[3]{27} \\
x + 5 = 3 \\
x = -2
\]

Check

\[
\frac{(x + 5)^3}{(-2 + 5)^3} = \frac{27}{27} \\
\frac{3^3}{3} = 27 \quad \checkmark
\]

B \(x^3 + 3x^2 = -2x\)

Add 2x to both sides.

\[
x^3 + 3x^2 + 2x = 0 \\
x(x^2 + 3x + 2) = 0 \\
x(x + 1)(x + 2) = 0
\]

Factor out x on the left side.

Factor the quadratic trinomial.

Zero Product Property

Solve each equation.

The solutions are 0, -1, and -2.

Check

<table>
<thead>
<tr>
<th>(x^3 + 3x^2 = -2x)</th>
<th>(x^3 + 3x^2 = -2x)</th>
<th>(x^3 + 3x^2 = -2x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^3 + 3(0)^2 = -2(0)</td>
<td>(-1)^3 + 3(-1)^2 = -2(-1)</td>
<td>(-2)^3 + 3(-2)^2 = -2(-2)</td>
</tr>
<tr>
<td>0 + 0 = 0</td>
<td>-1 + 3(1) = 2</td>
<td>-8 + 3(4) = 4</td>
</tr>
<tr>
<td>0 + 0 \checkmark</td>
<td>-1 + 3 = 2</td>
<td>-8 + 12 = 4</td>
</tr>
<tr>
<td>2 2 \checkmark</td>
<td>2 2 \checkmark</td>
<td>4 4 \checkmark</td>
</tr>
</tbody>
</table>

C \(x^3 - 3.125x = -1.25x^2\)

Add 1.25x^2 to both sides.

\[
x^3 + 1.25x^2 - 3.125x = 0 \\
x(x^2 + 1.25x - 3.125) = 0
\]

Factor out x on the left side.

Zero Product Property

Quadratic Formula

\[
x = \frac{-1.25 \pm \sqrt{(1.25)^2 - 4(1)(-3.125)}}{2(1)}
\]

Simplify.

\[
x = \frac{-1.25 \pm 3.75}{2}
\]

The solutions are -2.5, 0, and 1.25.
Check Use a graphing calculator.

Graph the related function and look for the zeros.
The solutions look reasonable.

Solve each equation. Check your answer.
3a. \((x + 2)^3 = 64\)  3b. \(4x^3 - 12x^2 + 4x = 0\)  3c. \(x^3 + 3x^2 = 10x\)

Graph each cubic function. Identify the intercepts and give the domain and range.
1. \(f(x) = x^3 - 2x^2 + 3x + 6\)
2. \(g(x) = -4x^3 + 2x - 2\)

Solve each equation by graphing. Check your answer.
3. \(2x^3 - 6x = -4x^2\)
4. \(-3x^3 + 12x^2 + 12x = 48\)

Solve each equation. Check your answer.
5. \((x - 9)^3 = 64\)
6. \(8x + 4x^2 = 4x^3\)
7. \(5x^3 + 3x^2 = 4x\)
8. The Send-It Store uses shipping labels that are \(x\) in. tall and \(2x\) in. wide. Six labels fit on the front of the store’s standard shipping box with an area of \(3x\) in\(^2\) left over. Three labels fit on the side of the box. The volume of the box is \(108x\) in\(^3\). What is the area of one label?

9. a. Graph the functions \(f(x) = x^3\), \(f(x) = x^3 + 1\), and \(f(x) = x^3 + 2\) on the same coordinate plane. Describe any patterns you observe. Predict the shape of the graph of \(f(x) = x^3 + c\).

b. Graph the functions \(g(x) = x^3\), \(g(x) = (x - 1)^3\), and \(g(x) = (x - 2)^3\) on the same coordinate plane. Describe any patterns you observe. Predict the shape of the graph of \(g(x) = (x - c)^3\).

Use a graphing calculator to find the approximate solution(s) of each cubic equation. Round to the nearest hundredth.

10. \(100x^3 - 40x^2 = 6x\)
11. \(2x^3 - 5x^2 + 4x = -3\)
12. \(-1.32x^3 - 3.65x^2 = -0.43x\)
13. \(\frac{3}{5} x^3 + x^2 - \frac{1}{2} x = 9\)

14. Critical Thinking How many zeros can a cubic function have? What does this tell you about the number of real solutions possible for a cubic equation?
Arithmetic Sequences

During a thunderstorm, you can estimate your distance from a lightning strike by counting the number of seconds from the time you see the lightning until the time you hear the thunder.

When you list the times and distances in order, each list forms a sequence. A sequence is a list of numbers that may form a pattern. Each number in a sequence is a term.

In the distance sequence, each distance is 0.2 mi greater than the previous distance. When the terms of a sequence differ by the same nonzero number \(d\), the sequence is an arithmetic sequence and \(d\) is the common difference. The distances in the table form an arithmetic sequence with \(d = 0.2\).

The variable \(a\) is often used to represent terms in a sequence. The variable \(a_9\), read “\(a\) sub 9,” is the ninth term in a sequence. To designate any term, or the \(n\)th term, in a sequence, you write \(a_n\), where \(n\) can be any number.

To find a term in an arithmetic sequence, add \(d\) to the previous term.

Finding a Term of an Arithmetic Sequence

The \(n\)th term of an arithmetic sequence with common difference \(d\) is

\[ a_n = a_{n-1} + d. \]

**Example 1**

Identifying Arithmetic Sequences

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms in the sequence.

**A** 12, 8, 4, 0, ...

Step 1 Find the difference between successive terms.

\[ 12, \quad 8, \quad 4, \quad 0, \quad \ldots \]

Add \(-4\) to each term to find the next term.

The common difference is \(-4\).

Step 2 Use the common difference to find the next 3 terms.

\[ 12, \quad 8, \quad 4, \quad 0, \quad -4, \quad -8, \quad -12 \]

\[ a_n = a_{n-1} + d \]

The sequence appears to be an arithmetic sequence with a common difference of \(-4\). The next 3 terms are \(-4, -8, -12\).
Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

1a. $\frac{-3}{4}, \frac{-1}{4}, \frac{1}{4}, \frac{3}{4}, \ldots$
1b. $-4, -2, 1, 5, \ldots$

To find the $n$th term of an arithmetic sequence when $n$ is a large number, you need an equation or rule. Look for a pattern to find a rule for the sequence below.

<table>
<thead>
<tr>
<th>Position</th>
<th>Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$</td>
<td>$3$</td>
</tr>
<tr>
<td>$2$</td>
<td>$5$</td>
</tr>
<tr>
<td>$3$</td>
<td>$7$</td>
</tr>
<tr>
<td>$4$</td>
<td>$9$</td>
</tr>
</tbody>
</table>

The sequence starts with $3$. The common difference $d$ is $2$. You can use the first term and the common difference to write a rule for finding $a_n$.

<table>
<thead>
<tr>
<th>Words</th>
<th>Numbers</th>
<th>Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st term</td>
<td>$3$</td>
<td>$a_1$</td>
</tr>
<tr>
<td>2nd term = 1st term plus common difference</td>
<td>$3 + (1)2 = 5$</td>
<td>$a_1 + 1d$</td>
</tr>
<tr>
<td>3nd term = 1st term plus 2 common differences</td>
<td>$3 + (2)2 = 7$</td>
<td>$a_1 + 2d$</td>
</tr>
<tr>
<td>4th term = 1st term plus 3 common differences</td>
<td>$3 + (3)2 = 9$</td>
<td>$a_1 + 3d$</td>
</tr>
<tr>
<td>\vdots</td>
<td>\vdots</td>
<td>\vdots</td>
</tr>
<tr>
<td>$n$th term = 1st term plus $(n - 1)$ common differences</td>
<td>$3 + (n-1)2$</td>
<td>$a_1 + (n-1)d$</td>
</tr>
</tbody>
</table>

The pattern in the table shows that to find the $n$th term, add the first term to the product of $(n-1)$ and the common difference.

**Finding the $n$th Term of an Arithmetic Sequence**

The $n$th term of an arithmetic sequence with common difference $d$ and first term $a_1$ is

$$a_n = a_1 + (n - 1)d.$$
Step 2 Find the 22nd term.

\[ a_n = a_1 + (n - 1)d \]

Write the rule to find the nth term.

\[ a_{22} = 5 + (22 - 1)(-3) \]

Substitute 5 for \( a_n \), 22 for \( n \), and \(-3\) for \( d \).

\[ = 5 + (21)(-3) \]

Simplify the expression in parentheses.

\[ = 5 - 63 \]

Multiply.

\[ = -58 \]

Subtract.

15th term: \( a_1 = 7; d = 3 \)

\[ a_n = a_1 + (n - 1)d \]

Write the rule to find the nth term.

\[ a_{15} = 7 + (15 - 1)3 \]

Substitute 7 for \( a_n \), 15 for \( n \), and 3 for \( d \).

\[ = 7 + (14)3 \]

Simplify the expression in parentheses.

\[ = 7 + 42 \]

Multiply.

\[ = 49 \]

Add.

Find the indicated term of each arithmetic sequence.

2a. 60th term: 11, 5, \(-1\), \(-7\), …

2b. 12th term: \( a_1 = 4.2; d = 1.4 \)

**Travel Application**

The odometer on a car reads 60,473 on day 1. Every day, the car is driven 54 miles. If this pattern continues, what is the odometer reading on day 20?

Notice that the sequence for the situation is arithmetic with \( d = 54 \) because the odometer reading will increase by 54 miles per day.

Since the odometer reading on day 1 is 60,473 miles, \( a_1 = 60,473 \).

Since you want to find the odometer reading on day 20, you will need to find the 20th term of the sequence, so \( n = 20 \).

\[ a_n = a_1 + (n - 1)d \]

Write the rule to find the nth term.

\[ a_{20} = 60,473 + (20 - 1)54 \]

Substitute 60,473 for \( a_n \), 54 for \( d \), and 21 for \( n \).

\[ = 60,473 + (19)54 \]

Simplify the expression in parentheses.

\[ = 60,473 + 1026 \]

Multiply.

\[ = 61,499 \]

Add.

The odometer will read 61,499 miles on day 20.

3. Each time a truck stops, it drops off 250 pounds of cargo. After stop 1, its cargo weighed 2000 pounds. How much does the load weigh after stop 6?

**THINK AND DISCUSS**

1. Explain how to determine if a sequence appears to be arithmetic.

2. GET ORGANIZED Copy and complete the graphic organizer with steps for finding the nth term of an arithmetic sequence.
GUIDED PRACTICE

1. **Vocabulary** When trying to find the \( n \)th term of an arithmetic sequence you must first know the ________? ________. (common difference or sequence)

**Multi-Step** Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

2. \( 2, 8, 14, 20, \ldots \)  
3. \( 2.1, 1.4, 0.7, 0, \ldots \)  
4. \( 1, 1, 2, 3, \ldots \)  
5. \( 0.1, 0.3, 0.9, 2.7, \ldots \)

**SEE EXAMPLE 1**

Find the indicated term of each arithmetic sequence.

6. 21st term: \( 3, 8, 13, 18, \ldots \)  
7. 18th term: \( a_1 = -2; d = -3 \)

**SEE EXAMPLE 2**

**SEE EXAMPLE 3**

**PRACTICE AND PROBLEM SOLVING**

**Multi-Step** Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

9. \( -1, 10, -100, 1,100, \ldots \)  
10. \( 0, -2, -4, -6, \ldots \)  
11. \( -22, -31, -40, -49, \ldots \)  
12. \( 0.2, 0.5, 0.9, 1.1, \ldots \)

Find the indicated term of each arithmetic sequence.

13. 31st term: \( 1.40, 1.55, 1.70, \ldots \)  
14. 50th term: \( a_1 = 2.2; d = 1.1 \)  
15. **Travel** Rachel signed up for a frequent-flier program. She receives 4300 frequent-flier miles for her first round trip and 1300 frequent-flier miles for each additional round-trip. How many frequent-flier miles will she have after 5 round-trips?

Find the common difference for each arithmetic sequence.

16. \( 0, 6, 12, 18, \ldots \)  
17. \( \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots \)  
18. \( 107, 105, 103, 101, \ldots \)  
19. \( 7.9, 5.7, 3.5, 1.3, \ldots \)  
20. \( \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \ldots \)  
21. \( 4.25, 4.32, 4.39, 4.46, \ldots \)

Find the next four terms in each arithmetic sequence.

22. \( -4, -7, -10, -13, \ldots \)  
23. \( \frac{1}{8}, 0, -\frac{1}{8}, -\frac{1}{4}, \ldots \)  
24. \( 505, 512, 519, 526, \ldots \)  
25. \( 1.8, 1.3, 0.8, 0.3, \ldots \)  
26. \( \frac{2}{3}, \frac{4}{3}, 2, \frac{8}{3}, \ldots \)  
27. \( -1.1, -0.9, -0.7, -0.5 \)

Find the given term of each arithmetic sequence.

28. \( 5, 10, 15, 20, \ldots ; 17 \)th term  
29. \( 121, 110, 99, 88, \ldots ; 10 \)th term  
30. \( -2, -5, -8, -11, \ldots ; 41 \)st term  
31. \( -30, -22, -14, -6, \ldots ; 20 \)th term  
32. **Critical Thinking** Is the sequence \( 5a - 1, 3a - 1, a - 1, -a - 1, \ldots \) arithmetic? If not, explain why not. If so, find the common difference and the next three terms.
33. **Recreation** The rates for a go-cart course are shown.
   a. Explain why the relationship described on the flyer could be represented by an arithmetic sequence.
   b. Find the cost for 1, 2, 3, and 4 laps. Write a rule to find the \( n \)th term of the sequence.
   c. How much would 15 laps cost?
   d. **What if...?** After 9 laps, you get the 10th one free. Will the sequence still be arithmetic? Explain.

Find the given term of each arithmetic sequence.

34. 2.5, 8.5, 14.5, 20.5, \( \ldots \); 30th term
35. 189.6, 172.3, 155, 137.7, \( \ldots \); 18th term
36. \( \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \ldots \); 15th term
37. \( \frac{2}{3}, \frac{11}{12}, \frac{7}{6}, \frac{17}{12}, \ldots \); 25th term

38. **Number Theory** The sequence 1, 1, 2, 3, 5, 8, 13, \( \ldots \) is a famous sequence called the Fibonacci sequence. After the first two terms, each term is the sum of the previous two terms.
   a. Write the first 10 terms of the Fibonacci sequence. Is the Fibonacci sequence arithmetic? Explain.
   b. Notice that the third term is divisible by 2. Are the 6th and 9th terms also divisible by 2? What conclusion can you draw about every third term? Why is this true?
   c. Can you find any other patterns? (**Hint:** Look at every 4th and 5th term.)

39. **Entertainment** Seats in a concert hall are arranged in the pattern shown.
   a. The numbers of seats in the rows form an arithmetic sequence. Write a rule for the arithmetic sequence.
   b. How many seats are in the 15th row?
   c. A ticket costs $40. Suppose every seat in the first 10 rows is filled. What is the total revenue from those seats?
   d. **What if...?** An extra chair is added to each row. Write the new rule for the arithmetic sequence and find the new total revenue from the first 10 rows.

40. **Write About It** Explain how to find the common difference of an arithmetic sequence. How can you determine whether the arithmetic sequence has a positive common difference or a negative common difference?

41. Juan is traveling to visit universities. He notices mile markers along the road. He records the mile marker every 10 minutes. His father is driving at a constant speed.
   a. Copy and complete the table.
   b. Write the rule for the sequence.
   c. What does the common difference represent?
   d. If this sequence continues, find the mile marker for time interval 10.

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Mile Marker</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>520</td>
</tr>
<tr>
<td>2</td>
<td>509</td>
</tr>
<tr>
<td>3</td>
<td>498</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
42. What are the next three terms in the arithmetic sequence $-21, -12, -3, 6, \ldots$?

- **A** 9, 12, 15  
- **B** 15, 24, 33  
- **C** 12, 21, 27  
- **D** 13, 20, 27

43. What is the common difference for the data listed in the second column?

- **A** $-1.8$  
- **B** 2.8  
- **C** 1.8  
- **D** $-3.6$

44. Which of the following sequences is NOT arithmetic?

- **A** $-4, 2, 8, 14, \ldots$  
- **B** 9, 4, $-1, -6, \ldots$  
- **C** 2, 4, 8, 16, \ldots  
- **D** $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \ldots$

### Challenge and Extend

45. The first term of an arithmetic sequence is 2, and the common difference is 9. Find two consecutive terms of the sequence that have a sum of 355. What positions in the sequence are the terms?

46. The 60th term of an arithmetic sequence is 106.5, and the common difference is 1.5. What is the first term of the sequence?

47. **Athletics** Verona is training for a marathon. The first part of her training schedule is shown below.

<table>
<thead>
<tr>
<th>Session</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance Run (mi)</td>
<td>3.5</td>
<td>5</td>
<td>6.5</td>
<td>8</td>
<td>9.5</td>
<td>11</td>
</tr>
</tbody>
</table>

a. If Verona continues this pattern, during which training session will she run 26 miles? Is her training schedule an arithmetic sequence? Explain.

b. If Verona's training schedule starts on a Monday and she runs every third day, on which day will she run 26 miles?
Geometric Sequences

**Objectives**
- Recognize and extend geometric sequences.
- Find the \( n \)th term of a geometric sequence.

**Vocabulary**
- geometric sequence
- common ratio

**Who uses this?**
Bungee jumpers can use geometric sequences to calculate how high they will bounce.

The table shows the heights of a bungee jumper’s bounces.

<table>
<thead>
<tr>
<th>Bounce</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (ft)</td>
<td>200</td>
<td>80</td>
<td>32</td>
</tr>
</tbody>
</table>

The height of the bounces shown in the table form a geometric sequence. In a geometric sequence, the ratio of successive terms is the same number \( r \), called the common ratio.

Geometric sequences can be thought of as functions. The term number, or position in the sequence, is the input, and the term itself is the output.

To find a term in a geometric sequence, multiply the previous term by \( r \).

**Finding a Term of a Geometric Sequence**

The \( n \)th term of a geometric sequence with common ratio \( r \) is

\[ a_n = a_{n-1}r \]

**Example 1**

Extending Geometric Sequences

Find the next three terms in each geometric sequence.

A \( 1, 3, 9, 27, \ldots \)

**Step 1** Find the value of \( r \) by dividing each term by the one before it.

\[
\frac{3}{1} = 3 \quad \frac{9}{3} = 3 \quad \frac{27}{9} = 3 \quad \text{← The value of } r \text{ is 3.}
\]

**Step 2** Multiply each term by 3 to find the next three terms.

\[
27 \times 3 = 81 \quad 81 \times 3 = 243 \quad 243 \times 3 = 729
\]

The next three terms are 81, 243, and 729.
### Helpful Hint

When the terms in a geometric sequence alternate between positive and negative, the value of \( r \) is negative.

### Step 1
Find the value of \( r \) by dividing each term by the one before it.

\[
\begin{align*}
-16 & \quad 4 \quad -1 \quad \frac{1}{4} \\
\frac{4}{-16} & = -\frac{1}{4} & \frac{-1}{4} & = -\frac{1}{4} & \frac{\frac{1}{4}}{-1} & = -\frac{1}{4} & \leftarrow \text{The value of } r \text{ is } -\frac{1}{4}.
\end{align*}
\]

### Step 2
Multiply each term by \(-\frac{1}{4}\) to find the next three terms.

\[
\begin{align*}
\frac{1}{4} & \quad \frac{-1}{16} & \quad \frac{1}{64} & \quad \frac{-1}{256} \\
\times \left(-\frac{1}{4}\right) & \quad \times \left(-\frac{1}{4}\right) & \quad \times \left(-\frac{1}{4}\right)
\end{align*}
\]

\[a_n = a_{n-1} \cdot r\]

The next three terms are \(-\frac{1}{16}, \frac{1}{64}, \text{ and } -\frac{1}{256}\).

### Find the next three terms in each geometric sequence.

1. \(5, -10, 20, -40, \ldots\)
2. \(512, 384, 288, \ldots\)

To find the output \(a_n\) of a geometric sequence when \(n\) is a large number, you need an equation, or function rule.

The pattern in the table shows that to get the \(n\)th term, multiply the first term by the common ratio raised to the power \(n - 1\).

If the first term of a geometric sequence is \(a_1\), the \(n\)th term is \(a_n\), and the common ratio is \(r\), then

\[a_n = a_1 \cdot r^{n-1}\]

### Example 2

#### Finding the \(n\)th Term of a Geometric Sequence

**A** The first term of a geometric sequence is 128, and the common ratio is 0.5. What is the 10th term of the sequence?

\[
a_n = a_1 \cdot r^{n-1}
\]

\[
a_{10} = 128 \cdot (0.5)^{10-1}
\]

\[
= 128 \cdot (0.5)^9
\]

\[
= 0.25
\]

**B** For a geometric sequence, \(a_1 = 8\) and \(r = 3\). Find the 5th term of this sequence.

\[
a_n = a_1 \cdot r^{n-1}
\]

\[
a_5 = 8 \cdot (3)^{5-1}
\]

\[
= 8 \cdot (3)^4
\]

\[
= 648
\]
**What is the 13th term of the geometric sequence 8, −16, 32, −64, ... ?**

The heights form a geometric sequence. What is the bungee jumper's height at the top of each bounce?

**Sports Application**

A bungee jumper jumps from a bridge. The diagram shows the bungee jumper's height above the ground at the top of each bounce. The heights form a geometric sequence. What is the bungee jumper's height at the top of the 5th bounce?

**Example 3**

The heights of the bungee jumper form a geometric sequence. What is the bungee jumper's height at the top of the 5th bounce?

**Think and Discuss**

1. How do you determine whether a sequence is geometric?
2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a way to represent the geometric sequence.
GUIDED PRACTICE

1. **Vocabulary** What is the common ratio of a geometric sequence?

Find the next three terms in each geometric sequence.

2. 2, 4, 8, 16, …  
3. 400, 200, 100, 50, …  
4. 4, -12, 36, -108, …

5. The first term of a geometric sequence is 1, and the common ratio is 10. What is the 10th term of the sequence?

6. What is the 11th term of the geometric sequence 3, 6, 12, 24, …?

7. **Sports** In the NCAA men’s basketball tournament, 64 teams compete in round 1. Fewer teams remain in each following round, as shown in the graph, until all but one team have been eliminated. The numbers of teams in each round form a geometric sequence. How many teams compete in round 5?

PRACTICE AND PROBLEM SOLVING

Find the next three terms in each geometric sequence.

8. -2, 10, -50, 250, …  
9. 32, 48, 72, 108, …  
10. 625, 500, 400, 320, …

11. 6, 42, 294, …  
12. 6, -12, 24, -48, …  
13. 40, 10, \(\frac{5}{2}\), \(\frac{5}{8}\), …

14. The first term of a geometric sequence is 18 and the common ratio is 3.5. What is the 5th term of the sequence?

15. What is the 14th term of the geometric sequence 1000, 100, 10, 1, …?

16. **Physical Science** A ball is dropped from a height of 500 meters. The table shows the height of each bounce, and the heights form a geometric sequence. How high does the ball bounce on the 8th bounce? Round your answer to the nearest tenth of a meter.

<table>
<thead>
<tr>
<th>Bounce</th>
<th>Height (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
</tr>
<tr>
<td>2</td>
<td>320</td>
</tr>
<tr>
<td>3</td>
<td>256</td>
</tr>
</tbody>
</table>

Find the missing term(s) in each geometric sequence.

17. 20, 40, \(\_,\_,\,\) …  
18. \(\_,\,6,\,18,\,\), …  
19. \(9,\,3,\,1,\,\), …

20. 3, 12, \(\_,\,192,\,\), …  
21. 7, 1, \(\_,\,\), \(\frac{1}{343}\), …  
22. \(\_,\,100,\,25,\,\), \(\frac{25}{16}\), …

23. -3, \(\_,\,\), -12, 24, \(\_,\,\), …  
24. \(\_,\,\), \(\_,\,1,\,\), -3, 9, …  
25. \(1,\,17,\,289,\,\), …

Determine whether each sequence could be geometric. If so, give the common ratio.

26. 2, 10, 50, 250, …  
27. 15, \(\_\), \(\frac{5}{3}\), \(\frac{5}{9}\), …  
28. 6, 18, 24, 38, …

29. 9, 3, -1, -5, …  
30. 7, 21, 63, 189, …  
31. 4, 1, -2, -4, …
32. **Multi-Step** Billy earns money by mowing lawns for the summer. He offers two payment plans, as shown at right.
   
a. Do the payments for plan 2 form a geometric sequence? Explain.
   
b. If you were one of Billy’s customers, which plan would you choose? (Assume that the summer is 10 weeks long.) Explain your choice.

33. **Measurement** When you fold a piece of paper in half, the thickness of the folded piece is twice the thickness of the original piece. A piece of copy paper is about 0.1 mm thick.
   
a. How thick is a piece of copy paper that has been folded in half 7 times?
   
b. Suppose that you could fold a piece of copy paper in half 12 times. How thick would it be? Write your answer in centimeters.

List the first four terms of each geometric sequence.

34. \(a_1 = 3, a_n = 3(2)^{n-1}\)

35. \(a_1 = -2, a_n = -2(4)^{n-1}\)

36. \(a_1 = 5, a_n = 5(-2)^{n-1}\)

37. \(a_1 = 2, a_n = 2(2)^{n-1}\)

38. \(a_1 = 2, a_n = 2(5)^{n-1}\)

39. \(a_1 = 12, a_n = 12\left(\frac{1}{4}\right)^{n-1}\)

40. **Critical Thinking** What happens to the terms of a geometric sequence when \(r\) is doubled? Use an example to support your answer.

41. **Geometry** The steps below describe how to make a geometric figure by repeating the same process over and over on a smaller and smaller scale.
   
   **Step 1** (stage 0) Draw a large square.
   
   **Step 2** (stage 1) Divide the square into four equal squares.
   
   **Step 3** (stage 2) Divide each small square into four equal squares.
   
   **Step 4** Repeat Step 3 indefinitely.
   
a. Draw stages 0, 1, 2, and 3.
   
b. How many small squares are in each stage? Organize your data relating stage and number of small squares in a table.
   
c. Does the data in part b form a geometric sequence? Explain.
   
d. Write a rule to find the number of small squares in stage \(n\).

42. **Write About It** Write a series of steps for finding the \(n\)th term of a geometric sequence when you are given the first several terms.

43. a. Three years ago, the annual tuition at a university was $3000. The following year, the tuition was $3300, and last year, the tuition was $3630. If the tuition has continued to grow in the same manner, what is the tuition this year? What do you expect it to be next year?
   
b. What is the common ratio?
   
c. What would you predict the tuition was 4 years ago? How did you find that value?
44. Which of the following is a geometric sequence?

- A. \( \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots \)
- B. \(-2, -6, -10, -14, \ldots \)
- C. \(3, 8, 13, 18, \ldots \)
- D. \(5, 10, 20, 40, \ldots \)

45. Which equation represents the \(n\)th term in the geometric sequence \(2, -8, 32, -128, \ldots\)?

- \(a_n = (-4)^n\)
- \(a_n = (-4)^{n-1}\)
- \(a_n = 2(-4)^n\)
- \(a_n = 2(-4)^{n-1}\)

46. The frequency of a musical note, measured in hertz (Hz), is called its pitch. The pitches of the A keys on a piano form a geometric sequence, as shown.

What is the frequency of A\(_7\)?

- A. 880 Hz
- B. 1760 Hz
- C. 3520 Hz
- D. 7040 Hz

**CHALLENGE AND EXTEND**

Find the next three terms in each geometric sequence.

47. \(x, x^2, x^3, \ldots\)
48. \(2x^2, 6x^3, 18x^4, \ldots\)
49. \(\frac{1}{y^3}, \frac{1}{y^2}, \frac{1}{y}, \ldots\)
50. \(\frac{1}{(x + 1)^2}, \frac{1}{x + 1}, 1, \ldots\)

51. The 10th term of a geometric sequence is 0.78125. The common ratio is \(-0.5\). Find the first term of the sequence.

52. The first term of a geometric sequence is 12 and the common ratio is \(\frac{1}{2}\). Is 0 a term in this sequence? Explain.

53. A geometric sequence starts with 14 and has a common ratio of 0.4. Colin finds that another number in the sequence is 0.057344. Which term in the sequence did Colin find?

54. The first three terms of a sequence are 1, 2, and 4. Susanna said the 8th term of this sequence is 128. Paul said the 8th term is 29. Explain how the students found their answers. Why could these both be considered correct answers?
Complete the sentences below with vocabulary words from the list above.

1. The set of x-coordinates of the ordered pairs of a relation is called the _____.
2. If one set of data values increases as another set of data values decreases, the relationship can be described as having a(n) _____.
3. A sequence is an ordered list of numbers where each number is a(n) _____.

**5-1 Graphing Relationships**

**Examples**

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

- A parking meter has a limit of 1 hour. The cost is $0.25 per 15 minutes and the meter accepts quarters only.

The graph is discrete.

- Ian bought a cup of coffee. At first, he sipped slowly. As it cooled, he drank more quickly. The last bit was cold, and he dumped it out.

As time passes the coffee was **sipped slowly, drank more quickly,** and then **dumped out.**

The graph is continuous.

**Exercises**

Sketch a graph for each situation. Tell whether the graph is continuous or discrete.

4. A girl was walking home at a steady pace. Then she stopped to talk to a friend. After her friend left, she jogged the rest of the way home.

5. A ball is dropped from a second story window and bounces to a stop on the patio below.

6. Jason was on the second floor when he got a call to attend a meeting on the sixth floor. He took the stairs. After the meeting, he took the elevator to the first floor.

Write a possible situation for each graph.

7. **Number of fish in tank**

   - **Week**

   - **Time**

   - **Amount of coffee in cup**

   - **Time**

The graph is discrete.

8. **Height**

   - **Time**

   - **Week**

The graph is continuous.
1. Express each relation as a table, as a graph, and as a mapping diagram.

9. \{(-1, 0), (0, 1), (2, 1)\}

10. \{(-2, -1), (-1, 1), (2, 3), (3, 4)\}

Give the domain and range of each relation.

11. \{(-4, 5), (-2, 3), (0, 1), (2, -1)\}

12. \{(-2, -1), (-1, 0), (0, -1), (1, 0), (2, -1)\}

13. 

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>4</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

14. 

Give the domain and range of each relation. Tell whether the relation is a function. Explain.

15. \{(-5, -3), (-3, -2), (-1, -1), (1, 0)\}

16. 

17. 

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

18. A local parking garage charges $5.00 for the first hour plus $1.50 for each additional hour or part of an hour. Write a relation as a set of ordered pairs in which the x-value represents the number of hours and the y-value represents the cost for x hours. Use a domain of 1, 2, 3, 4, 5. Is this relation a function? Explain.

19. A baseball coach is taking the team for ice cream. Four students can ride in each car. Create a mapping diagram to show the number of cars needed to transport 8, 10, 14, and 16 students. Is this relation a function? Explain.
5-3 Writing Functions

**Examples**

- Determine a relationship between the $x$- and $y$-values in the table. Write an equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>−3</td>
<td>−6</td>
<td>−9</td>
<td>−12</td>
</tr>
</tbody>
</table>

1. $1 - 4 = -3$  
2. $2 - 4 \neq -6$  
3. $3(-3) = -9$  
4. $4(-3) = -12$

$y = -3x$

- Nia earns $5.25 per hour. Identify the independent and dependent variables. Write an equation in function notation for the situation.

Nia's pay depends on the number of hours she works.

Dependent: pay  
Independent: hours

Let $h$ represent the number of hours Nia works.

The function for Nia’s pay is $f(h) = 5.25h$.

5-4 Graphing Functions

**Example**

- Graph the function $y = 3x - 1$.

**Step 1** Choose several values of $x$ to generate ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 3x - 1$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>$3(-1) - 1 = -4$</td>
<td>−4</td>
</tr>
<tr>
<td>0</td>
<td>$3(0) - 1 = -1$</td>
<td>−1</td>
</tr>
<tr>
<td>1</td>
<td>$3(1) - 1 = 2$</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>$3(2) - 1 = 5$</td>
<td>5</td>
</tr>
</tbody>
</table>

**Step 2** Plot enough points to see a pattern.

**Step 3** Draw a line through the points to show all the ordered pairs that satisfy this function.

**Exercises**

- Determine the relationship between the $x$- and $y$-values. Write an equation.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>−6</td>
<td>−5</td>
<td>−4</td>
<td>−3</td>
<td>−2</td>
</tr>
</tbody>
</table>

21. $\{(1, 9), (2, 18), (3, 27), (4, 36)\}$

Identify the independent and dependent variables. Write an equation in function notation for the situation.

22. A baker spends $6 on ingredients for each cake he bakes.

23. Tim will buy twice as many CDs as Raul.

Evaluate each function for the given input values.

24. For $f(x) = -2x + 4$, find $f(x)$ when $x = -5$.

25. For $g(n) = -n^2 - 2$, find $g(n)$ when $n = -3$.

26. For $h(t) = 7 - |t + 3|$, find $h(t)$ when $t = -4$ and when $t = 5$.

**Exercises**

Graph each function.

27. $4x + y = 2$  
28. $y = (1 - x)^2$

Graph each function.

29. $3x - y = 1$  
30. $y = 2 - |x|$  
31. $y = x^2 - 6$  
32. $y = |x + 5| + 1$

33. The function $y = 6.25x$ describes the amount of money $y$ Peter gets paid after $x$ hours. Graph the function. Use the graph to estimate how much money Peter gets paid after 7 hours.
The table shows the value of a car for the given years. Graph a scatter plot using the given data. Describe the correlation illustrated by the scatter plot.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (thousand $)</td>
<td>28</td>
<td>25</td>
<td>23</td>
<td>20</td>
</tr>
</tbody>
</table>

The graph shows the results of a 2003–2004 survey on class size at the given grade levels. Based on this relationship, predict the class size for the 9th grade.

Based on the data, $90 is a reasonable prediction.

5-6 Arithmetic Sequences

**Examples**

- Determine whether the sequence appears to be arithmetic. If so, find the common difference and the next three terms.
  - $-8, -5, -2, 1, ...$

  **Step 1** Find the difference between successive terms.
  
  $-8, -5, -2, 1, ...$  
  
  The common difference is $3$.

  **Step 2** Use the common difference to find the next 3 terms.
  
  $-8, -5, -2, 1, 4, 7, 10$  
  
  Find the 18th term of the arithmetic sequence for which $a_1 = -4$ and $d = 6$.

  $$a_n = a_1 + (n - 1)d$$  
  
  Write the rule.

  $$a_{18} = -4 + (18 - 1)6$$  
  
  Substitute.

  $$= -4 + 102$$  
  
  Simplify.

  $$= 98$$  
  
  The 18th term is 98.

**Exercises**

Determine whether each sequence appears to be arithmetic. If so, find the common difference and the next three terms.

36. $20, 14, 8, 2, ...$

37. $-15, -12, -9, -4, ...$

38. $5, 4, 2, -1, ...$

39. $-8, -5.5, -3, -0.5, ...$

Find the indicated term of each arithmetic sequence.

40. 31st term: $-15, -11, -7, -3, ...$

41. 24th term: $a_1 = 7; d = -3$

42. 17th term: $a_1 = -20; d = 2.5$

43. Marie has $180 in a savings account in week 1. She plans to deposit $12 each following week. Assuming that she does not withdraw any money from her account, what will her balance be in week 20?

44. The table shows the temperature at the given heights above sea level. Use an arithmetic sequence to find the temperature at 8000 feet above sea level.

<table>
<thead>
<tr>
<th>Height Above Sea Level (thousand feet)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>30</td>
<td>23.5</td>
<td>17</td>
<td>10.5</td>
</tr>
</tbody>
</table>
Choose the graph that best represents each situation.

1. A person walks leisurely, stops, and then continues walking.

2. A person jogs, then runs, and then jogs again.

Give the domain and range for each relation. Tell whether the relation is a function. Explain.

3. 

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

4. 

5. Bowling costs $3 per game plus $2.50 for shoe rental. Identify the independent and dependent variables. Write an equation in function notation for the situation.

Evaluate each function for the given input values.

6. For \( f(x) = -3x + 4 \), find \( f(x) \) when \( x = -2 \).

7. For \( f(x) = 2x^2 \), find \( f(x) \) when \( x = -3 \).

8. An engraver charges a $10 fee plus $6 for each line of engraving. Write a function to describe the situation. Find a reasonable domain and range for the function for up to 8 lines.

Graph each function for the given domain.

9. \( 3x + y = 4 \); \( D: \{-2, -1, 0, 1, 2\} \)

10. \( y = |x - 1| \); \( D: \{-3, 0, 1, 3, 5\} \)

11. \( y = x^2 - 1 \); \( D: \{-2, -1, 0, 1, 2\} \)

Graph each function.

12. \( y = x - 5 \)

13. \( y = x^2 - 5 \)

14. \( y = |x| + 3 \)

15. The function \( y = 30x \) describes the amount of interest \( y \) earned in a savings account in \( x \) years. Graph the function. Use the graph to estimate the total amount of interest earned in 7 years.

The table shows possible recommendations for the amount of sleep that children should get every day.

16. Graph a scatter plot of the given data.

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleep Needed (h)</td>
<td>14</td>
<td>13</td>
<td>12</td>
<td>12</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

17. Describe the correlation illustrated by the scatter plot.

18. Predict how many hours of sleep a 16-year-old needs.

Determine whether each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms.

19. 11, 6, 1, -4, ...

20. -4, -3, -1, 2, ...

21. 7, 21, 30, 45, ...

Find the indicated term of the arithmetic sequence.

22. 32nd term: 18, 11, 4, -3, ...

23. 24th term: \( a_1 = 4; d = 6 \)

24. Mandy's new job has a starting salary of $16,000 and annual increases of $800. How much will she earn during her fifth year?
Chapter 6

Linear Functions

6-1 Identifying Linear Functions
EXT Linear and Nonlinear Rates of Change
6-2 Using Intercepts
6-3 Rate of Change and Slope
LAB Explore Constant Changes
6-4 The Slope Formula
6-5 Direct Variation
6-6 Slope-Intercept Formula
6-7 Point-Slope Formula
LAB Graph Linear Functions
6-8 Slopes of Parallel and Perpendicular Lines

Chapter Focus
- Translate among different representations of linear functions.
- Find and interpret slopes and intercepts of linear equations that model real-world problems.
- Solve real-world problems involving linear equations.

Take Flight
You can use linear functions to describe patterns and relationships in flight times.

Learn It Online
Chapter Project Online
**Study Strategy: Use Multiple Representations**

Representing a math concept in more than one way can help you understand it more clearly. As you read the explanations and example problems in your text, note the use of tables, lists, graphs, diagrams, and symbols, as well as words to explain a concept.

In this example, the given function is described using an equation, a table, ordered pairs, and a graph.

**Graphing Functions**

Graph each function.

**Equation**

\[ 2x + 1 = y \]

**Step 1** Choose several values of \( x \) and generate ordered pairs.

**Table**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2x + 1 = y )</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( 2(-3) + 1 = -5 )</td>
<td>(-3, -5)</td>
</tr>
<tr>
<td>-2</td>
<td>( 2(-2) + 1 = -3 )</td>
<td>(-2, -3)</td>
</tr>
<tr>
<td>-1</td>
<td>( 2(-1) + 1 = -1 )</td>
<td>(-1, -1)</td>
</tr>
<tr>
<td>0</td>
<td>( 2(0) + 1 = 1 )</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>1</td>
<td>( 2(1) + 1 = 3 )</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>2</td>
<td>( 2(2) + 1 = 5 )</td>
<td>(2, 5)</td>
</tr>
<tr>
<td>3</td>
<td>( 2(3) + 1 = 7 )</td>
<td>(3, 7)</td>
</tr>
</tbody>
</table>

**Step 2** Plot enough points to see a pattern.

**Graph**

**Ordered Pairs**

Step 3 The ordered pairs appear to form a line. Draw a line through all the points to show all the ordered pairs that satisfy the function. Draw arrowheads on both “ends” of the line.

**Try This**

1. If an employee earns $8.00 an hour, \( y = 8x \) gives the total pay \( y \) the employee will earn for working \( x \) hours. For this equation, make a table of ordered pairs and a graph. Explain the relationships between the equation, the table, and the graph. How does each one describe the situation?

2. What situations might make one representation more useful than another?
Identifying Linear Functions

**Objectives**
Identify linear functions and linear equations.
Graph linear functions that represent real-world situations and give their domain and range.

**Vocabulary**
linear function
linear equation

**Why learn this?**
Linear functions can describe many real-world situations, such as distances traveled at a constant speed.

Most people believe that there is no speed limit on the German autobahn. However, many stretches have a speed limit of 120 km/h. If a car travels continuously at this speed, \( y = 120x \) gives the number of kilometers \( y \) that the car would travel in \( x \) hours. Solutions are shown in the graph.

The graph represents a function because each domain value (\( x \)-value) is paired with exactly one range value (\( y \)-value). Notice that the graph is a straight line. A function whose graph forms a straight line is called a **linear function**.

**Example 1**
Identifying a Linear Function by Its Graph

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

**A**
Each domain value is paired with exactly one range value. The graph forms a line.
linear function

**B**
Each domain value is paired with exactly one range value. The graph is not a line.
not a linear function

**C**
The only domain value, 3, is paired with many different range values.
not a function

**Check It Out!**
Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

1a.
1b.
1c.
You can sometimes identify a linear function by looking at a table or a list of ordered pairs. In a linear function, a constant change in $x$ corresponds to a constant change in $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>7</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
</tr>
</tbody>
</table>

In this table, a constant change of +1 in $x$ corresponds to a constant change of -3 in $y$. These points satisfy a linear function.

The points from this table lie on a line.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>6</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

In this table, a constant change of +1 in $x$ does not correspond to a constant change in $y$. These points do not satisfy a linear function.

The points from this table do not lie on a line.

**Example 2**

### Identifying a Linear Function by Using Ordered Pairs

Tell whether each set of ordered pairs satisfies a linear function. Explain.

**A** \(\{(2, 4), (5, 3), (8, 2), (11, 1)\}\)

Write the ordered pairs in a table. Look for a pattern. A constant change of +3 in $x$ corresponds to a constant change of -1 in $y$.

These points satisfy a linear function.

**B** \(\{(-10, 10), (-5, 4), (0, 2), (5, 0)\}\)

Write the ordered pairs in a table. Look for a pattern. A constant change of +5 in $x$ corresponds to different changes in $y$.

These points do not satisfy a linear function.

2. Tell whether the set of ordered pairs \(\{(3, 5), (5, 4), (7, 3), (9, 2), (11, 1)\}\) satisfies a linear function. Explain.
Another way to determine whether a function is linear is to look at its equation. A function is linear if it is described by a linear equation. A linear equation is any equation that can be written in the standard form shown below.

\[ Ax + By = C \]

where \( A \), \( B \), and \( C \) are real numbers and \( A \) and \( B \) are not both 0

**Standard Form of a Linear Equation**

Notice that when a linear equation is written in standard form

- \( x \) and \( y \) both have exponents of 1.
- \( x \) and \( y \) are not multiplied together.
- \( x \) and \( y \) do not appear in denominators, exponents, or radical signs.

<table>
<thead>
<tr>
<th>Linear</th>
<th>Not Linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 3x + 2y = 10 )</td>
<td>Standard form</td>
</tr>
<tr>
<td>( y - 2 = 3x )</td>
<td>Can be written as ( 3x - y = -2 )</td>
</tr>
<tr>
<td>( -y = 5x )</td>
<td>Can be written as ( 5x + y = 0 )</td>
</tr>
</tbody>
</table>

For any two points, there is exactly one line that contains them both. This means you need only two ordered pairs to graph a line.

**Example 3**

Tell whether each function is linear. If so, graph the function.

**A**

\( y = x + 3 \)

Write the equation in standard form.

\[ y = x + 3 \]

\[ -x \]

Subtraction Property of Equality

\[ y - x = 3 \]

\[-x + y = 3 \]

The equation is in standard form (\( A = -1 \), \( B = 1 \), \( C = 3 \)).

The equation can be written in standard form, so the function is linear.

To graph, choose three values of \( x \), and use them to generate ordered pairs.

(You only need two, but graphing three points is a good check.)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x + 3 )</th>
<th>(( x ), ( y ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( y = 0 + 3 = 3 )</td>
<td>(0, 3)</td>
</tr>
<tr>
<td>1</td>
<td>( y = 1 + 3 = 4 )</td>
<td>(1, 4)</td>
</tr>
<tr>
<td>2</td>
<td>( y = 2 + 3 = 5 )</td>
<td>(2, 5)</td>
</tr>
</tbody>
</table>

Plot the points and connect them with a straight line.

**B**

\( y = x^2 \)

This is not linear, because \( x \) has an exponent other than 1.

**Check It Out!**

Tell whether each function is linear. If so, graph the function.

3a. \( y = 5x - 9 \)  
3b. \( y = 12 \)  
3c. \( y = 2^x \)
For linear functions whose graphs are not horizontal, the domain and range are all real numbers. However, in many real-world situations, the domain and range must be restricted. For example, some quantities cannot be negative, such as time.

Sometimes domain and range are restricted even further to a set of points. For example, a quantity such as number of people can only be whole numbers. When this happens, the graph is not actually connected because every point on the line is not a solution. However, you may see these graphs shown connected to indicate that the linear pattern, or trend, continues.

Example 4

Career Application

Sue rents a manicure station in a salon and pays the salon owner $5.50 for each manicure she gives. The amount Sue pays each day is given by $f(x) = 5.50x$, where $x$ is the number of manicures. Graph this function and give its domain and range.

Choose several values of $x$ and make a table of ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 5.50x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$f(0) = 5.50(0) = 0$</td>
</tr>
<tr>
<td>1</td>
<td>$f(1) = 5.50(1) = 5.50$</td>
</tr>
<tr>
<td>2</td>
<td>$f(2) = 5.50(2) = 11.00$</td>
</tr>
<tr>
<td>3</td>
<td>$f(3) = 5.50(3) = 16.50$</td>
</tr>
<tr>
<td>4</td>
<td>$f(4) = 5.50(4) = 22.00$</td>
</tr>
<tr>
<td>5</td>
<td>$f(5) = 5.50(5) = 27.50$</td>
</tr>
</tbody>
</table>

The number of manicures must be a whole number, so the domain is \{0, 1, 2, 3, \ldots\}. The range is \{0, 5.50, 11.00, 16.50, \ldots\}.

4. What if...? At another salon, Sue can rent a station for $10.00 per day plus $3.00 per manicure. The amount she would pay each day is given by $f(x) = 3x + 10$, where $x$ is the number of manicures. Graph this function and give its domain and range.

Think and Discuss

1. Suppose you are given five ordered pairs that satisfy a function. When you graph them, four lie on a straight line, but the fifth does not. Is the function linear? Why or why not?

2. In Example 4, why is every point on the line not a solution?

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how to use the information to identify a linear function. Include an example.
GUIDED PRACTICE

1. **Vocabulary** Is the linear equation $3x - 2 = y$ in standard form? Explain.

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

2. \[ 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \]
   \[ x \quad y \]

3. \[ 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \]
   \[ x \quad y \]

4. \[ 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \]
   \[ x \quad y \]

Tell whether the given ordered pairs satisfy a linear function. Explain.

5. \begin{array}{|c|c|c|c|c|}
   \hline
   x & 5 & 4 & 3 & 2 & 1 \\
   \hline
   y & 0 & 2 & 4 & 6 & 8 \\
   \hline
\end{array}

6. \begin{array}{|c|c|c|c|c|c|}
   \hline
   x & 1 & 4 & 9 & 16 & 25 \\
   \hline
   y & 1 & 2 & 3 & 4 & 5 \\
   \hline
\end{array}

7. \{(0, 5), (-2, 3), (-4, 1), (-6, -1), (-8, -3)\}

8. \{(2, -2), (-1, 0), (-4, 1), (-7, 3), (-10, 6)\}

Tell whether each function is linear. If so, graph the function.

9. $2x + 3y = 5$
10. $2y = 8$
11. $\frac{x^2 + 3}{5} = y$
12. $\frac{x}{5} = \frac{y}{3}$

13. **Transportation** A train travels at a constant speed of 75 mi/h. The function $f(x) = 75x$ gives the distance that the train travels in $x$ hours. Graph this function and give its domain and range.

14. **Entertainment** A movie rental store charges a $6.00 membership fee plus $2.50 for each movie rented. The function $f(x) = 2.50x + 6$ gives the cost of renting $x$ movies. Graph this function and give its domain and range.

PRACTICE AND PROBLEM SOLVING

Identify whether each graph represents a function. Explain. If the graph does represent a function, is the function linear?

15. \[ 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \]
   \[ x \quad y \]

16. \[ 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \]
   \[ x \quad y \]

17. \[ 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \quad 2 \quad 4 \]
   \[ x \quad y \]

Tell whether the given ordered pairs satisfy a linear function. Explain.

18. \begin{array}{|c|c|c|c|c|}
   \hline
   x & -3 & 0 & 3 & 6 & 9 \\
   \hline
   y & -2 & -1 & 0 & 2 & 4 \\
   \hline
\end{array}

19. \begin{array}{|c|c|c|c|c|}
   \hline
   x & -1 & 0 & 1 & 2 & 3 \\
   \hline
   y & -3 & -2 & -1 & 0 & 1 \\
   \hline
\end{array}

20. \{(3, 4), (0, 2), (-3, 0), (-6, -2), (-9, -4)\}
Tell whether each function is linear. If so, graph the function.

21. \( y = 5 \)  
22. \( 4y - 2x = 0 \)  
23. \( \frac{3}{x} + 4y = 10 \)  
24. \( 5 + 3y = 8 \)

25. **Transportation** The gas tank in Tony’s car holds 15 gallons, and the car can travel 25 miles for each gallon of gas. When Tony begins with a full tank of gas, the function \( f(x) = -\frac{1}{25}x + 15 \) gives the amount of gas \( f(x) \) that will be left in the tank after traveling \( x \) miles (if he does not buy more gas). Graph this function and give its domain and range.

Tell whether the given ordered pairs satisfy a function. If so, is it a linear function?

26. \( \{(2, 5), (2, 4), (2, 3), (2, 2), (2, 1)\} \)  
27. \( \{-8, 2\}, (-6, 0), (-4, -2), (-2, -4), (0, -6)\} \)

Tell whether each equation is linear. If so, write the equation in standard form and give the values of \( A, B, \) and \( C. \)

30. \( 2x - 8y = 16 \)  
31. \( y = 4x + 2 \)  
32. \( 2x = \frac{y}{3} - 4 \)  
33. \( \frac{4}{x} = y \)

34. \( \frac{x + 4}{2} = \frac{y - 4}{3} \)  
35. \( x = 7 \)  
36. \( xy = 6 \)  
37. \( 3x - 5 + y = 2y - 4 \)

38. \( y = -x + 2 \)  
39. \( 5x = 2y - 3 \)  
40. \( 2y = -6 \)  
41. \( y = \sqrt{x} \)

Graph each linear function.

42. \( y = 3x + 7 \)  
43. \( y = x + 25 \)  
44. \( y = 8 - x \)  
45. \( y = 2x \)

46. \( -2y = -3x + 6 \)  
47. \( y - x = 4 \)  
48. \( y - 2x = -3 \)  
49. \( x = 5 + y \)

Tell whether each equation is linear. If so, write the equation in standard form and give the values of \( A, B, \) and \( C. \)

50. **Measurement** One inch is equal to approximately 2.5 centimeters. Let \( x \) represent inches and \( y \) represent centimeters. Write an equation in standard form relating \( x \) and \( y. \) Give the values of \( A, B, \) and \( C. \)

51. **Wages** Molly earns $8.00 an hour at her job.
   a. Let \( x \) represent the number of hours that Molly works. Write a function using \( x \) and \( f(x) \) that describes Molly’s pay for working \( x \) hours.
   b. Graph this function and give its domain and range.

52. **Write About It** For \( y = 2x - 1, \) make a table of ordered pairs and a graph. Describe the relationships between the equation, the table, and the graph.

53. **Critical Thinking** Describe a real-world situation that can be represented by a linear function whose domain and range must be limited. Give your function and its domain and range.

54. a. Juan is running on a treadmill. The table shows the number of Calories Juan burns as a function of time. Explain how you can tell that this relationship is linear by using the table.
   
<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
</tr>
<tr>
<td>9</td>
<td>81</td>
</tr>
<tr>
<td>12</td>
<td>108</td>
</tr>
<tr>
<td>15</td>
<td>135</td>
</tr>
<tr>
<td>18</td>
<td>162</td>
</tr>
<tr>
<td>21</td>
<td>189</td>
</tr>
</tbody>
</table>

   b. Create a graph of the data.
   c. How can you tell from the graph that the relationship is linear?
55. **Physical Science** A ball was dropped from a height of 100 meters. Its height above the ground in meters at different times after its release is given in the table. Do these ordered pairs satisfy a linear function? Explain.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (m)</td>
<td>100</td>
<td>90.2</td>
<td>60.8</td>
<td>11.8</td>
</tr>
</tbody>
</table>

56. **Critical Thinking** Is the equation \( x = 9 \) a linear equation? Does it describe a linear function? Explain.

57. Which is NOT a linear function?

- A. \( y = 8x \)
- B. \( y = x + 8 \)
- C. \( y = \frac{8}{x} \)
- D. \( y = 8 - x \)

58. The speed of sound in 0 °C air is about 331 feet per second. Which function could be used to describe the distance in feet \( d \) that sound will travel in air in \( s \) seconds?

- F. \( d = s + 331 \)
- G. \( d = 331s \)
- H. \( s = 331d \)
- I. \( s = 331 - d \)

59. **Extended Response** Write your own linear function. Show that it is a linear function in at least three different ways. Explain any connections you see between your three methods.

### Challenge and Extend

60. What equation describes the \( x \)-axis? the \( y \)-axis? Do these equations represent linear functions?

**Geometry** Copy and complete each table below. Then tell whether the table shows a linear relationship.

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Side Length</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>
RECALL that a rate of change is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

rate of change = \frac{\text{change in dependent variable}}{\text{change in independent variable}}

The table shows the price of one ounce of gold in 2005 and 2008. The year is the independent variable and the price is the dependent variable. The rate of change is \( \frac{870 - 513}{2008 - 2005} = \frac{357}{3} = 119 \), or $119 per year.

<table>
<thead>
<tr>
<th>Price of Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>2005</td>
</tr>
<tr>
<td>2008</td>
</tr>
</tbody>
</table>

**EXAMPLE 1**

Identifying Constant and Variable Rates of Change

Determine whether each function has a constant or variable rate of change.

**A** \( \{(0, 0), (1, 4), (3, 8), (6, 8), (8, 6)\} \)

Find the ratio of the amount of change in the dependent variable \( y \) to the corresponding amount of change in the independent variable \( x \).

\[
\begin{array}{c|c}
    x & y \\
    \hline
    0 & 0 \\
    1 & 4 \\
    3 & 8 \\
    6 & 8 \\
    8 & 6 \\
\end{array}
\]

The rates of change are \( \frac{4}{1} = 4, \frac{4}{2} = 2, \frac{0}{3} = 0 \), and \( \frac{-2}{2} = -1 \).

The function has a variable rate of change.

**B** \( \{(0, 1), (1, 2), (4, 5), (6, 7), (7, 8)\} \)

Find the ratio of the amount of change in the dependent variable \( y \) to the corresponding amount of change in the independent variable \( x \).

\[
\begin{array}{c|c}
    x & y \\
    \hline
    0 & 1 \\
    1 & 2 \\
    4 & 5 \\
    6 & 7 \\
    7 & 8 \\
\end{array}
\]

The rates of change are \( \frac{1}{1} = 1, \frac{3}{3} = 1, \frac{2}{2} = 1 \), and \( \frac{1}{1} = 1 \).

The function has a constant rate of change.

**CHECK IT OUT!**

Determine whether each function has a constant or variable rate of change.

1a. \( \{(-3, 10), (0, 7), (1, 6), (4, 3), (7, 0)\} \)

1b. \( \{(-2, -3), (2, 5), (3, 7), (5, 9), (8, 12)\} \)
The functions in Examples 1A and 1B are graphed below.

A function is a **linear function** if and only if the function has a constant rate of change. The graph of such a function is a straight line and the rate of change is the slope of the line, as in Example 1B.

A function with a variable rate of change, as in Example 1A, is a **nonlinear function**. Examples of nonlinear functions include quadratic functions and exponential functions.

**Example 2**

**Identifying Linear and Nonlinear Functions**

Use rates of change to determine whether each function is linear or nonlinear.

**A**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
</tr>
</tbody>
</table>

Find the rates of change.

\[
\begin{align*}
\frac{1}{2} &= \frac{0.5}{1} \\
\frac{1.5}{3} &= \frac{1}{2} \\
\frac{3}{6} &= \frac{1}{2}
\end{align*}
\]

There is a constant rate of change, \(\frac{1}{2}\), so this function is linear.

**B**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>18</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Find the rates of change.

\[
\begin{align*}
\frac{-16}{4} &= \frac{-4}{4} = 0 \\
\frac{-2}{-2} &= 1 \\
\frac{8}{4} &= \frac{1}{2}
\end{align*}
\]

The rates of change are not constant, so this function is nonlinear.

**Check it Out!**

Use rates of change to determine whether each function is linear or nonlinear.

**2a.**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1/4</td>
</tr>
<tr>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
</tbody>
</table>

**2b.**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

When you are given a verbal description of a function, you can determine whether the function is linear or nonlinear by making a table of values and examining the rates of change. You can compare two functions by comparing their rates of change.
**Physical Science Application**

Two water tanks contain 512 gallons of water each. Tank A begins to drain, losing half of its volume of water every hour. Tank B begins to drain at the same time and loses 40 gallons of water every hour. Identify the function that gives the volume of water in each tank as linear or nonlinear. Which tank loses water more quickly between hour 4 and hour 5?

Use the verbal descriptions to make a table for the volume of water in each tank.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water in Tank A (gal)</td>
<td>512</td>
<td>256</td>
<td>128</td>
<td>64</td>
<td>32</td>
<td>16</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water in Tank B (gal)</td>
<td>512</td>
<td>472</td>
<td>432</td>
<td>392</td>
<td>352</td>
<td>312</td>
</tr>
</tbody>
</table>

For tank A, the rates of change are \(-256, -128, -64, -32,\) and \(-16\), so the rate of change is variable and the function is nonlinear.

For tank B, the rates of change are all \(-40\), so the rate of change is constant and the function is linear.

Between hours 4 and 5, the volume of water in tank A decreases at a rate of 16 gallons per hour. The volume of water in tank B decreases at a rate of 40 gallons per hour. Tank B loses water more quickly.

3. Reka and Charlotte each invest $500. Each month, Charlotte’s investment grows by $25, while Reka’s investment grows by 5% of the previous month’s amount. Identify the function that gives the value of each investment as linear or nonlinear. Who is earning money more quickly between month 3 and month 4?

Use rates of change to determine whether each function is linear or nonlinear.

### Exercises

1. 

<table>
<thead>
<tr>
<th>(x)</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-2</td>
<td>-1</td>
<td>1</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

2. 

<table>
<thead>
<tr>
<th>(x)</th>
<th>-2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-4</td>
<td>6</td>
<td>8</td>
<td>14</td>
<td>20</td>
</tr>
</tbody>
</table>

3. 

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>3</th>
<th>9</th>
<th>12</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>14</td>
<td>12</td>
<td>8</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

4. 

<table>
<thead>
<tr>
<th>(x)</th>
<th>-8</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-3</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>
5. **Hobbies** Caitlin and Greg collect stamps. Each starts with a collection of 50 stamps. Caitlin adds 15 stamps to her collection each week. Greg adds 1 stamp to his collection the first week, 3 stamps the second week, 5 stamps the third week, and so on. Identify the function that gives the number of stamps in each collection as linear or nonlinear. Which collection is growing more quickly between week 5 and week 6?

Determine whether each function has a constant or variable rate of change.

6. [Graph 1]

7. [Graph 2]

8. [Graph 3]

9. \( y = 2x^2 \)
10. \( y + 1 = 3x \)
11. \( y = -7 \)
12. \( y = \frac{1}{5}x \)
13. \( y = 5^x \)
14. \( y = x^2 + 1 \)
15. \( y = 3\sqrt{x} \)
16. \( y = \frac{x - 3}{2x} \)
17. \( x + y = 6.25 \)

Determine whether each statement is sometimes, always, or never true.

18. A function whose graph is a straight line has a variable rate of change.
19. A quadratic function has a constant rate of change.
20. The rate of change of a linear function is negative.
21. The rate of change between two points on the graph of a nonlinear function is 0.

22. **Critical Thinking** The figure shows the graph of the exponential function \( y = \left(\frac{1}{2}\right)^x \).
   a. Find the rates of change between points A and B, between points B and C, and between points A and D.
   b. What do you notice about the rates of change you found in part a? Do you think this would be true for the rate of change between any two points on the graph?
   c. How do your findings about the rates of change relate to the shape of the graph?

23. A model rocket is launched from the ground. The graph shows the height of the rocket at various times.
   a. Find the rates of change between points A and B and between points B and C.
   b. Which rate of change is greater? What does this tell you about the motion of the rocket?
   c. Find the rates of change between points C and D and between points D and E.
   d. What does the sign of the rates of change you found in part c tell you about the motion of the rocket? Explain.
**Objectives**
Find \( x \)- and \( y \)-intercepts and interpret their meanings in real-world situations.
Use \( x \)- and \( y \)-intercepts to graph lines.

**Vocabulary**
- \( y \)-intercept
- \( x \)-intercept

---

**Who uses this?**
Divers can use intercepts to determine the time a safe ascent will take.

A diver explored the ocean floor 120 feet below the surface and then ascended at a rate of 30 feet per minute. The graph shows the diver's elevation below sea level during the ascent.

The \( y \)-intercept is the \( y \)-coordinate of the point where the graph intersects the \( y \)-axis. The \( x \)-coordinate of this point is always 0.

The \( x \)-intercept is the \( x \)-coordinate of the point where the graph intersects the \( x \)-axis. The \( y \)-coordinate of this point is always 0.

---

**Example 1**
Finding Intercepts

Find the \( x \)- and \( y \)-intercepts.

**A**
The graph intersects the \( x \)-axis at \((-4, 0)\).
The \( x \)-intercept is \(-4\).

The graph intersects the \( y \)-axis at \((0, -3)\).
The \( y \)-intercept is \(-3\).

**B**
\[ 3x - 2y = 12 \]
To find the \( x \)-intercept, replace \( y \) with 0 and solve for \( x \).
\[
3x - 2(0) = 12 \\
3x = 12 \\
x = 4
\]
The \( x \)-intercept is 4.

To find the \( y \)-intercept, replace \( x \) with 0 and solve for \( y \).
\[
3(0) - 2y = 12 \\
-2y = 12 \\
y = -6
\]
The \( y \)-intercept is \(-6\).

---

**Check it Out!**
Find the \( x \)- and \( y \)-intercepts.

1a. \[ 4x - y = 4 \]
1b. \[-3x + 5y = 30 \]
1c. \[ 4x + 2y = 16 \]
EXAMPLE 2

Travel Application

The Sandia Peak Tramway in Albuquerque, New Mexico, travels a distance of about 4500 meters to the top of Sandia Peak. Its speed is 300 meters per minute. The function \( f(x) = 4500 - 300x \) gives the tram’s distance in meters from the top of the peak after \( x \) minutes. Graph this function and find the intercepts. What does each intercept represent?

Neither time nor distance can be negative, so choose several nonnegative values for \( x \). Use the function to generate ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = 4500 - 300x )</td>
<td>4500</td>
<td>3900</td>
<td>3000</td>
<td>1500</td>
<td>0</td>
</tr>
</tbody>
</table>

Graph the ordered pairs. Connect the points with a line.

- **y-intercept:** 4500. This is the starting distance from the top (time = 0).
- **x-intercept:** 15. This is the time when the tram reaches the peak (distance = 0).

Caution!
The graph is not the path of the tram. Even though the line is descending, the graph describes the distance from the peak as the tram goes up the mountain.

CHECK IT OUT!

2. The school store sells pens for $2.00 and notebooks for $3.00. The equation \( 2x + 3y = 60 \) describes the number of pens \( x \) and notebooks \( y \) that you can buy for $60.

   a. Graph the function and find its intercepts.
   
   b. What does each intercept represent?
Remember, to graph a linear function, you need to plot only two ordered pairs. It is often simplest to find the ordered pairs that contain the intercepts.

**Example 3**

**Graphing Linear Equations by Using Intercepts**

Use intercepts to graph the line described by each equation.

**A**

\[ 2x - 4y = 8 \]

**Step 1** Find the intercepts.

\[
\begin{align*}
\text{x-intercept:} & \quad 2x - 4y = 8 \\
2x - 4(0) & = 8 \\
x & = 4 \\
\text{y-intercept:} & \quad 2x - 4y = 8 \\
2(0) - 4y & = 8 \\
y & = -2 \\
\end{align*}
\]

**Step 2** Graph the line.

Plot \((4, 0)\) and \((0, -2)\). Connect with a straight line.

**B**

\[ \frac{2}{3}y = 4 - \frac{1}{2}x \]

**Step 1** Write the equation in standard form.

\[
\begin{align*}
6\left( \frac{2}{3}y \right) & = 6\left( 4 - \frac{1}{2}x \right) \\
4y & = 24 - 3x \\
3x + 4y & = 24
\end{align*}
\]

**Step 2** Find the intercepts.

\[
\begin{align*}
\text{x-intercept:} & \quad 3x + 4y = 24 \\
3x + 4(0) & = 24 \\
x & = 8 \\
\text{y-intercept:} & \quad 3x + 4y = 24 \\
3(0) + 4y & = 24 \\
y & = 6 \\
\end{align*}
\]

**Step 3** Graph the line.

Plot \((8, 0)\) and \((0, 6)\). Connect with a straight line.

**Use intercepts to graph the line described by each equation.**

**3a.** \(-3x + 4y = -12\)  

**3b.** \(y = \frac{1}{3}x - 2\)

**Think and Discuss**

1. A function has x-intercept 4 and y-intercept 2. Name two points on the graph of this function.

2. What is the y-intercept of \(2.304x + y = 4.318\)? What is the x-intercept of \(x - 92.4920y = -21.5489\)?

3. **Get Organized** Copy and complete the graphic organizer.

   - **Know it!**
     - **Graphing \(Ax + By = C\) Using Intercepts**
     - 1. Find the x-intercept by \(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.\)
     - 2. Find the y-intercept by \(\_\_\_\_\_\_\_\_\_\_\_\_\_\_.\)
     - 3. Graph the line by \(\_\_\_\_\_\_\_\_\_\_\_\_\_\_.\)
GUIDED PRACTICE

1. **Vocabulary** The ___?___ is the y-coordinate of the point where a graph crosses the y-axis. (x-intercept or y-intercept)

Find the x- and y-intercepts.

2.  
3.  
4.  

5. $2x - 4y = 4$
6. $-2y = 3x - 6$
7. $4y + 5x = 2y - 3x + 16$

8. **Biology** To thaw a specimen stored at $-25^\circ C$, the temperature of a refrigeration tank is raised 5 °C every hour. The temperature in the tank after $x$ hours can be described by the function $f(x) = -25 + 5x$.
   a. Graph the function and find its intercepts.
   b. What does each intercept represent?

SEE EXAMPLE 2

Use intercepts to graph the line described by each equation.

9. $4x - 5y = 20$
10. $y = 2x + 4$
11. $\frac{1}{3}x - \frac{1}{4}y = 2$
12. $-5y + 2x = -10$

SEE EXAMPLE 3

PRACTICE AND PROBLEM SOLVING

Find the x- and y-intercepts.

13.  
14.  
15.  

16. $6x + 3y = 12$
17. $4y - 8 = 2x$
18. $-2y + x = 2y - 8$
19. $4x + y = 8$
20. $y - 3x = -15$
21. $2x + y = 10x - 1$

22. **Environmental Science** A fishing lake was stocked with 300 bass. Each year, the population decreases by 25. The population of bass in the lake after $x$ years is represented by the function $f(x) = 300 - 25x$.
   a. Graph the function and find its intercepts.
   b. What does each intercept represent?

23. **Sports** Julie is running a 5-kilometer race. She runs 1 kilometer every 5 minutes. Julie’s distance from the finish line after $x$ minutes is represented by the function $f(x) = 5 - \frac{1}{5}x$.
   a. Graph the function and find its intercepts.
   b. What does each intercept represent?
Use intercepts to graph the line described by each equation.

24. \(4x - 6y = 12\)  
25. \(2x + 3y = 18\)  
26. \(\frac{1}{2}x - 4y = 4\)  
27. \(y - x = -1\)  
28. \(5x + 3y = 15\)  
29. \(x - 3y = -1\)

30. **Biology** A bamboo plant is growing 1 foot per day. When you first measure it, it is 4 feet tall.
   
a. Write an equation to describe the height \(y\), in feet, of the bamboo plant \(x\) days after you measure it.

b. What is the \(y\)-intercept?

b. What is the meaning of the \(y\)-intercept in this problem?

31. **Estimation** Look at the scatter plot and trend line.

   a. Estimate the \(x\)- and \(y\)-intercepts.

   b. What is the real-world meaning of each intercept?

32. **Personal Finance** A bank employee notices an abandoned checking account with a balance of $412. If the bank charges a $4 monthly fee for the account, the function \(b = 412 - 4m\) shows the balance \(b\) in the account after \(m\) months.

   a. Graph the function and give its domain and range. (Hint: The bank will keep charging the monthly fee even after the account is empty.)

   b. Find the intercepts. What does each intercept represent?

   c. When will the bank account balance be 0?

33. **Critical Thinking** Complete the following to learn about intercepts and horizontal and vertical lines.

   a. Graph \(x = -6\), \(x = 1\), and \(x = 5\). Find the intercepts.

   b. Graph \(y = -3\), \(y = 2\), and \(y = 7\). Find the intercepts.

   c. Write a rule describing the intercepts of linear equations whose graphs are horizontal and vertical lines.

Match each equation with a graph.

34. \(-2x - y = 4\)  
35. \(y = 4 - 2x\)  
36. \(2y + 4x = 8\)  
37. \(4x - 2y = 8\)
38. Kristyn rode a stationary bike at the gym. She programmed the timer for 20 minutes. The display counted backward to show how much time remained in her workout. It also showed her mileage.
   a. What are the intercepts?
   b. What do the intercepts represent?

<table>
<thead>
<tr>
<th>Time Remaining (min)</th>
<th>Distance Covered (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>16</td>
<td>0.35</td>
</tr>
<tr>
<td>12</td>
<td>0.70</td>
</tr>
<tr>
<td>8</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>1.40</td>
</tr>
<tr>
<td>0</td>
<td>1.75</td>
</tr>
</tbody>
</table>

39. **Write About It** Write a real-world problem that could be modeled by a linear function whose $x$-intercept is 5 and whose $y$-intercept is 60.

40. Which is the $x$-intercept of $-2x = 9y - 18$?
   A) $-9$  B) $-2$  C) 2  D) 9

41. Which of the following situations could be represented by the graph?
   F) Jamie owed her uncle $200. Each week for 40 weeks she paid him $5.
   G) Jamie owed her uncle $200. Each week for 5 weeks she paid him $40.
   H) Jamie owed her uncle $40. Each week for 5 weeks she paid him $200.
   I) Jamie owed her uncle $40. Each week for 200 weeks she paid him $5.

42. **Gridded Response** What is the $y$-intercept of $60x + 55y = 660$?

43. \( \frac{1}{2}x + \frac{1}{5}y = 1 \)
44. \( 0.5x - 0.2y = 0.75 \)
45. \( y = \frac{3}{8}x + 6 \)

46. For any linear equation \( Ax + By = C \), what are the intercepts?

47. Find the intercepts of \( 22x - 380y = 20,900 \). Explain how to use the intercepts to determine appropriate scales for the graph.
Area in the Coordinate Plane

Lines in the coordinate plane can form the sides of polygons. You can use points on these lines to help you find the areas of these polygons.

**Example**

Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $3x + 2y = 18$.

**Step 1** Find the intercepts of $3x + 2y = 18$.

- **x-intercept:** $3x + 2y = 18$
  
  $3x + 2(0) = 18$
  
  $3x = 18$
  
  $x = 6$

- **y-intercept:** $3x + 2y = 18$
  
  $3(0) + 2y = 18$
  
  $2y = 18$
  
  $y = 9$

**Step 2** Use the intercepts to graph the line. The x-intercept is 6, so plot (6, 0). The y-intercept is 9, so plot (0, 9). Connect with a straight line. Then shade the triangle formed by the line and the axes, as described.

**Step 3** Recall that the area of a triangle is given by $A = \frac{1}{2}bh$.

- The length of the base is 6.
- The height is 9.

**Step 4** Substitute these values into the formula.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(9) \quad \text{Substitute into the area formula.}$$

$$= \frac{1}{2}(54) \quad \text{Simplify.}$$

$$= 27$$

The area of the triangle is 27 square units.

**Try This**

1. Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $3x + 2y = 12$.

2. Find the area of the triangle formed by the x-axis, the y-axis, and the line described by $y = 6 - x$.

3. Find the area of the polygon formed by the x-axis, the y-axis, the line described by $y = 6$, and the line described by $x = 4$. 

**Connecting Algebra to Geometry** 305
Objectives
Find rates of change and slopes.
Relate a constant rate of change to the slope of a line.

Vocabulary
rate of change
rise
run
slope

Why learn this?
Rates of change can be used to find how quickly costs have increased.

In 1985, the cost of sending a 1-ounce letter was 22 cents. In 1988, the cost was 25 cents. How fast did the cost change from 1985 to 1988? In other words, at what rate did the cost change?

A rate of change is a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable.

rate of change = \frac{\text{change in dependent variable}}{\text{change in independent variable}}

Example 1
Consumer Application
The table shows the cost of mailing a 1-ounce letter in different years. Find the rate of change in cost for each time interval. During which time interval did the cost increase at the greatest rate?

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (¢)</td>
<td>25</td>
<td>25</td>
<td>29</td>
<td>37</td>
<td>42</td>
</tr>
</tbody>
</table>

Step 1 Identify the dependent and independent variables.

dependent: cost   independent: year

Step 2 Find the rates of change.

1988 to 1990 \frac{\text{change in cost}}{\text{change in years}} = \frac{25 - 25}{1990 - 1988} = \frac{0}{2} = 0 \text{ cents/year}

1990 to 1991 \frac{\text{change in cost}}{\text{change in years}} = \frac{29 - 25}{1991 - 1990} = \frac{4}{1} = 4 \text{ cents/year}

1991 to 2004 \frac{\text{change in cost}}{\text{change in years}} = \frac{37 - 29}{2004 - 1991} = \frac{8}{13} \approx 0.62 \approx \frac{0.62}{\text{cents/year}}

2004 to 2008 \frac{\text{change in cost}}{\text{change in years}} = \frac{42 - 37}{2008 - 2004} = \frac{5}{4} = 1.25 \text{ cents/year}

The cost increased at the greatest rate from 1990 to 1991.

Check It Out!
1. The table shows the balance of a bank account on different days of the month. Find the rate of change for each time interval. During which time interval did the balance decrease at the greatest rate?

<table>
<thead>
<tr>
<th>Day</th>
<th>1</th>
<th>6</th>
<th>16</th>
<th>22</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance ($)</td>
<td>550</td>
<td>285</td>
<td>210</td>
<td>210</td>
<td>175</td>
</tr>
</tbody>
</table>
**Example 2**

**Finding Rates of Change from a Graph**

Graph the data from Example 1 and show the rates of change.

Graph the ordered pairs. The vertical blue segments show the changes in the dependent variable, and the horizontal green segments show the changes in the independent variable.

Notice that the greatest rate of change is represented by the steepest of the red line segments.

Also notice that between 1988 and 1990, when the cost did not change, the red line segment is horizontal.

**Check It Out!**

2. Graph the data from Check It Out Problem 1 and show the rates of change.

If all of the connected segments have the same rate of change, then they all have the same steepness and together form a straight line. The constant rate of change of a nonvertical line is called the slope of the line.

**Slope of a Line**

The rise is the difference in the y-values of two points on a line.

The run is the difference in the x-values of two points on a line.

The slope of a line is the ratio of rise to run for any two points on the line.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}
\]

(Remember that y is the dependent variable and x is the independent variable.)

**Example 3**

**Finding Slope**

Find the slope of the line.

Begin at one point and count vertically to find the rise.

Then count horizontally to the second point to find the run.

It does not matter which point you start with. The slope is the same.

\[
\text{slope} = \frac{2}{1} = 2
\]

\[
\text{slope} = \frac{-2}{-1} = 2
\]

**Check It Out!**

3. Find the slope of the line that contains (0, -3) and (5, -5).
**Example 4**

**Finding Slopes of Horizontal and Vertical Lines**

Find the slope of each line.

**A**

\[
\begin{align*}
\text{rise} & \quad = \quad 0 \\
\text{run} & \quad = \quad 4
\end{align*}
\]

\[
\frac{\text{rise}}{\text{run}} = \frac{0}{4} = 0
\]

The slope is 0.

**B**

\[
\begin{align*}
\text{rise} & \quad = \quad 2 \\
\text{run} & \quad = \quad 0
\end{align*}
\]

You cannot divide by 0.

The slope is undefined.

**Check It Out**

Find the slope of each line.

4a.

4b.

As shown in the previous examples, slope can be positive, negative, zero, or undefined. You can tell which of these is the case by looking at the graph of a line—you do not need to calculate the slope.

**Example 5**

**Describing Slope**

Tell whether the slope of each line is positive, negative, zero, or undefined.

**A**

The line falls from left to right.

The slope is negative.

**B**

The line is horizontal.

The slope is 0.
Tell whether the slope of each line is positive, negative, zero, or undefined.

5a. 
5b. 

A line’s slope is a measure of its steepness. Some lines are steeper than others. As the absolute value of the slope increases, the line becomes steeper. As the absolute value of the slope decreases, the line becomes less steep.

**Comparing Slopes**

<table>
<thead>
<tr>
<th>Slope</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>![Graph 1]</td>
</tr>
<tr>
<td>4</td>
<td>![Graph 2]</td>
</tr>
<tr>
<td>$1_2$</td>
<td>![Graph 3]</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>![Graph 4]</td>
</tr>
</tbody>
</table>

The line with slope 4 is steeper than the line with slope $\frac{1}{2}$.

$|4| > \frac{1}{2}$

The line with slope $-2$ is steeper than the line with slope $-1$.

$|-2| > |-1|$

The line with slope $-3$ is steeper than the line with slope $\frac{3}{4}$.

$|-3| > \frac{3}{4}$

**THINK AND DISCUSS**

1. What is the rise shown in the graph? What is the run? What is the slope?

2. The rate of change of the profits of a company over one year is negative. How have the profits of the company changed over that year?

3. Would you rather climb a hill with a slope of 4 or a hill with a slope of $\frac{5}{2}$? Explain your answer.

4. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, sketch a line whose slope matches the given description.
GUIDED PRACTICE

1. **Vocabulary** The slope of any nonvertical line is _____. (positive or constant)

2. The table shows the volume of gasoline in a gas tank at different times. Find the rate of change for each time interval. During which time interval did the volume decrease at the greatest rate?

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume (gal)</td>
<td>12</td>
<td>9</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

3. The table shows a person's heart rate over time. Graph the data and show the rates of change.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heart Rate (beats/min)</td>
<td>64</td>
<td>92</td>
<td>146</td>
<td>84</td>
<td>64</td>
</tr>
</tbody>
</table>

Find the slope of each line.

4. **See Example 3**

5. **See Example 4**

6. **See Example 5**

Tell whether the slope of each line is positive, negative, zero, or undefined.

8. ****

9. ****

10. ****

11. ****
12. The table shows the length of a baby at different ages. Find the rate of change for each time interval. Round your answers to the nearest tenth. During which time interval did the baby have the greatest growth rate?

<table>
<thead>
<tr>
<th>Age (mo)</th>
<th>3</th>
<th>9</th>
<th>18</th>
<th>26</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.)</td>
<td>23.5</td>
<td>27.5</td>
<td>31.6</td>
<td>34.5</td>
<td>36.7</td>
</tr>
</tbody>
</table>

13. The table shows the distance of an elevator from the ground floor at different times. Graph the data and show the rates of change.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>15</th>
<th>23</th>
<th>30</th>
<th>35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (m)</td>
<td>30</td>
<td>70</td>
<td>0</td>
<td>45</td>
<td>60</td>
</tr>
</tbody>
</table>

Find the slope of each line.

14.

15.

16.

17.

Tell whether the slope of each line is positive, negative, zero, or undefined.

18.

19.

20. Travel The Lookout Mountain Incline Railway in Chattanooga, Tennessee, is the steepest passenger railway in the world. A section of the railway has a slope of about 0.73. In this section, a vertical change of 1 unit corresponds to a horizontal change of what length? Round your answer to the nearest hundredth.

21. Critical Thinking In Lesson 5-1, you learned that in a linear function, a constant change in x corresponds to a constant change in y. How is this related to slope?
22. a. The graph shows a relationship between a person's age and his or her estimated maximum heart rate in beats per minute. Find the slope.
   b. Describe the rate of change in this situation.

23. Construction Most staircases in use today have 9-inch treads and $8\frac{1}{2}$-inch risers. What is the slope of a staircase with these measurements?

24. A ladder is leaned against a building. The bottom of the ladder is 9 feet from the building. The top of the ladder is 16 feet above the ground.
   a. Draw a diagram to represent this situation.
   b. What is the slope of the ladder?

25. Write About It Why will the slope of any horizontal line be 0? Why will the slope of any vertical line be undefined?

26. The table shows the distance traveled by a car during a five-hour road trip.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (mi)</td>
<td>0</td>
<td>40</td>
<td>80</td>
<td>80</td>
<td>110</td>
<td>160</td>
</tr>
</tbody>
</table>

   a. Graph the data and show the rates of change.
   b. The rate of change represents the average speed. During which hour was the car's average speed the greatest?

27. Estimation The graph shows the number of files scanned by a computer virus detection program over time.
   a. Estimate the coordinates of point $A$.
   b. Estimate the coordinates of point $B$.
   c. Use your answers from parts a and b to estimate the rate of change (in files per second) between points $A$ and $B$.

28. Data Collection Use a graphing calculator and a motion detector for the following. Set the equipment so that the graph shows distance on the $y$-axis and time on the $x$-axis.
   a. Experiment with walking in front of the motion detector. How must you walk to graph a straight line? Explain.
   b. Describe what you must do differently to graph a line with a positive slope vs. a line with a negative slope.
   c. How can you graph a line with slope 0? Explain.
29. The slope of which line has the greatest absolute value?

A. line A  C. line C
B. line B  D. line D

30. For which line is the run equal to 0?

A. line A  C. line C
B. line B  D. line D

31. Which line has a slope of 4?

[Graphs of lines A, B, C, D with options F, G, H, J]

32. **Recreation** Tara and Jade are hiking up a hill. Each has a different stride. The run for Tara's stride is 32 inches, and the rise is 8 inches. The run for Jade's stride is 36 inches. What is the rise of Jade's stride?

33. **Economics** The table shows cost in dollars charged by an electric company for various amounts of energy in kilowatt-hours.

<table>
<thead>
<tr>
<th>Energy (kWh)</th>
<th>0</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>1000</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>3</td>
<td>3</td>
<td>31</td>
<td>59</td>
<td>115</td>
<td>150</td>
</tr>
</tbody>
</table>

a. Graph the data and show the rates of change.
b. Compare the rates of change for each interval. Are they all the same? Explain.
c. What do the rates of change represent?
d. Describe in words the electric company's billing plan.
Explore Constant Changes

There are many real-life situations in which the amount of change is constant. In these activities, you will explore what happens when

• a quantity increases by a constant amount.
• a quantity decreases by a constant amount.

Activity 1

Janice has read 7 books for her summer reading club. She plans to read 2 books each week for the rest of the summer. The table shows the total number of books that Janice will have read after different numbers of weeks have passed.

1. What number is added to the number of books in each row to get the number of books in the next row?

2. What does your answer to Problem 1 represent in Janice’s situation? Describe the meaning of the constant change.

3. Graph the ordered pairs from the table. Describe how the points are related.

4. Look again at your answer to Problem 1. Explain how this number affects your graph.

Try This

At a particular college, a full-time student must take at least 12 credit hours per semester and may take up to 18 credit hours per semester. Tuition costs $200 per credit hour.

1. Copy and complete the table by using the information above.

2. What number is added to the cost in each row to get the cost in the next row?

3. What does your answer to Problem 2 above represent in the situation? Describe the meaning of the constant change.

4. Graph the ordered pairs from the table. Describe how the points are related.

5. Look again at your answer to Problem 2. Explain how this number affects your graph.

6. Compare your graphs from Activity 1 and Problem 4. How are they alike? How are they different?

7. Make a Conjecture Describe the graph of any situation that involves repeated addition of a positive number. Why do you think your description is correct?
Activity 2
An airplane is 3000 miles from its destination. The plane is traveling at a rate of 540 miles per hour. The table shows how far the plane is from its destination after various amounts of time have passed.

1. What number is subtracted from the distance in each row to get the distance in the next row?

2. What does your answer to Problem 1 represent in the situation? Describe the meaning of the constant change.

3. Graph the ordered pairs from the table. Describe how the points are related.

4. Look again at your answer to Problem 1. Explain how this number affects your graph.

Try This
A television game show begins with 20 contestants. Each week, the players vote 2 contestants off the show.

8. Copy and complete the table by using the information above.

9. What number is subtracted from the number of contestants in each row to get the number of contestants in the next row?

10. What does your answer to Problem 9 represent in the situation? Describe the meaning of the constant change.

11. Graph the ordered pairs from the table. Describe how the points are related.

12. Look again at your answer to Problem 9. Explain how this number affects your graph.

13. Compare your graphs from Activity 2 and Problem 11. How are they alike? How are they different?

14. Make a Conjecture Describe the graph of any situation that involves repeated subtraction of a positive number. Why do you think your description is correct?

15. Compare your two graphs from Activity 1 with your two graphs from Activity 2. How are they alike? How are they different?

16. Make a Conjecture How are graphs of situations involving repeated subtraction different from graphs of situations involving repeated addition? Explain your answer.
The Slope Formula

**Why learn this?**

You can use the slope formula to find how quickly a quantity, such as the amount of water in a reservoir, is changing. (See Example 3.)

In a previous lesson, slope was described as the constant rate of change of a line. You saw how to find the slope of a line by using its graph.

There is also a formula you can use to find the slope of a line, which is usually represented by the letter \( m \). To use this formula, you need the coordinates of two different points on the line.

**WORDS FORMULA EXAMPLE**

The slope of a line is the ratio of the difference in \( y \)-values to the difference in \( x \)-values between any two different points on the line.

If \((x_1, y_1)\) and \((x_2, y_2)\) are any two different points on a line, the slope of the line is \( m = \frac{y_2 - y_1}{x_2 - x_1} \). If \((2, -3)\) and \((1, 4)\) are two points on a line, the slope of the line is \( m = \frac{4 - (-3)}{1 - 2} = \frac{7}{-1} = -7 \).

**Finding Slope by Using the Slope Formula**

Find the slope of the line that contains \((4, -2)\) and \((-1, 2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Use the slope formula.}
\]

\[
= \frac{2 - (-2)}{-1 - 4} \quad \text{Substitute \((4, -2)\) for \((x_1, y_1)\) and \((-1, 2)\) for \((x_2, y_2)\).}
\]

\[
= \frac{4}{-5} \quad \text{Simplify.}
\]

\[
= -\frac{4}{5}
\]

The slope of the line that contains \((4, -2)\) and \((-1, 2)\) is \(-\frac{4}{5}\).

**Example 1a.** Find the slope of the line that contains \((-2, -2)\) and \((7, -2)\).

**Example 1b.** Find the slope of the line that contains \((5, -7)\) and \((6, -4)\).

**Example 1c.** Find the slope of the line that contains \(\left(\frac{3}{4}, \frac{7}{5}\right)\) and \(\left(\frac{1}{4}, \frac{2}{5}\right)\).
Sometimes you are not given two points to use in the formula. You might have to choose two points from a graph or a table.

**Example 2** Finding Slope from Graphs and Tables

Each graph or table shows a linear relationship. Find the slope.

**A**

Let \((2, 2)\) be \((x_1, y_1)\) and \((-2, -1)\) be \((x_2, y_2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Use the slope formula.

\[
= \frac{-1 - 2}{-2 - 2}
\]

Substitute \((2, 2)\) for \((x_1, y_1)\) and \((-2, -1)\) for \((x_2, y_2)\).

\[
= \frac{-3}{-4}
\]

Simplify.

\[
= \frac{3}{4}
\]

**B**

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

Step 1 Choose any two points from the table. Let \((2, 0)\) be \((x_1, y_1)\) and \((2, 3)\) be \((x_2, y_2)\).

Step 2 Use the slope formula.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Use the slope formula.

\[
= \frac{3 - 0}{2 - 2}
\]

Substitute \((2, 0)\) for \((x_1, y_1)\) and \((2, 3)\) for \((x_2, y_2)\).

\[
= \frac{3}{0}
\]

Simplify.

The slope is undefined.

**Check It Out!**

Each graph or table shows a linear relationship. Find the slope.

2a. \[(8, 6)\] 2b. \[(-2, 4)\]

2c. \[
\begin{array}{lllll}
\text{x} & 0 & 2 & 5 & 6 \\
\text{y} & 1 & 5 & 11 & 13 \\
\end{array}
\]

2d. \[
\begin{array}{lllll}
\text{x} & -2 & 0 & 2 & 4 \\
\text{y} & 3 & 0 & -3 & -6 \\
\end{array}
\]

Remember that slope is a rate of change. In real-world problems, finding the slope can give you information about how a quantity is changing.
Environmental Science Application

The graph shows how much water is in a reservoir at different times. Find the slope of the line. Then tell what the slope represents.

**Step 1** Use the slope formula.

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
= \frac{2000 - 3000}{60 - 20}
\]

\[
= \frac{-1000}{40}
\]

\[
= -25
\]

**Step 2** Tell what the slope represents.

In this situation, \( y \) represents volume of water and \( x \) represents time. So slope represents change in volume in units of thousands of cubic feet per hour.

A slope of -25 means the amount of water in the reservoir is decreasing (negative change) at a rate of 25 thousand cubic feet each hour.

3. The graph shows the height of a plant over a period of days. Find the slope of the line. Then tell what the slope represents.

If you know the equation that describes a line, you can find its slope by using any two ordered-pair solutions. It is often easiest to use the ordered pairs that contain the intercepts.

Finding Slope from an Equation

Find the slope of the line described by \( 6x - 5y = 30 \).

**Step 1** Find the \( x \)-intercept.

\[
6x - 5y = 30
\]

\[
6x - 5(0) = 30 \quad \text{Let } y = 0.
\]

\[
6x = 30
\]

\[
\frac{6x}{6} = \frac{30}{6}
\]

\[
x = 5
\]

**Step 2** Find the \( y \)-intercept.

\[
6x - 5y = 30 \quad \text{Let } x = 0.
\]

\[
-5y = 30
\]

\[
\frac{-5y}{-5} = \frac{30}{-5}
\]

\[
y = -6
\]

**Step 3** The line contains \((5, 0)\) and \((0, -6)\). Use the slope formula.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{0 - 5} = \frac{-6}{-5} = \frac{6}{5}
\]

4. Find the slope of the line described by \( 2x + 3y = 12 \).
THINK AND DISCUSS

1. The slope of a line is the difference of the \( y \) values divided by the difference of the \( x \) values for any two points on the line.

2. Two points lie on a line. When you substitute their coordinates into the slope formula, the value of the denominator is 0. Describe this line.

3. GET ORGANIZED  Copy and complete the graphic organizer. In each box, describe how to find slope using the given method.

GUIDED PRACTICE

1. Find the slope of the line that contains each pair of points.
   1. \((3, 6)\) and \((6, 9)\)
   2. \((2, 7)\) and \((4, 4)\)
   3. \((-1, -5)\) and \((-9, -1)\)

2. Each graph or table shows a linear relationship. Find the slope.
   4. [Graph with points \((-2, -1)\) and \((4, 2)\)]
   5. \[
   \begin{array}{|c|c|}
   \hline
   x & y \\
   \hline
   0 & 25 \\
   2 & 45 \\
   4 & 65 \\
   6 & 85 \\
   \hline
   \end{array}
   \]

3. Find the slope of each line. Then tell what the slope represents.
   6. [Graph with points \((4, 80)\) and \((12, 160)\)]
   7. [Graph with points \((1620, 3)\) and \((4860, 9)\)]

4. Find the slope of the line described by each equation.
   8. \(8x + 2y = 96\)
   9. \(5x = 90 - 9y\)
   10. \(5y = 160 + 9x\)
PRACTICE AND PROBLEM SOLVING

Find the slope of the line that contains each pair of points.

11. (2, 5) and (3, 1)  
12. (−9, −5) and (6, −5)  
13. (3, 4) and (3, −1)

Each graph or table shows a linear relationship. Find the slope.

14. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.5</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>25.5</td>
</tr>
<tr>
<td>4</td>
<td>29</td>
</tr>
</tbody>
</table>

15. 

Find the slope of each line. Then tell what the slope represents.

16. **Temperature Conversion**

17. **Boiling Point of Water**

Find the slope of the line described by each equation.

18. \(7x + 13y = 91\)  
19. \(5y = 130 - 13x\)  
20. \(7 - 3y = 9x\)

21. **ERROR ANALYSIS**  
   Two students found the slope of the line that contains (−6, 3) and (2, −1). Who is incorrect? Explain the error.

\[A\] \(m = \frac{-1 - 3}{2 - (-6)} = \frac{-4}{8} = -\frac{1}{2}\)

\[B\] \(m = \frac{-1 - 3}{-6 - 2} = \frac{-4}{-8} = \frac{1}{2}\)

22. **Environmental Science**  
The table shows how the number of cricket chirps per minute changes with the air temperature.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chirps per minute</td>
<td>0</td>
<td>40</td>
<td>80</td>
<td>120</td>
<td>160</td>
<td>200</td>
</tr>
</tbody>
</table>

a. Find the rates of change.
b. Is the graph of the data a line? If so, what is the slope? If not, explain why not.

23. **Critical Thinking**  
The graph shows the distance traveled by two cars.
a. Which car is going faster? How much faster?
b. How are the speeds related to slope?
c. At what rate is the distance between the cars changing?

24. **Write About It**  
You are given the coordinates of two points on a line. Describe two different ways to find the slope of that line.
26. The equation $2y + 3x = -6$ describes a line with what slope?

\[ \text{A} \quad \frac{3}{2} \quad \text{B} \quad 0 \quad \text{C} \quad \frac{1}{2} \quad \text{D} \quad -\frac{3}{2} \]

27. A line with slope $-\frac{1}{3}$ could pass through which of the following pairs of points?

\[ \text{F} \quad (0, -\frac{1}{3}) \text{ and } (1, 1) \quad \text{H} \quad (0, 0) \text{ and } \left(-\frac{1}{3}, -\frac{1}{3}\right) \]
\[ \text{G} \quad (-6, 5) \text{ and } (-3, 4) \quad \text{I} \quad (5, -6) \text{ and } (4, 3) \]

28. **Gridded Response** Find the slope of the line that contains $(-1, 2)$ and $(5, 5)$.

**CHALLENGE AND EXTEND**

Find the slope of the line that contains each pair of points.

29. $(a, 0)$ and $(0, b)$
30. $(2x, y)$ and $(x, 3y)$
31. $(x, y)$ and $(x + 2, 3 - y)$

Find the value of $x$ so that the points lie on a line with the given slope.

32. $(x, 2)$ and $(-5, 8)$, $m = -1$
33. $(4, x)$ and $(6, 3x)$, $m = \frac{1}{2}$
34. $(1, -3)$ and $(3, x)$, $m = -1$
35. $(-10, -4)$ and $(x, x)$, $m = \frac{1}{7}$

36. A line contains the point $(1, 2)$ and has a slope of $\frac{1}{2}$. Use the slope formula to find another point on this line.

37. The points $(-2, 4)$, $(0, 2)$, and $(3, x - 1)$ all lie on the same line. What is the value of $x$? *(Hint: Remember that the slope of a line is constant for any two points on the line.)*
Direct Variation

Who uses this?

Chefs can use direct variation to determine ingredients needed for a certain number of servings.

A recipe for paella calls for 1 cup of rice to make 5 servings. In other words, a chef needs 1 cup of rice for every 5 servings.

<table>
<thead>
<tr>
<th>Rice (c)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servings</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

The equation \( y = 5x \) describes this relationship. In this relationship, the number of servings varies directly with the number of cups of rice.

A **direct variation** is a special type of linear relationship that can be written in the form \( y = kx \), where \( k \) is a nonzero constant called the **constant of variation**.

**Example 1**

**Identifying Direct Variations from Equations**

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

**A** \( y = 4x \)

This equation represents a direct variation because it is in the form \( y = kx \). The constant of variation is 4.

**B** \(-3x + 5y = 0\)

\[-3x + 5y = 0\]
\[+3x \quad +3x\]
\[5y = 3x\]
\[\frac{5y}{5} = \frac{3x}{5}\]
\[y = \frac{3}{5}x\]

This equation represents a direct variation because it can be written in the form \( y = kx \). The constant of variation is \( \frac{3}{5} \).

**C** \(2x + y = 10\)

\[2x + y = 10\]
\[-2x \quad -2x\]
\[y = -2x + 10\]

This equation does not represent a direct variation because it cannot be written in the form \( y = kx \).

**Check It Out!**

Tell whether each equation represents a direct variation. If so, identify the constant of variation.

1a. \(3y = 4x + 1\)  
1b. \(3x = -4y\)  
1c. \(y + 3x = 0\)
What happens if you solve $y = kx$ for $k$?

$$y = kx$$

$$\frac{y}{x} = \frac{kx}{x}$$  \textit{Divide both sides by } x \neq 0.

$$\frac{y}{x} = k$$

So, in a direct variation, the ratio $\frac{y}{x}$ is equal to the constant of variation. Another way to identify a direct variation is to check whether $\frac{y}{x}$ is the same for each ordered pair (except where $x = 0$).

**Example 2**  Identifying Direct Variations from Ordered Pairs

Tell whether each relationship is a direct variation. Explain.

**A**

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>18</td>
<td>30</td>
</tr>
</tbody>
</table>

**Method 1** Write an equation.

$y = 6x$  \textit{Each } $y$-value \textit{is 6 times the corresponding } $x$-value.

This is a direct variation because it can be written as $y = kx$, where $k = 6$.

**Method 2** Find $\frac{y}{x}$ for each ordered pair.

$$\frac{6}{1} = 6 \quad \frac{18}{3} = 6 \quad \frac{30}{5} = 6$$

This is a direct variation because $\frac{y}{x}$ is the same for each ordered pair.

**B**

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-2</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

**Method 1** Write an equation.

$y = x - 4$  \textit{Each } $y$-value \textit{is 4 less than the corresponding } $x$-value.

This is not a direct variation because it cannot be written as $y = kx$.

**Method 2** Find $\frac{y}{x}$ for each ordered pair.

$$\frac{-2}{2} = -1 \quad \frac{0}{4} = 0 \quad \frac{4}{8} = \frac{1}{2}$$

This is not a direct variation because $\frac{y}{x}$ is not the same for all ordered pairs.

**Check it Out!**

Tell whether each relationship is a direct variation. Explain.

2a.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

2b.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>-10</td>
</tr>
<tr>
<td>5</td>
<td>-20</td>
</tr>
<tr>
<td>7.5</td>
<td>-30</td>
</tr>
</tbody>
</table>

2c.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

If you know one ordered pair that satisfies a direct variation, you can write the equation. You can also find other ordered pairs that satisfy the direct variation.
**Example 3**

**Writing and Solving Direct Variation Equations**

The value of $y$ varies directly with $x$, and $y = 6$ when $x = 12$. Find $y$ when $x = 27$.

**Method 1** Find the value of $k$ and then write the equation.

\[
y = kx \quad \text{Write the equation for a direct variation.}
\]

\[
6 = k(12) \quad \text{Substitute 6 for } y \text{ and 12 for } x. \text{ Solve for } k.
\]

\[
\frac{1}{2} = k \quad \text{Since } k \text{ is multiplied by 12, divide both sides by 12.}
\]

The equation is $y = \frac{1}{2}x$. When $x = 27$, $y = \frac{1}{2}(27) = 13.5$.

**Method 2** Use a proportion.

\[
\frac{6}{12} = \frac{y}{27} \quad \text{In a direct variation, } \frac{y}{x} \text{ is the same for all values of } x \text{ and } y.
\]

\[
12y = 162 \quad \text{Use cross products.}
\]

\[
y = 13.5 \quad \text{Since } y \text{ is multiplied by 12, divide both sides by 12.}
\]

**Check It Out!**

3. The value of $y$ varies directly with $x$, and $y = 4.5$ when $x = 0.5$. Find $y$ when $x = 10$.

**Example 4**

**Graphing Direct Variations**

The three-toed sloth is an extremely slow animal. On the ground, it travels at a speed of about 6 feet per minute. Write a direct variation equation for the distance $y$ a sloth will travel in $x$ minutes. Then graph.

**Step 1** Write a direct variation equation.

\[
\text{distance} = 6 \text{ feet per minute}
\]

\[
y = 6x
\]

**Step 2** Choose values of $x$ and generate ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 6x$</th>
<th>$(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$y = 6(0) = 0$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>1</td>
<td>$y = 6(1) = 6$</td>
<td>$(1, 6)$</td>
</tr>
<tr>
<td>2</td>
<td>$y = 6(2) = 12$</td>
<td>$(2, 12)$</td>
</tr>
</tbody>
</table>

**Step 3** Graph the points and connect.

4. The perimeter $y$ of a square varies directly with its side length $x$. Write a direct variation equation for this relationship. Then graph.

Look at the graph in Example 4. It passes through $(0, 0)$ and has a slope of 6. The graph of any direct variation $y = kx$ has a line through $(0, 0)$. It has a slope of $k$. 

324 Chapter 6 Linear Functions
**THINK AND DISCUSS**

1. How do you know that a direct variation is linear?

2. How does the graph of a direct variation differ from the graphs of other types of linear relationships?

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe how you can use the given information to identify a direct variation.

<table>
<thead>
<tr>
<th>Recognizing a Direct Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>From an Equation</td>
</tr>
</tbody>
</table>

---

**GUIDED PRACTICE**

1. **Vocabulary** If \( x \) varies directly with \( y \), then the relationship between the two variables is said to be a _____. (direct variation or constant of variation)

2. Tell whether each equation represents a direct variation. If so, identify the constant of variation.
   - \( y = 4x + 9 \)
   - \( 2y = -8x \)
   - \( x + y = 0 \)

3. **SEE EXAMPLE 1**
   - Tell whether each relationship is a direct variation. Explain.
   - \( x \) \hspace{1cm} 10 \hspace{1cm} 5 \hspace{1cm} 2
   - \( y \) \hspace{1cm} 12 \hspace{1cm} 7 \hspace{1cm} 4

4. **SEE EXAMPLE 2**
   - \( x \) \hspace{1cm} 3 \hspace{1cm} -1 \hspace{1cm} -4
   - \( y \) \hspace{1cm} -6 \hspace{1cm} 2 \hspace{1cm} 8

5. **SEE EXAMPLE 3**
   - The value of \( y \) varies directly with \( x \), and \( y = -3 \) when \( x = 1 \). Find \( y \) when \( x = -6 \).

6. **SEE EXAMPLE 4**
   - The value of \( y \) varies directly with \( x \), and \( y = 6 \) when \( x = 18 \). Find \( y \) when \( x = 12 \).

7. **Wages** Cameron earns $7 per hour at her after-school job. The total amount of her paycheck varies directly with the amount of time she works. Write a direct variation equation for the amount of money \( y \) that she earns for working \( x \) hours. Then graph.

---

**PRACTICE AND PROBLEM SOLVING**

1. Tell whether each equation represents a direct variation. If so, identify the constant of variation.
   - \( y = \frac{1}{6}x \)
   - \( 4y = x \)
   - \( x = 2y - 12 \)

2. Tell whether each relationship is a direct variation. Explain.
   - \( x \) \hspace{1cm} 6 \hspace{1cm} 9 \hspace{1cm} 17
   - \( y \) \hspace{1cm} 13.2 \hspace{1cm} 19.8 \hspace{1cm} 37.4

3. \( x \) \hspace{1cm} -6 \hspace{1cm} 3 \hspace{1cm} 12
   - \( y \) \hspace{1cm} 4 \hspace{1cm} -2 \hspace{1cm} -8
15. The value of \( y \) varies directly with \( x \), and \( y = 8 \) when \( x = -32 \). Find \( y \) when \( x = 64 \).

16. The value of \( y \) varies directly with \( x \), and \( y = \frac{1}{2} \) when \( x = 3 \). Find \( y \) when \( x = 1 \).

17. While on his way to school, Norman saw that the cost of gasoline was $2.50 per gallon. Write a direct variation equation to describe the cost \( y \) of \( x \) gallons of gas. Then graph.

Tell whether each relationship is a direct variation. Explain your answer.

18. The equation \(-15x + 4y = 0\) relates the length of a videotape in inches \( x \) to its approximate playing time in seconds \( y \).

19. The equation \( y - 2.00x = 2.50 \) relates the cost \( y \) of a taxicab ride to distance \( x \) of the cab ride in miles.

Each ordered pair is a solution of a direct variation. Write the equation of direct variation. Then graph your equation and show that the slope of the line is equal to the constant of variation.

20. \((2, 10)\) 21. \((-3, 9)\) 22. \((8, 2)\) 23. \((1.5, 6)\)

24. \((7, 21)\) 25. \((1, 2)\) 26. \((2, -16)\) 27. \(\left(\frac{1}{7}, 1\right)\)

28. \((-2, 9)\) 29. \((9, -2)\) 30. \((4, 6)\) 31. \((3, 4)\)

32. \((5, 1)\) 33. \((1, -6)\) 34. \(\left(-1, \frac{1}{2}\right)\) 35. \((7, 2)\)

36. **Astronomy**  Weight varies directly with gravity. A Mars lander weighed 767 pounds on Earth but only 291 pounds on Mars. Its accompanying Mars rover weighed 155 pounds on Mars. How much did it weigh on Earth? Round your answer to the nearest pound.

37. **Environment**  Mischa bought an energy-efficient washing machine. She will save about 15 gallons of water per wash load.

   a. Write an equation of direct variation to describe how many gallons of water \( y \) Mischa saves for \( x \) loads of laundry she washes.

   b. Graph your direct variation from part a. Is every point on the graph a solution in this situation? Why or why not?

   c. If Mischa does 2 loads of laundry per week, how many gallons of water will she have saved at the end of a year?

38. **Critical Thinking**  If you double an \( x \)-value in a direct variation, will the corresponding \( y \)-value double? Explain.

39. **Write About It**  In a direct variation \( y = kx \), \( k \) is sometimes called the “constant of proportionality.” How are proportions related to direct variations?

40. Rhea exercised on a treadmill at the gym. When she was finished, the display showed that she had walked at an average speed of 3 miles per hour.

   a. Write an equation that gives the number of miles \( y \) that Rhea would cover in \( x \) hours if she walked at this speed.

   b. Explain why this is a direct variation and find the value of \( k \). What does this value represent in Rhea’s situation?
41. Which equation does NOT represent a direct variation?

- A) \( y = \frac{1}{3}x \)
- B) \( y = -2x \)
- C) \( y = 4x + 1 \)
- D) \( 6x - y = 0 \)

42. Identify which set of data represents a direct variation.

- F) 
<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
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<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

- H) 
<table>
<thead>
<tr>
<th>x</th>
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<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
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<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

- G) 
<table>
<thead>
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<th>3</th>
</tr>
</thead>
<tbody>
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<td>2</td>
</tr>
</tbody>
</table>

- J) 
<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

43. Two yards of fabric cost $13, and 5 yards of fabric cost $32.50. Which equation relates the cost of the fabric \( c \) to its length \( \ell \)?

- A) \( c = 2.6\ell \)
- B) \( c = 6.5\ell \)
- C) \( c = 13\ell \)
- D) \( c = 32.5\ell \)

44. **Gridded Response** A car is traveling at a constant speed. After 3 hours, the car has traveled 180 miles. If the car continues to travel at the same constant speed, how many hours will it take to travel a total of 270 miles?

45. **Transportation** The function \( y = 20x \) gives the number of miles \( y \) that a sport-utility vehicle (SUV) can travel on \( x \) gallons of gas. The function \( y = 60x \) gives the number of miles \( y \) that a hybrid car can travel on \( x \) gallons of gas.

- a. If you drive 120 miles, how much gas will you save by driving the hybrid instead of the SUV?

- b. Graph both functions on the same coordinate plane. Will the lines ever meet other than at the origin? Explain.

- c. **What if...?** Shannon drives 15,000 miles in one year. How many gallons of gas will she use if she drives the SUV? the hybrid?

46. Suppose the equation \( ax + by = c \), where \( a \), \( b \), and \( c \) are real numbers, describes a direct variation. What do you know about the value of \( c \)?
You have seen that you can graph a line if you know two points on the line. Another way is to use the slope of the line and the point that contains the \(y\)-intercept.

**EXAMPLE 1**

**Graphing by Using Slope and \(y\)-intercept**

Graph the line with slope \(-2\) and \(y\)-intercept \(4\).

**Step 1**
The \(y\)-intercept is \(4\), so the line contains (0, 4). Plot (0, 4).

**Step 2**
Slope \[ m = \frac{\text{change in } y}{\text{change in } x} = \frac{-2}{1} \]

Count 2 units down and 1 unit right from (0, 4) and plot another point.

**Step 3**
Draw the line through the two points.

Graph each line given the slope and \(y\)-intercept.

1a. slope = 2, \(y\)-intercept = -3
1b. slope = \(-\frac{2}{3}\), \(y\)-intercept = 1

If you know the slope of a line and the \(y\)-intercept, you can write an equation that describes the line.

**Step 1**
If a line has slope 2 and the \(y\)-intercept is 3, then \(m = 2\) and (0, 3) is on the line. Substitute these values into the slope formula.

\[
Slope \text{ formula } \rightarrow \quad m = \frac{y_2 - y_1}{x_2 - x_1} \\
2 = \frac{y - 3}{x - 0} \quad \text{Since you don’t know (}x_2, y_2\text{), use (}x, y\text{).}
\]

**Step 2**
Solve for \(y\):
\[
2 = \frac{y - 3}{x} \\
2 = \frac{y - 3}{x} \quad \text{Simplify the denominator.}
\]
\[
2x = (y - 3) \cdot x \\
2x = y - 3 \quad \text{Multiply both sides by } x.
\]
\[
2x + 3 = +3 \\
2x + 3 = y, \text{ or } y = 2x + 3 \quad \text{Add 3 to both sides.}
\]
If a line has slope $m$ and the $y$-intercept is $b$, then the line is described by the equation $y = mx + b$.

Any linear equation can be written in slope-intercept form by solving for $y$ and simplifying. In this form, you can immediately see the slope and $y$-intercept. Also, you can quickly graph a line when the equation is written in slope-intercept form.

**Example 2**

**Writing Linear Equations in Slope-Intercept Form**

Write the equation that describes each line in slope-intercept form.

**A** slope = $\frac{1}{3}$, $y$-intercept = 6

$y = mx + b$

$y = \frac{1}{3}x + 6$

Substitute the given values for $m$ and $b$.

**B** slope = 0, $y$-intercept = $-5$

$y = mx + b$

$y = 0x + (-5)$

Simplify if necessary.

**C**

Step 1 Find the $y$-intercept. The graph crosses the $y$-axis at $(0, 1)$, so $b = 1$.

Step 2 Find the slope. The line contains the points $(0, 1)$ and $(1, 3)$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{3 - 1}{1 - 0} = 2$

Use the slope formula.

Step 3 Write the equation.

$y = mx + b$

$y = 2x + 1$

Substitute 2 for $m$ and 1 for $b$.

**D** slope = 4, $(2, 5)$ is on the line

Step 1 Find the $y$-intercept.

$y = mx + b$

$y = 4x + b$

Substitute 4 for $m$, 2 for $x$, and 5 for $y$.

$5 = 4(2) + b$

Solve for $b$. Since 8 is added to $b$, subtract 8 from both sides to undo the addition.

$-3 = b$

Step 2 Write the equation.

$y = mx + b$

$y = 4x + (-3)$

Substitute 4 for $m$ and $-3$ for $b$.

Write the equation that describes each line in slope-intercept form.

2a. slope = $-12$, $y$-intercept = $-\frac{1}{2}$

2b. slope = 1, $y$-intercept = 0

2c. slope = 8, $(-3, 1)$ is on the line.
Using Slope-Intercept Form to Graph

Write each equation in slope-intercept form. Then graph the line described by the equation.

A. \( y = 4x - 3 \)
   - \( y = 4x - 3 \) is in the form \( y = mx + b \).
     - slope: \( m = 4 \)
     - \( y \)-intercept: \( b = -3 \)
   - Step 1: Plot \((0, -3)\).
   - Step 2: Count 4 units up and 1 unit right and plot another point.
   - Step 3: Draw the line connecting the two points.

B. \( y = -\frac{2}{3}x + 2 \)
   - \( y = -\frac{2}{3}x + 2 \) is in the form \( y = mx + b \).
     - slope: \( m = -\frac{2}{3} \)
     - \( y \)-intercept: \( b = 2 \)
   - Step 1: Plot \((0, 2)\).
   - Step 2: Count 2 units down and 3 units right and plot another point.
   - Step 3: Draw the line connecting the two points.

C. \( 3x + 2y = 8 \)
   - Step 1: Write the equation in slope-intercept form by solving for \( y \).
     \[
     3x + 2y = 8 \\
     -3x
     
     2y = 8 - 3x \\
     2y = 8 - 3x \\
     \frac{2y}{2} = \frac{8 - 3x}{2} \\
     y = 4 - \frac{3}{2}x \\
     y = -\frac{3}{2}x + 4
     
     \text{Subtract 3x from both sides.} \\
     \text{Since } y \text{ is multiplied by 2, divide both sides by 2.} \\
     \text{Write the equation in the form } y = mx + b.
     
   - Step 2: Graph the line.
     - slope: \( m = -\frac{3}{2} \)
     - \( y \)-intercept: \( b = 4 \)
     - Plot \((0, 4)\).
     - Then count 3 units down and 2 units right and plot another point.
     - Draw the line connecting the two points.

HELPFUL HINT

To divide \((8 - 3x)\) by 2, you can multiply by \(\frac{1}{2}\) and use the Distributive Property.

\[
\frac{8 - 3x}{2} = \frac{1}{2}(8 - 3x) \\
= \frac{1}{2}(8) + \frac{1}{2}(-3x) \\
= 4 - \frac{3}{2}x
\]

CHECK IT OUT

Write each equation in slope-intercept form. Then graph the line described by the equation.

3a. \( y = \frac{2}{3}x \)  \hspace{1cm} 3b. \( 6x + 2y = 10 \)  \hspace{1cm} 3c. \( y = -4 \)
**Example 4**

**Consumer Application**

To rent a van, a moving company charges $30.00 plus $0.50 per mile. The cost as a function of the number of miles driven is shown in the graph.

a. Write an equation that represents the cost as a function of the number of miles.

<table>
<thead>
<tr>
<th>Cost</th>
<th>is $0.50 per mile</th>
<th>times</th>
<th>miles</th>
<th>plus $30.00</th>
</tr>
</thead>
</table>

An equation is \( y = 0.5x + 30 \).

b. Identify the slope and \( y \)-intercept and describe their meanings.

The \( y \)-intercept is 30. This is the cost for 0 miles, or the initial fee of $30.00.

The slope is 0.5. This is the rate of change of the cost: $0.50 per mile.

c. Find the cost of the van for 150 miles.

\[
y = 0.5x + 30
\]

\[
= 0.5(150) + 30 = 105
\]

Substitute 150 for \( x \) in the equation.

The cost of the van for 150 miles is $105.

---

4. A caterer charges a $200 fee plus $18 per person served. The cost as a function of the number of guests is shown in the graph.

a. Write an equation that represents the cost as a function of the number of guests.

b. Identify the slope and \( y \)-intercept and describe their meanings.

c. Find the cost of catering an event for 200 guests.

---

**Think and Discuss**

1. If a linear function has a \( y \)-intercept of \( b \), at what point does its graph cross the \( y \)-axis?

2. Where does the line described by \( y = 4.395x - 23.75 \) cross the \( y \)-axis?

3. **Get Organized** Copy and complete the graphic organizer.

---

**Know it!**

- Graphing the Line Described by \( y = mx + b \)
  1. Plot the point __?__.
  2. Find a second point on the line by __?__.
  3. Draw __?__.
GUIDED PRACTICE

Graph each line given the slope and y-intercept.
1. slope = $\frac{1}{3}$, y-intercept = $-3$
2. slope = 0.5, y-intercept = 3.5
3. slope = 5, y-intercept = $-1$
4. slope = $-2$, y-intercept = 2

Write the equation that describes each line in slope-intercept form.
5. slope = 8, y-intercept = 2
6. slope = 0, y-intercept = $-3$
7. slope = 5, (2, 7) is on the line.
8. slope = $-2$, (1, $-3$) is on the line.

Write each equation in slope-intercept form. Then graph the line described by the equation.
10. $y = \frac{2}{3}x - 6$
11. $3x - y = 1$
12. $2x + y = 4$
13. Helen is in a bicycle race. She has already biked 10 miles and is now biking at a rate of 18 miles per hour. Her distance as a function of time is shown in the graph.
   a. Write an equation that represents the distance Helen has biked as a function of time.
   b. Identify the slope and y-intercept and describe their meanings.
   c. How far will Helen have biked after 2 hours?

PRACTICE AND PROBLEM SOLVING

Graph each line given the slope and y-intercept.
14. slope = $\frac{1}{4}$, y-intercept = 7
15. slope = $-6$, y-intercept = $-3$
16. slope = 1, y-intercept = $-4$
17. slope = $-\frac{4}{5}$, y-intercept = 6

Write the equation that describes each line in slope-intercept form.
18. slope = 5, y-intercept = $-9$
19. slope = $-\frac{2}{3}$, y-intercept = 2
20. slope = $-\frac{1}{2}$, (6, 4) is on the line.
21. slope = 0, (6, $-8$) is on the line.
Write each equation in slope-intercept form. Then graph the line described by the equation.

23. \(- \frac{1}{2}x + y = 4\)
24. \(\frac{2}{3}x + y = 2\)
25. \(2x + y = 8\)

26. **Fitness**  Pauline's health club has an enrollment fee of $175 and costs $35 per month. Total cost as a function of number of membership months is shown in the graph.
   a. Write an equation that represents the total cost as a function of months.
   b. Identify the slope and \(y\)-intercept and describe their meanings.
   c. Find the cost of one year of membership.

27. A company rents video games. The table shows the linear relationship between the number of games a customer can rent at one time and the monthly cost of the service.
   a. Graph the relationship.
   b. Write an equation that represents the monthly cost as a function of games rented at one time.

<table>
<thead>
<tr>
<th>Games Rented at One Time</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Cost ($)</td>
<td>14</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>

**Critical Thinking**  Tell whether each situation is possible or impossible. If possible, draw a sketch of the graphs. If impossible, explain.

28. Two different lines have the same slope.
29. Two different linear functions have the same \(y\)-intercept.
30. Two intersecting lines have the same slope.
31. A linear function does not have a \(y\)-intercept.

Match each equation with its corresponding graph.

32. \(y = 2x - 1\)
33. \(y = \frac{1}{2}x - 1\)
34. \(y = -\frac{1}{2}x + 1\)

35. **Write About It**  Write an equation that describes a vertical line. Can you write this equation in slope-intercept form? Why or why not?

36. a. Ricardo and Sam walk from Sam's house to school. Sam lives 3 blocks from Ricardo's house. The graph shows their distance from Ricardo's house as they walk to school. Create a table of these values.
   b. Find an equation for the distance as a function of time.
   c. What are the slope and \(y\)-intercept? What do they represent in this situation?
37. Which function has the same y-intercept as \( y = \frac{1}{2}x - 2 \)?

- A \( 2x + 3y = 6 \)
- B \( x + 4y = -8 \)
- C \( -\frac{1}{2}x + y = 4 \)
- D \( \frac{1}{2}x - 2y = -2 \)

38. What is the slope-intercept form of \( x - y = -8 \)?

- F \( y = -x - 8 \)
- G \( y = x - 8 \)
- H \( y = -x + 8 \)
- J \( y = x + 8 \)

39. Which function has a y-intercept of 3?

- A \( 2x - y = 3 \)
- B \( 2x + y = 3 \)
- C \( 2x + y = 6 \)
- D \( y = 3x \)

40. **Gridded Response** What is the slope of the line described by \(-6x = -2y + 5\)?

41. **Short Response** Write a function whose graph has the same slope as the line described by \(3x - 9y = 9\) and the same y-intercept as \(8x - 2y = 6\). Show your work.

**CHALLENGE AND EXTEND**

42. The standard form of a linear equation is \( Ax + By = C \). Rewrite this equation in slope-intercept form. What is the slope? What is the y-intercept?

43. What value of \( n \) in the equation \( nx + 5 = 3y \) would give a line with slope \(-2\)?

44. If \( b \) is the y-intercept of a linear function whose graph has slope \( m \), then \( y = mx + b \) describes the line. Below is an incomplete justification of this statement. Fill in the missing information.

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( m = \frac{y_2 - y_1}{x_2 - x_1} )</td>
<td>1. Slope formula</td>
</tr>
<tr>
<td>2. ( m = \frac{y - b}{x - 0} )</td>
<td>2. By definition, if ( b ) is the y-intercept, then ((x, y)) is any other point on the line.  ((x_1, y_1)) is a point on the line.</td>
</tr>
<tr>
<td>3. ( m = \frac{y - b}{x} )</td>
<td>3. ( \frac{y - b}{x} )</td>
</tr>
<tr>
<td>4. ( m ) = ( y - b )</td>
<td>4. Multiplication Property of Equality (Multiply both sides of the equation by ( x ).)</td>
</tr>
<tr>
<td>5. ( mx + b = y ), or ( y = mx + b )</td>
<td>5. ( \frac{y - b}{x} )</td>
</tr>
</tbody>
</table>

334  Chapter 6 Linear Functions
Objectives
Graph a line and write a linear equation using point-slope form.
Write a linear equation given two points.

Why learn this?
You can use point-slope form to represent a cost function, such as the cost of placing a newspaper ad. (See Example 5.)

If you know the slope and any point on the line, you can write an equation of the line by using the slope formula. For example, suppose a line has a slope of 3 and contains (2, 1). Let (x, y) be any other point on the line.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ 3 = \frac{y - 1}{x - 2} \]

Substitute into the slope formula.

\[ 3(x - 2) = \left( \frac{y - 1}{x - 2} \right)(x - 2) \]

Multiplication Property of Equality

\[ 3(x - 2) = y - 1 \]

Simplify.

\[ y - 1 = 3(x - 2) \]

Point-Slope Form of a Linear Equation

The line with slope \( m \) that contains the point \((x_1, y_1)\) can be described by the equation \( y - y_1 = m(x - x_1) \).

**EXAMPLE 1** Writing Linear Equations in Point-Slope Form

Write an equation in point-slope form for the line with the given slope that contains the given point.

A. slope = \( \frac{5}{2} \); \((-3, 0)\)

\[ y - y_1 = m(x - x_1) \]

Write the point-slope form.

\[ y - 0 = \frac{5}{2} [x - (-3)] \]

Substitute \( \frac{5}{2} \) for \( m \), \(-3\) for \( x_1 \), and \( 0 \) for \( y_1 \).

\[ y - 0 = \frac{5}{2} (x + 3) \]

Rewrite subtraction of negative numbers as addition.

B. slope = \(-7\); \((4, 2)\)

\[ y - y_1 = m(x - x_1) \]

\[ y - 2 = -7(x - 4) \]

C. slope = \( 0 \); \((-2, -3)\)

\[ y - y_1 = m(x - x_1) \]

\[ y - (-3) = 0[x - (-2)] \]

\[ y + 3 = 0(x + 2) \]

Write an equation in point-slope form for the line with the given slope that contains the given point.

1a. slope = \( \frac{1}{2} \); \((\frac{1}{2}, 1)\)  

1b. slope = \( 0 \); \((3, -4)\)
Previously, you graphed a line given its equation in slope-intercept form. You can also graph a line when given its equation in point-slope form. Start by using the equation to identify a point on the line. Then use the slope of the line to identify a second point.

**Example 2**

Using Point-Slope Form to Graph

Graph the line described by each equation.

**A** \( y - 1 = 3(x - 1) \)

\( y - 1 = 3(x - 1) \) is in the form \( y - y_1 = m(x - x_1) \).

The line contains the point \((1, 1)\).

**Slope**:
\[
m = \frac{3}{1}
\]

**Step 1** Plot \((1, 1)\).

**Step 2** Count 3 units up and 1 unit right and plot another point.

**Step 3** Draw the line connecting the two points.

**B** \( y + 2 = \frac{1}{2}(x - 3) \)

Step 1 Write the equation in point-slope form:
\[
y - y_1 = m(x - x_1)
\]
\[
y - (-2) = \frac{1}{2}(x - 3)
\]
**Rewrite addition of 2 as subtraction of \(-2\).**

**Step 2** Graph the line.

The line contains the point \((3, -2)\).

**Slope**:
\[
m = \frac{-1}{2} = \frac{1}{-2}
\]

• Plot \((3, -2)\).
• Count 1 unit up and 2 units left and plot another point.
• Draw the line connecting the two points.

**Check It Out!**

Graph the line described by each equation.

2a. \( y + 2 = -(x - 2) \)

2b. \( y + 3 = -2(x - 1) \)

**Example 3**

Writing Linear Equations in Slope-Intercept Form

Write the equation that describes each line in slope-intercept form.

**A** slope = \(-4\), \((-1, -2)\) is on the line.

Step 1 Write the equation in point-slope form:
\[
y - y_1 = m(x - x_1)
\]
\[
y - (-2) = -4[x - (-1)]
\]

Step 2 Write the equation in slope-intercept form by solving for \(y\).
\[
y - (-2) = -4[x - (-1)]
\]
\[
y + 2 = -4(x + 1) \quad \text{Rewrite subtraction of negative numbers as addition. Distribute} \ -4 \ \text{on the right side.}
\]
\[
y + 2 = -4x - 4
\]
\[
\frac{-2}{-2} \quad \frac{-2}{-2}
\]
\[
y = -4x - 6 \quad \text{Subtract 2 from both sides.}
\]
B (1, −4) and (3, 2) are on the line.

Step 1 Find the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{3 - 1} = \frac{6}{2} = 3 \]

Step 2 Substitute the slope and one of the points into the point-slope form. Then write the equation in slope-intercept form.

\[ y - y_1 = m(x - x_1) \]

\[ y - 2 = 3(x - 3) \quad \text{Use (3, 2).} \]

\[ y - 2 = 3x - 9 \quad \text{Distribute 3 on the right side.} \]

\[ y = 3x - 7 \quad \text{Add 2 to both sides.} \]

C \( x \)-intercept = 2, \( y \)-intercept = 4

Step 1 Use the intercepts to find two points: (−2, 0) and (0, 4).

Step 2 Find the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{0 - (-2)} = \frac{4}{-2} = -2 \]

Step 3 Write the equation in slope-intercept form.

\[ y = mx + b \quad \text{Write the slope-intercept form.} \]

\[ y = 2x + 4 \quad \text{Substitute 2 for } m \text{ and } 4 \text{ for } b. \]

**EXAMPLE 4 Using Two Points to Find Intercepts**

The points (4, 8) and (−1, −12) are on a line. Find the intercepts.

Step 1 Find the slope.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 8}{-1 - 4} = \frac{-20}{-5} = 4 \]

Step 2 Write the equation in slope-intercept form.

\[ y - y_1 = m(x - x_1) \quad \text{Write the point-slope form.} \]

\[ y - 8 = 4(x - 4) \quad \text{Substitute (4, 8) for } (x_1, y_1) \text{ and } 4 \text{ for } m. \]

\[ y - 8 = 4x - 16 \quad \text{Distribute 4 on the right side.} \]

\[ y = 4x - 8 \quad \text{Add 8 to both sides.} \]

Step 3 Find the intercepts.

**x-intercept:**

\[ y = 4x - 8 \quad \text{Replace } y \text{ with } 0 \text{ and solve for } x. \]

\[ 0 = 4x - 8 \quad y = 4x - 8 \quad \text{Use the slope-intercept form to identify the } \]

\[ 8 = 4x \quad \text{Use the } y \text{-intercept.} \]

\[ 2 = x \quad y \text{-intercept:} \]

The \( x \)-intercept is 2, and the \( y \)-intercept is −8.

4. The points (2, 15) and (−4, −3) are on a line. Find the intercepts.
**Problem-Solving Application**

The cost to place an ad in a newspaper for one week is a linear function of the number of lines in the ad. The costs for 3, 5, and 10 lines are shown. Write an equation in slope-intercept form that represents the function. Then find the cost of an ad that is 18 lines long.

1. **Understand the Problem**
   - The answer will have two parts—an equation in slope-intercept form and the cost of an ad that is 18 lines long.
   - The ordered pairs given in the table satisfy the equation.

2. **Make a Plan**
   First, find the slope. Then use point-slope form to write the equation. Finally, write the equation in slope-intercept form.

3. **Solve**

   **Step 1** Choose any two ordered pairs from the table to find the slope.
   \[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18.50 - 13.50}{5 - 3} = \frac{5}{2} = 2.5 \quad \text{Use (3, 13.50) and (5, 18.50).} \]

   **Step 2** Substitute the slope and any ordered pair from the table into the point-slope form.
   \[ y - y_1 = m(x - x_1) \]
   \[ y - 31 = 2.5(x - 10) \quad \text{Use (10, 31).} \]

   **Step 3** Write the equation in slope-intercept form by solving for \( y \).
   \[ y - 31 = 2.5(x - 10) \]
   \[ y - 31 = 2.5x - 25 \]
   \[ y = 2.5x + 6 \quad \text{Distribute 2.5.} \]
   \[ y = 2.5x + 6 \quad \text{Add 31 to both sides.} \]

   **Step 4** Find the cost of an ad containing 18 lines by substituting 18 for \( x \).
   \[ y = 2.5x + 6 \]
   \[ y = 2.5(18) + 6 = 51 \]
   The cost of an ad containing 18 lines is $51.

4. **Look Back**

   Check the equation by substituting the ordered pairs (3, 13.50) and (5, 18.50).

   \[ \begin{array}{c|c|c|c|c}
   \text{Lines} & \text{3} & \text{5} & \text{10} \\
   \hline
   \text{Cost ($)} & 13.50 & 18.50 & 31 \\
   \end{array} \]

   \[ \begin{array}{c|c|c|c}
   \text{Lines} & \text{3} & \text{5} & \text{10} \\
   \hline
   \text{Cost ($)} & 12.75 & 17.25 & 28.50 \\
   \end{array} \]

5. **What if...?** At a different newspaper, the costs to place an ad for one week are shown. Write an equation in slope-intercept form that represents this linear function. Then find the cost of an ad that is 21 lines long.
THINK AND DISCUSS
1. How are point-slope form and slope-intercept form alike? different?
2. When is point-slope form useful? When is slope-intercept form useful?
3. GET ORGANIZED Copy and complete the graphic organizer. In each box, describe how to find the equation of a line by using the given method.

Writing the Equation of a Line
If you know two points on the line
If you know the slope and y-intercept
If you know the slope and a point on the line

GUIDED PRACTICE
SEE EXAMPLE 1
Write an equation in point-slope form for the line with the given slope that contains the given point.
1. slope = \frac{1}{5}; (2, -6)  
2. slope = -4; (1, 5)  
3. slope = 0; (3, -7)

SEE EXAMPLE 2
Graph the line described by each equation.
4. \( y - 1 = -(x - 3) \)  
5. \( y + 2 = -2(x + 4) \)  
6. \( y + 1 = -\frac{1}{2}(x + 4) \)

SEE EXAMPLE 3
Write the equation that describes each line in slope-intercept form.
7. slope = -\frac{1}{3}, (-3, 8) is on the line.  
8. slope = 2; (1, 1) is on the line.  
9. \((-2, 2) \) and \((2, -2) \) are on the line.  
10. \((1, 1) \) and \((-5, 3) \) are on the line.  
11. \(x\)-intercept = 8, \(y\)-intercept = 4  
12. \(x\)-intercept = -2, \(y\)-intercept = 3

SEE EXAMPLE 4
Each pair of points is on a line. Find the intercepts.
13. (5, 2) and (7, 4)  
14. \((-1, 5) \) and \((-3, -5) \)  
15. (2, 9) and (-4, -9)

SEE EXAMPLE 5
16. Measurement An oil tank is being filled at a constant rate. The depth of the oil is a function of the number of minutes the tank has been filling, as shown in the table. Write an equation in slope-intercept form that represents this linear function. Then find the depth of the oil after one-half hour.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Depth (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
</tr>
</tbody>
</table>

PRACTICE AND PROBLEM SOLVING
Write an equation in point-slope form for the line with the given slope that contains the given point.
17. slope = \frac{2}{9}; (-1, 5)  
18. slope = 0; (4, -2)  
19. slope = 8; (1, 8)
Graph the line described by each equation.

20. \( y - 4 = -\frac{1}{2}(x + 3) \)
21. \( y + 2 = \frac{3}{5}(x - 1) \)
22. \( y - 0 = 4(x - 1) \)

Write the equation that describes each line in slope-intercept form.

23. slope = \(-\frac{2}{7}\), (14, -3) is on the line.
24. slope = \(\frac{4}{5}\), (-15, 1) is on the line.
25. slope = -6, (9, 3) is on the line.
26. (7, 8) and (-7, 6) are on the line.
27. (2, 7) and (4, -4) are on the line.
28. (-1, 2) and (4, -23) are on the line.
29. \( x \)-intercept = 3, \( y \)-intercept = -6
30. \( x \)-intercept = 4, \( y \)-intercept = -1

Each pair of points is on a line. Find the intercepts.

31. (-1, -4) and (6, 10)
32. (3, 4) and (-6, 16)
33. (4, 15) and (-2, 6)

34. **History** The amount of fresh water left in the tanks of a 19th-century clipper ship is a linear function of the time since the ship left port, as shown in the table. Write an equation in slope-intercept form that represents the function. Then find the amount of water that will be left in the ship's tanks 50 days after leaving port.

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Amount (gal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3555</td>
</tr>
<tr>
<td>8</td>
<td>3240</td>
</tr>
<tr>
<td>15</td>
<td>2925</td>
</tr>
</tbody>
</table>

35. **Science** At higher altitudes, water boils at lower temperatures. This relationship between altitude and boiling point is linear. At an altitude of 1000 feet, water boils at 210 °F. At an altitude of 3000 feet, water boils at 206 °F. Write an equation in slope-intercept form that represents this linear function. Then find the boiling point at 6000 feet.

36. **Consumer Economics** Lora has a gift card from an online music store where all downloads cost the same amount. After downloading 2 songs, the balance on her card was $18.10. After downloading a total of 5 songs, the balance was $15.25.
   a. Write an equation in slope-intercept form that represents the amount in dollars remaining on the card as a function of songs downloaded.
   b. Identify the slope of the line and tell what the slope represents.
   c. Identify the \( y \)-intercept of the line and tell what it represents.
   d. How many additional songs can Lora download when there is $15.25 left on the card?

Graph the line with the given slope that contains the given point.

37. slope = -3; (2, 4)
38. slope = \(-\frac{1}{4}\); (0, 0)
39. slope = \(\frac{1}{2}\); (-2, -1)

Tell whether each statement is sometimes, always, or never true.

40. A line described by the equation \( y = mx + b \) contains the point (0, b).
41. The slope of the line that contains the points (0, 0) and (c, d) is negative if both c and d are negative.
42. The \( y \)-intercept of the graph of \( y - y_1 = m(x - x_1) \) is negative if \( y_1 \) is negative.
43. **Meteorology** Snowfall accumulates at an average rate of 2.5 inches per hour during a snowstorm. Two hours after the snowstorm begins, the average depth of snow on the ground is 11 inches.
   a. Write an equation in point-slope form that represents the depth of the snow in inches as a function of hours since the snowstorm began.
   b. How much snow is on the ground when the snowstorm starts?
   c. The snowstorm begins at 2:15 P.M. and continues until 6:30 P.M. How much snow is on the ground at the end of the storm?
Write an equation in point-slope form that describes each graph.

44. \[ y - 2 = -3(x - 5) \]
45. \[ y - 3 = -3(x - 3) \]
46. \[ y - 1 = -3(x + 1) \]

The tables show linear relationships between \( x \) and \( y \). Copy and complete the tables.

47. | \( x \) | -2 | 0 | 7 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-18</td>
<td>12</td>
<td>27</td>
</tr>
</tbody>
</table>

48. | \( x \) | -4 | 1 | 0 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>14</td>
<td>4</td>
<td>-6</td>
</tr>
</tbody>
</table>

49. **ERROR ANALYSIS** Two students used point-slope form to find an equation that describes the line with slope \(-3\) through \((-5, 2)\). Who is incorrect? Explain the error.

**A**

\[
\begin{align*}
y - y_1 &= m(x - x_1) \\
y - 2 &= -3(x - 5)
\end{align*}
\]

**B**

\[
\begin{align*}
y - y_1 &= m(x - x_1) \\
y - 2 &= -3[x - (-5)] \\
y - 2 &= -3(x + 5)
\end{align*}
\]

50. **Critical Thinking** Compare the methods for finding the equation that describes a line when you know

- a point on the line and the slope of the line.
- two points on the line.

How are the methods alike? How are they different?

51. **Write About It** Explain why the first statement is false but the second is true.

- All linear equations can be written in point-slope form.
- All linear equations that describe functions can be written in point-slope form.

52. **Multi-Step** The table shows the mean scores on a standardized test for several different years.

<table>
<thead>
<tr>
<th>Years Since 1985</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>17</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Combined Score</td>
<td>994</td>
<td>1009</td>
<td>1001</td>
<td>1016</td>
<td>1020</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the data and add a trend line to your graph.
b. Use your trend line to estimate the slope and \( y \)-intercept, and write an equation in slope-intercept form.
c. What do the slope and \( y \)-intercept represent in this situation?

53. **Multi-Step Test Prep**

a. Stephen is walking from his house to his friend Sharon's house. When he is 12 blocks away, he looks at his watch. He looks again when he is 8 blocks away and finds that 6 minutes have passed. Write two ordered pairs for these data in the form (time, blocks).
b. Write a linear equation for these two points.
c. What is the total amount of time it takes Stephen to reach Sharon's house? Explain how you found your answer.
54. Which equation describes the line through \((-5, 1)\) with slope of 1? 
   - A) \(y + 1 = x - 5\)
   - B) \(y + 5 = x - 1\)
   - C) \(y - 1 = -5(x - 1)\)
   - D) \(y - 1 = x + 5\)

55. A line contains \((4, 4)\) and \((5, 2)\). What are the slope and \(y\)-intercept? 
   - F) slope = \(-2\); \(y\)-intercept = 2
   - G) slope = \(1.2\); \(y\)-intercept = \(-2\)
   - H) slope = \(-2\); \(y\)-intercept = 12
   - I) slope = \(12\); \(y\)-intercept = \(1.2\)

**CHALLENGE AND EXTEND**

56. A linear function has the same \(y\)-intercept as \(x + 4y = 8\) and its graph contains the point \((2, 7)\). Find the slope and \(y\)-intercept.

57. Write the equation of a line in slope-intercept form that contains \(\left(\frac{3}{4}, \frac{1}{2}\right)\) and has the same slope as the line described by \(y + 3x = 6\).

58. Write the equation of a line in slope-intercept form that contains \(\left(-\frac{1}{2}, -\frac{1}{3}\right)\) and \(\left(1\frac{1}{2}, 1\right)\).

---

**Career Path**

**Q:** What math classes did you take in high school?
**A:** Algebra 1 and 2, Geometry, and Statistics

**Q:** What math classes have you taken in college?
**A:** Applied Statistics, Data Mining Methods, Web Mining, and Artificial Intelligence

**Q:** How do you use math?
**A:** Once for a class, I used software to analyze basketball statistics. What I learned helped me develop strategies for our school team.

**Q:** What are your future plans?
**A:** There are many options for people with data mining skills. I could work in banking, pharmaceuticals, or even the military. But my dream job is to develop game strategies for an NBA team.
Graph Linear Functions

You can use a graphing calculator to quickly graph lines whose equations are in point-slope form. To enter an equation into your calculator, it must be solved for \( y \), but it does not necessarily have to be in slope-intercept form.

**Activity**

Graph the line with slope 2 that contains the point \((2, 6.09)\).

1. Use point-slope form.
   \[
   y - y_1 = m(x - x_1)
   \]
   \[
   y - 6.09 = 2(x - 2)
   \]

2. Solve for \( y \) by adding 6.09 to both sides of the equation.
   \[
   y = 2(x - 2) + 6.09
   \]

3. Enter this equation into your calculator.

4. Graph in the standard viewing window by pressing \( \text{ZOOM} \) and selecting 6:ZStandard. In this window, both the \( x \)- and \( y \)-axes go from –10 to 10.

5. Notice that the scale on the \( y \)-axis is smaller than the scale on the \( x \)-axis. This is because the width of the calculator screen is about 50% greater than its height. To see a more accurate graph of this line, use the square viewing window. Press \( \text{ZOOM} \) and select 5:ZSquare.

**Try This**

1. Graph the function represented by the line with slope \(-1.5\) that contains the point \((2.25, -3)\). View the graph in the standard viewing window.

2. Now view the graph in the square viewing window. Press \( \text{WINDOW} \) and write down the minimum and maximum values on the \( x \)- and \( y \)-axes.

3. In which graph does the line appear steeper? Why?

4. Explain why it might sometimes be useful to look at a graph in a square window.
Interpreting Trend Lines

Previously, you learned how to draw trend lines on scatter plots. Now you will learn how to find the equations of trend lines and write them in slope-intercept form.

**Example**

Write an equation for the trend line on the scatter plot.

Two points on the trend line are (30, 75) and (60, 90).

To find the slope of the line that contains (30, 75) and (60, 90), use the slope formula.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]

Use the slope formula.

\[ m = \frac{90 - 75}{60 - 30} \]

Substitute (30, 75) for \((x_1, y_1)\) and (60, 90) for \((x_2, y_2)\).

\[ m = \frac{15}{30} \]

Simplify.

\[ m = \frac{1}{2} \]

Use the slope and the point (30, 75) to find the y-intercept of the line.

\[ y = mx + b \]

Slope-intercept form

\[ 75 = \frac{1}{2}(30) + b \]

Substitute \(\frac{1}{2}\) for \(m\), 30 for \(x\), and 75 for \(y\).

\[ 75 = 15 + b \]

Solve for \(b\).

\[ 60 = b \]

Write the equation.

\[ y = \frac{1}{2}x + 60 \]

Substitute \(\frac{1}{2}\) for \(m\) and 60 for \(b\).

**Try This**

1. In the example above, what is the meaning of the slope?
2. What does the y-intercept represent?
3. Use the equation to predict the test score of a student who spent 25 minutes studying.
4. Use the table to create a scatter plot. Draw a trend line and find the equation of your trend line. Tell the meaning of the slope and y-intercept. Then use your equation to predict the race time of a runner who ran 40 miles in training.

<table>
<thead>
<tr>
<th>Distance Run in Training (mi)</th>
<th>12</th>
<th>15</th>
<th>16</th>
<th>18</th>
<th>21</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Race Time (min)</td>
<td>65</td>
<td>64</td>
<td>55</td>
<td>58</td>
<td>55</td>
<td>50</td>
<td>50</td>
<td>47</td>
<td>36</td>
</tr>
</tbody>
</table>
**Objectives**
Identify and graph parallel and perpendicular lines.
Write equations to describe lines parallel or perpendicular to a given line.

**Vocabulary**
parallel lines
perpendicular lines

---

**Why learn this?**
Parallel lines and their equations can be used to model costs, such as the cost of a booth at a farmers’ market.

To sell at a particular farmers’ market for a year, there is a $100 membership fee. Then you pay $3 for each hour that you sell at the market. However, if you were a member the previous year, the membership fee is reduced to $50.

- The **red** line shows the total cost if you are a new member.
- The **blue** line shows the total cost if you are a returning member.

These two lines are parallel. **Parallel lines** are lines in the same plane that have no points in common. In other words, they do not intersect.

---

**Parallel Lines**

<table>
<thead>
<tr>
<th>WORDS</th>
<th>Two different nonvertical lines are parallel if and only if they have the same slope.</th>
<th>All different vertical lines are parallel.</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRAPH</td>
<td><img src="image" alt="Graph of parallel lines" /></td>
<td><img src="image" alt="Graph of perpendicular lines" /></td>
</tr>
</tbody>
</table>

---

**Example 1**

Identify which lines are parallel.

**A**

\[ y = \frac{4}{3}x + 3; \quad y = 2; \quad y = \frac{4}{3}x - 5; \quad y = -3 \]

The lines described by \( y = \frac{4}{3}x + 3 \) and \( y = \frac{4}{3}x - 5 \) both have slope \( \frac{4}{3} \). These lines are parallel. The lines described by \( y = 2 \) and \( y = -3 \) both have slope 0. These lines are parallel.
Identify which lines are parallel.

B \( y = 3x + 2; y = -\frac{1}{2}x + 4; x + 2y = -4; y - 5 = 3(x - 1) \)

Write all equations in slope-intercept form to determine the slopes.

\[
\begin{align*}
\text{y} & \quad \text{slope-intercept form} \quad \text{y} \\
3x + 2 & \quad \checkmark \\
x + 2y = -4 & \\
-x & \quad -x \\
2y & = -x - 4 \\
\frac{2y}{2} & = \frac{-x - 4}{2} \\
y & = -\frac{1}{2}x - 2
\end{align*}
\]

\[
\begin{align*}
\text{y} & \quad \text{slope-intercept form} \quad \text{y} \\
-\frac{1}{2}x + 4 & \quad \checkmark \\
y - 5 & = 3(x - 1) \\
y - 5 & = 3x - 3 \\
+5 & \quad +5 \\
y & = 3x + 2
\end{align*}
\]

The lines described by \( y = 3x + 2 \) and \( y - 5 = 3(x - 1) \) have the same slope, but they are not parallel lines. They are the same line.

The lines described by \( y = -\frac{1}{2}x + 4 \) and \( x + 2y = -4 \) represent parallel lines. They each have slope \(-\frac{1}{2}\).

Identify which lines are parallel.

1a. \( y = 2x + 2; y = 2x + 1; y = -4; x = 1 \)

1b. \( y = \frac{3}{4}x + 8; -3x + 4y = 32; y = 3x; y - 1 = 3(x + 2) \)

Geometry Application

Show that \(ABCD\) is a parallelogram.

Use the ordered pairs and the slope formula to find the slopes of \(\overline{AB}\) and \(\overline{CD}\).

\[
\begin{align*}
\text{slope of } \overline{AB} & = \frac{7 - 5}{4 - (-1)} = \frac{2}{5} \\
\text{slope of } \overline{CD} & = \frac{3 - 1}{4 - (-1)} = \frac{2}{5}
\end{align*}
\]

\(\overline{AB}\) is parallel to \(\overline{CD}\) because they have the same slope.

\(\overline{AD}\) is parallel to \(\overline{BC}\) because they are both vertical.

Therefore, \(ABCD\) is a parallelogram because both pairs of opposite sides are parallel.

2. Show that the points \(A(0, 2), B(4, 2), C(1, -3), \) and \(D(-3, -3)\) are the vertices of a parallelogram.
Perpendicular lines are lines that intersect to form right angles (90°).

**Example 3**

Identifying Perpendicular Lines

Identify which lines are perpendicular: \(x = -2; y = 1; y = -4x; y + 2 = \frac{1}{4}(x + 1)\).

The graph described by \(x = -2\) is a vertical line, and the graph described by \(y = 1\) is a horizontal line. These lines are perpendicular.

The slope of the line described by \(y = -4x\) is \(-4\). The slope of the line described by \(y + 2 = \frac{1}{4}(x - 1)\) is \(\frac{1}{4}\).

\[-4 \cdot \frac{1}{4} = -1\]

These lines are perpendicular because the product of their slopes is \(-1\).

**Example 4**

Geometry Application

Show that \(PQR\) is a right triangle.

If \(PQR\) is a right triangle, \(\overline{PQ}\) will be perpendicular to \(\overline{QR}\).

slope of \(\overline{PQ}\) = \(\frac{3 - 1}{3 - 0} = \frac{2}{3}\)

slope of \(\overline{QR}\) = \(\frac{3 - 0}{3 - 5} = \frac{3}{-2} = -\frac{3}{2}\)

\(\overline{PQ}\) is perpendicular to \(\overline{QR}\) because \(\frac{2}{3} \left( -\frac{3}{2} \right) = -1\).

Therefore, \(PQR\) is a right triangle because it contains a right angle.

**Check It Out!**

3. Identify which lines are perpendicular: \(y = -4; y - 6 = 5(x + 4); x = 3; y = -\frac{1}{5}x + 2\).

4. Show that \(P(1, 4), Q(2, 6),\) and \(R(7, 1)\) are the vertices of a right triangle.
Writing Equations of Parallel and Perpendicular Lines

**Example 5**

**A** Write an equation in slope-intercept form for the line that passes through \((4, 5)\) and is parallel to the line described by \(y = 5x + 10\).

**Step 1** Find the slope of the line.
\[
y = 5x + 10 \quad \text{The slope is 5.}
\]
The parallel line also has a slope of 5.

**Step 2** Write the equation in point-slope form.
\[
y - y_1 = m(x - x_1) \quad \text{Use point-slope form.}
\]
\[
y - 5 = 5(x - 4) \quad \text{Substitute 5 for } m, 4 \text{ for } x_1, \text{ and 5 for } y_1.
\]

**Step 3** Write the equation in slope-intercept form.
\[
y - 5 = 5(x - 4) \quad \text{Distributive Property}
\]
\[
y - 5 = 5x - 20 \quad \text{Addition Property of Equality}
\]
\[
y = 5x - 15
\]

**B** Write an equation in slope-intercept form for the line that passes through \((3, 2)\) and is perpendicular to the line described by \(y = 3x - 1\).

**Step 1** Find the slope of the line.
\[
y = 3x - 1 \quad \text{The slope is 3.}
\]
The perpendicular line has a slope of \(-\frac{1}{3}\), because \(3\left(-\frac{1}{3}\right) = -1\).

**Step 2** Write the equation in point-slope form.
\[
y - y_1 = m(x - x_1) \quad \text{Use point-slope form.}
\]
\[
y - 2 = -\frac{1}{3}(x - 3) \quad \text{Substitute } -\frac{1}{3} \text{ for } m, 3 \text{ for } x_1, \text{ and 2 for } y_1.
\]

**Step 3** Write the equation in slope-intercept form.
\[
y - 2 = -\frac{1}{3}(x - 3) \quad \text{Distributive Property}
\]
\[
y - 2 = -\frac{1}{3}x + 1
\]
\[
y = -\frac{1}{3}x + 3 \quad \text{Addition Property of Equality}
\]

5a. Write an equation in slope-intercept form for the line that passes through \((5, 7)\) and is parallel to the line described by \(y = \frac{4}{5}x - 6\).

5b. Write an equation in slope-intercept form for the line that passes through \((-5, 3)\) and is perpendicular to the line described by \(y = 5x\).

**THINK AND DISCUSS**

1. Are the lines described by \(y = \frac{1}{2}x\) and \(y = 2x\) perpendicular? Explain.

2. Describe the slopes and \(y\)-intercepts when two nonvertical lines are parallel.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch an example and describe the slopes.
**GUIDED PRACTICE**

1. **Vocabulary** ______ lines have the same slope. (Parallel or Perpendicular)

   Identify which lines are parallel.
   2. \( y = 6; y = 6x + 5; y = 6x - 7; y = -8 \)
   3. \( y = \frac{3}{4}x - 1; y = -2x; y - 3 = \frac{3}{4}(x - 5); y - 4 = -2(x + 2) \)

4. **Geometry** Show that \(ABCD\) is a trapezoid. 
   (Hint: In a trapezoid, exactly one pair of opposite sides is parallel.)

   Identify which lines are perpendicular.
   5. \( y = \frac{2}{3}x - 4; y = -\frac{3}{2}x + 2; y = -1; x = 3 \)
   6. \( y = -\frac{3}{7}x - 4; y - 4 = -7(x + 2); y - 1 = \frac{1}{7}(x - 4); y - 7 = \frac{7}{3}(x - 3) \)

7. **Geometry** Show that \(PQRS\) is a rectangle. (Hint: In a rectangle, all four angles are right angles.)

8. Write an equation in slope-intercept form for the line that passes through \((5, 0)\) and is perpendicular to the line described by \(y = -\frac{5}{2}x + 6\).

**PRACTICE AND PROBLEM SOLVING**

Identify which lines are parallel.
9. \( x = 7; y = -\frac{5}{6}x + 8; y = -\frac{5}{6}x - 4; x = -9 \)
10. \( y = -x; y - 3 = -1(x + 9); y - 6 = \frac{1}{2}(x - 14); y + 1 = \frac{1}{2}x \)
11. \( y = -3x + 2; y = \frac{1}{2}x - 1; -x + 2y = 17; 3x + y = 27 \)
12. **Geometry** Show that \(LMNP\) is a parallelogram.

Identify which lines are perpendicular.
13. \( y = 6x; y = \frac{1}{6}x; y = -\frac{1}{6}x; y = -6x \)
14. \( y - 9 = 3(x + 1); y = -\frac{1}{3}x + 5; y = 0; x = 6 \)
15. \( x - 6y = 15; y = 3x - 2; y = -3x - 3; y = -6x - 8; 3y = -x - 11 \)
16. **Geometry** Show that $ABC$ is a right triangle.

17. Write an equation in slope-intercept form for the line that passes through $(0, 0)$ and is parallel to the line described by $y = -\frac{6}{7}x + 1$.

Without graphing, tell whether each pair of lines is parallel, perpendicular, or neither.

18. $x = 2$ and $y = -5$
19. $y = 7x$ and $y - 28 = 7(x - 4)$
20. $y = 2x - 1$ and $y = \frac{1}{2}x + 2$
21. $y - 3 = \frac{1}{4}(x - 3)$ and $y + 13 = \frac{1}{4}(x + 1)$

Write an equation in slope-intercept form for the line that is parallel to the given line and that passes through the given point.

22. $y = 3x - 7; (0, 4)$
23. $y = \frac{1}{2}x + 5; (4, -3)$
24. $4y = x; (4, 0)$
25. $y = 2x + 3; (1, 7)$
26. $5x - 2y = 10; (3, -5)$
27. $y = 3x - 4; ( -2, 7)$
28. $y = 7; (2, 4)$
29. $x + y = 1; (2, 3)$
30. $2x + 3y = 7; (4, 5)$
31. $y = 4x + 2; (5, -3)$
32. $y = \frac{1}{2}x - 1; (0, -4)$
33. $3x + 4y = 8; (4, -3)$

Write an equation in slope-intercept form for the line that is perpendicular to the given line and that passes through the given point.

34. $y = -3x + 4; (6, -2)$
35. $y = x - 6; (-1, 2)$
36. $3x - 4y = 8; (-6, 5)$
37. $5x + 2y = 10; (3, -5)$
38. $y = 5 - 3x; (2, -4)$
39. $-10x + 2y = 8; (4, -3)$
40. $2x + 3y = 7; (4, 5)$
41. $4x - 2y = -6; (3, -2)$
42. $-2x - 8y = 16; (4, 5)$
43. $y = -2x + 4; (-2, 5)$
44. $y = x - 5; (0, 5)$
45. $x + y = 2; (8, 5)$

46. Write an equation describing the line that is parallel to the $y$-axis and that is 6 units to the right of the $y$-axis.
47. Write an equation describing the line that is perpendicular to the $y$-axis and that is 4 units below the $x$-axis.

48. **Critical Thinking** Is it possible for two linear functions whose graphs are parallel lines to have the same $y$-intercept? Explain.

49. **Estimation** Estimate the slope of a line that is perpendicular to the line through $(2.07, 8.95)$ and $(-1.9, 25.07)$.

50. **Write About It** Explain in words how to write an equation in slope-intercept form that describes a line parallel to $y - 3 = -6(x - 3)$.

51. **a.** Flora walks from her home to the bus stop at a rate of 50 steps per minute. Write a rule that gives her distance from home (in steps) as a function of time.

   **b.** Flora’s neighbor Dan lives 30 steps closer to the bus stop. He begins walking at the same time and at the same pace as Flora. Write a rule that gives Dan’s distance from Flora’s house as a function of time.

   **c.** Will Flora meet Dan along the walk? Use a graph to help explain your answer.
52. Which describes a line parallel to the line described by \( y = -3x + 2 \)?

- (A) \( y = -3x \)  
- (B) \( y = \frac{1}{3}x \)  
- (C) \( y = 2 - 3x \)  
- (D) \( y = \frac{1}{3}x + 2 \)

53. Which describes a line passing through \((3, 3)\) that is perpendicular to the line described by \( y = \frac{3}{5}x + 2 \)?

- (F) \( y = \frac{5}{3}x - 2 \)  
- (H) \( y = \frac{3}{5}x + \frac{6}{5} \)

54. **Gridded Response** The graph of a linear function \( f(x) \) is parallel to the line described by \( 2x + y = 5 \) and contains the point \((6, -2)\). What is the \( y \)-intercept of \( f(x) \)?

**Challenge and Extend**

55. Three or more points that lie on the same line are called **collinear points**. Explain why the points \( A, B, \) and \( C \) must be collinear if the line containing \( A \) and \( B \) has the same slope as the line containing \( B \) and \( C \).

56. The lines described by \( y = (a + 12)x + 3 \) and \( y = 4ax \) are parallel. What is the value of \( a \)?

57. The lines described by \( y = (5a + 3)x \) and \( y = -\frac{1}{2}x \) are perpendicular. What is the value of \( a \)?

58. **Geometry** The diagram shows a square in the coordinate plane. Use the diagram to show that the diagonals of a square are perpendicular.
**Vocabulary**

constant of variation  
correlation coefficient  
direct variation  
family of functions  
least-squares line  
line of best fit  
linear equation  
linear function  
linear regression  
parallel lines  
parent function  
perpendicular lines  
rate of change  
reflection  
residual  
rise  
rotation  
run  
slope  
transformation  
translation  
x-intercept  
y-intercept

Complete the sentences below with vocabulary words from the list above. Words may be used more than once.

1. A(n) __?__ is a “slide,” a(n) __?__ is a “turn,” and a(n) __?__ is a “flip.”

2. The __?__ is always 0.

3. In the equation __?__, the value of __?__ is the __?__, and the value of __?__ is the __?__.

**6-1 Identifying Linear Functions**

**Examples**

Tell whether each function is linear. If so, graph the function.

- **y = –3x + 2**
  
  Write the equation in standard form.
  
  $3x + y = 2$
  
  This is a linear function.

  Generate ordered pairs.

<table>
<thead>
<tr>
<th>x</th>
<th>y = –3x + 2</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>–2</td>
<td>y = –3(–2) + 2 = 8</td>
<td>(–2, 8)</td>
</tr>
<tr>
<td>x</td>
<td>y = –3(0) + 2 = 2</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>x</td>
<td>y = –3(2) + 2 = –4</td>
<td>(2, –4)</td>
</tr>
</tbody>
</table>

  Plot the points and connect them with a straight line.

- **y = 2x^3**
  
  This is not a linear function because x has an exponent other than 1.

**Exercises**

Tell whether the given ordered pairs satisfy a linear function. Explain.

4. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>–3</td>
<td>3</td>
</tr>
<tr>
<td>–1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

5. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>–3</td>
</tr>
<tr>
<td>1</td>
<td>–1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

6. \{(–2, 5), (–1, 3), (0, 1), (1, –1), (2, –3)\}

7. \{(1, 7), (3, 6), (6, 5), (9, 4), (13, 3)\}

Each equation below is linear. Write each equation in standard form and give the values of A, B, and C.

8. $y = –5x + 1$

9. $\frac{x + 2}{2} = –3y$

10. $4y = 7x$

11. $9 = y$

12. Helene is selling cupcakes for $0.50 each. The function $f(x) = 0.5x$ gives the total amount of money Helene makes after selling x cupcakes. Graph this function and give its domain and range.
6-2 Using Intercepts

**Example**

Find the \(x\)- and \(y\)-intercepts of \(2x + 5y = 10\).

Let \(y = 0\).
\[2x + 5(0) = 10\]
\[2x = 10\]
\[x = 5\]

Let \(x = 0\).
\[2(0) + 5y = 10\]
\[5y = 10\]
\[y = 2\]

The \(x\)-intercept is 5. The \(y\)-intercept is 2.

**Exercises**

Find the \(x\)- and \(y\)-intercepts.

13. \(42 - 4 - 2 - 2 - 4\)  
14. \(44 - 2\)  
15. \(3x - y = 9\)  
16. \(-2x + y = 1\)  
17. \(-x + 6y = 18\)  
18. \(3x - 4y = 1\)

6-3 Rate of Change and Slope

**Example**

Find the slope of the line.

**Conversion of Measurement**

<table>
<thead>
<tr>
<th>Length (ft)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (yd)</td>
<td>1</td>
<td>3</td>
<td>(1, 3)</td>
<td>(2, 6)</td>
<td>(3, 9)</td>
</tr>
</tbody>
</table>

\[\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{3}{1} = 3\]

**Exercises**

19. Graph the data and show the rates of change.
20. Find the slope of the line graphed below.

21. \(4x + 3y = 24\)  
22. \(y = -3x + 6\)  
23. \(x + 2y = 10\)  
24. \(3x = y + 3\)  
25. \(y + 2 = 7x\)  
26. \(16x = 4y + 1\)

6-4 The Slope Formula

**Example**

Find the slope of the line described by \(2x - 3y = 6\).

**Step 1** Find the \(x\)- and \(y\)-intercepts.

Let \(y = 0\).
\[2x - 3(0) = 6\]
\[2x = 6\]
\[x = 3\]

Let \(x = 0\).
\[2(0) - 3y = 6\]
\[-3y = 6\]
\[y = -2\]

The line contains (3, 0) and (0, -2).

**Step 2** Use the slope formula.

\[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 0}{0 - 3} = \frac{-2}{-3} = \frac{2}{3}\]

**Exercises**

Find the slope of the line described by each equation.

21. \(4x + 3y = 24\)  
22. \(y = -3x + 6\)  
23. \(x + 2y = 10\)  
24. \(3x = y + 3\)  
25. \(y + 2 = 7x\)  
26. \(16x = 4y + 1\)

Find the slope of the line that contains each pair of points.

27. (1, 2) and (2, -3)  
28. (4, -2) and (-5, 7)  
29. (-3, -6) and (4, 1)  
30. \(\left(\frac{1}{2}, 2\right)\) and \(\left(\frac{3}{4}, \frac{5}{2}\right)\)  
31. (2, 2) and (2, 7)  
32. (1, -3) and (5, -3)
6-5 Direct Variation

**EXAMPLE**

Tell whether $6x = -4y$ represents a direct variation. If so, identify the constant of variation.

$$
6x = -4y \\
\frac{6x}{-4} = \frac{-4y}{-4} \quad \text{Solve the equation for } y. \\
-\frac{3}{2}x = y \\
y = -\frac{3}{2}x \quad \text{Simplify.}
$$

This equation represents a direct variation because it can be written in the form $y = kx$, where $k = -\frac{3}{2}$.

**EXERCISES**

Tell whether each equation is a direct variation. If so, identify the constant of variation.

33. $y = -6x$  
34. $x - y = 0$  
35. $y + 4x = 3$  
36. $2x = -4y$

37. The value of $y$ varies directly with $x$, and $y = -8$ when $x = 2$. Find $y$ when $x = 3$.

38. Maleka charges $8 per hour for baby-sitting. The amount of money she makes varies directly with the number of hours she baby-sits. Write a direct variation equation for the amount $y$ Maleka earns for baby-sitting $x$ hours. Then graph.

6-6 Slope-Intercept Form

**EXAMPLE**

Graph the line with slope $= -\frac{4}{5}$ and $y$-intercept $= 8$.

Step 1 Plot $(0, 8)$.

Step 2 For a slope of $-\frac{4}{5}$, count 4 down and 5 right from $(0, 8)$. Plot another point.

Step 3 Connect the two points with a line.

**EXERCISES**

Graph each line given the slope and $y$-intercept.

39. slope $= -\frac{1}{2}$; $y$-intercept $= 4$  
40. slope $= 3$; $y$-intercept $= -7$

Write the equation that describes each line in slope-intercept form.

41. slope $= \frac{1}{3}$, $y$-intercept $= 5$  
42. slope $= 4$, $(1, -5)$ is on the line.

6-7 Point-Slope Form

**EXAMPLE**

Write an equation in slope-intercept form for the line through $(4, -1)$ and $(-2, 8)$.

Step 1 Find the slope.

$$
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-1)}{-2 - 4} = \frac{9}{-6} = -\frac{3}{2}
$$

Step 2 Write the point-slope form.

$$
y - 8 = -\frac{3}{2}[x - (-2)] \quad \text{Substitute into the point-slope form.}
$$

$$
y - 8 = -\frac{3}{2}(x + 2)
$$

Step 3 Write the slope-intercept form.

$$
y = -\frac{3}{2}x + 5 \quad \text{Solve for } y.
$$

**EXERCISES**

Graph the line described by each equation.

43. $y + 3 = \frac{1}{2}(x - 4)$  
44. $y - 1 = -(x + 3)$

Write the equation that describes each line in slope-intercept form.

45. slope $= 2$, $(1, 3)$ is on the line.  
46. slope $= -5$, $(-6, 4)$ is on the line.  
47. $(1, 4)$ and $(3, 8)$ are on the line.  
48. $(-2, 4)$ and $(-1, 6)$ are on the line.
6-8 Line of Best Fit

**Example**

Two lines of fit for the data in the table are 
\( y = -x + 5 \) and \( y = -0.5x + 4 \). For each line, find the sum of the squares of the residuals. Which line is a better fit?

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

**EXERCISES**

49. Two lines of fit for the data in the table are \( y = 0.5x \) and \( y = x - 1 \). For each line, find the sum of the squares of the residuals. Which line is a better fit?

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

50. The lengths and weights of 6 koi in a pond are shown in the table. Find an equation of a line of best fit. How well does the line fit the data?

<table>
<thead>
<tr>
<th>Length (in.)</th>
<th>9</th>
<th>12</th>
<th>11</th>
<th>15</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (oz)</td>
<td>5</td>
<td>11</td>
<td>9</td>
<td>20</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

6-9 Slopes of Parallel and Perpendicular Lines

**Example**

Write an equation in slope-intercept form for the line that passes through \( (4, -2) \) and is perpendicular to the line described by \( y = -4x + 3 \).

The slope of \( y = -4x + 3 \) is -4.
The perpendicular line has a slope of \( \frac{1}{4} \) and contains \( (4, -2) \).

\[
\begin{align*}
y + 2 &= \frac{1}{4}(x - 4) & \text{Substitute into the point-slope form.} \\
y &= \frac{1}{4}x - 3 & \text{Solve for } y.
\end{align*}
\]

**EXERCISES**

Identify which lines are parallel.

51. \( y = -\frac{1}{3}x; y = 3x + 2; y = -\frac{1}{3}x - 6; y = 3 \)

52. \( y - 2 = -4(x - 1); y = 4x - 4; y = \frac{1}{4}x; y = -4x - 2 \)

Identify which lines are perpendicular.

53. \( y - 1 = -5(x - 6); y = \frac{1}{5}x + 2; y = 5; y = 5x + 8 \)

54. \( y = 2x; y - 2 = 3(x + 1); y = \frac{2}{3}x - 4; y = -\frac{1}{3}x \)

55. Write an equation in slope-intercept form for the line that passes through \( (1, -2) \) and is parallel to the line described by \( y = 2x - 4 \).

6-10 Transforming Linear Functions

**Example**

Graph \( f(x) = \frac{1}{2}x \) and \( g(x) = 4x + 2 \). Then describe the transformation(s) from the graph of \( f(x) \) to the graph of \( g(x) \).

- Multiply \( f(x) = \frac{1}{2}x \) by 8 to get \( h(x) = 4x \).
  This rotates the graph about \( (0, 0) \), making it steeper.
- Then add 2 to \( h(x) = 4x \) to get \( g(x) = 4x + 2 \).
  This translates the graph 2 units up.

**EXERCISES**

Graph \( f(x) \) and \( g(x) \). Then describe the transformation(s) from the graph of \( f(x) \) to the graph of \( g(x) \).

56. \( f(x) = x, g(x) = x + 4 \)

57. \( f(x) = 4x, g(x) = -4x \)

58. \( f(x) = \frac{1}{3}x - 2, g(x) = -\frac{1}{3}x - 2 \)

59. The entrance fee at a carnival is $3 and each ride costs $1. The total cost for \( x \) rides is \( f(x) = x + 3 \). How will the graph of this function change if the entrance fee is increased to $5? If the cost per ride is increased to $2?
Tell whether each function is linear. If so, graph the function.

1. $3y = 2x + 3$
2. $y = x(4 + x)$

3. Lily plans to volunteer at the tutoring center for 45 hours. She can tutor 3 hours per week. The function $f(x) = 45 - 3x$ gives the number of hours she will have left to tutor after $x$ weeks. Graph the function and find its intercepts. What does each intercept represent?

4. The table shows the number of guppies in an aquarium over time. Graph the data and show the rates of change.

<table>
<thead>
<tr>
<th>Time (mo)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guppies</td>
<td>4</td>
<td>4</td>
<td>10</td>
<td>25</td>
<td>31</td>
</tr>
</tbody>
</table>

Find the slope of each line. Then tell what the slope represents.

5. Ticket Costs
6. Water in Tank
7. Temperature of Specimen

8. A space shuttle travels at a speed of about 5 miles per second. Write a direct variation equation for the distance $y$ the space shuttle will travel in $x$ seconds. Then graph.

9. Write the equation $2x - 2y = 4$ in slope-intercept form. Then graph the line described by the equation.

Write the equation that describes each line in slope-intercept form.

10. slope = 4, $(–3, 3)$ is on the line.
11. $x$-intercept = 3, $y$-intercept = $–3$

Write an equation in point-slope form for the line with the given slope that contains the given point.

12. slope = $–1$; $(1, 3)$
13. slope = 5; $(–3, 2)$

14. Four friends recorded the numbers of CDs and video games their families purchased in the last month, as shown in the table. Find an equation of a line of best fit. How well does the line fit the data?

<table>
<thead>
<tr>
<th>CDs</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Games</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

15. Write an equation in slope-intercept form for the line that passes through $(0, 6)$ and is parallel to the line described by $y = 2x + 3$.

Graph $f(x)$ and $g(x)$. Then describe the transformation(s) from the graph of $f(x)$ to the graph of $g(x)$.

16. $f(x) = 8x$, $g(x) = 4x$
17. $f(x) = –x + 2$, $g(x) = –x – 1$
18. $f(x) = 3x$, $g(x) = 6x – 1$
Where's the Money?

You can solve a system of equations to determine how many basketball game tickets you can buy at different price levels.

**Chapter Focus**

- Solve real-world problems involving systems of linear equations and inequalities.

---

**Where's the Money?**

You can solve a system of equations to determine how many basketball game tickets you can buy at different price levels.
Writing Strategy: Write a Convincing Argument/Explanation

The Write About It icon appears throughout the book. These icons identify questions that require you to write a complete argument or explanation. Writing a convincing argument or explanation shows that you have a solid understanding of a concept.

To be effective, an argument or explanation should include
- evidence, work, or facts.
- a complete response that will answer or explain.

23. Write About It Lewis invested $1000 at 3% simple interest for 4 years. Lisa invested $1000 at 4% simple interest for 3 years. Explain why Lewis and Lisa earned the same amount of interest.

Step 1 Identify what you need to answer or explain.
Explain why Lewis and Lisa earned the same amount of interest.

Step 2 Give evidence, work, or facts that are needed to answer the question.
Use the formula for simple interest to find the amount of interest earned: \( I = Prt \).

\[
\begin{align*}
\text{Lewis: } & P = 1000, \ r = 0.03, \ t = 4 \\
& I = Prt = 1000(0.03)(4) = 120 \\
& I = 1000(0.12) = $120 \\
\text{Lisa: } & P = 1000, \ r = 0.04, \ t = 3 \\
& I = Prt = 1000(0.04)(3) = 120 \\
& I = 1000(0.12) = $120
\end{align*}
\]

Step 3 Write a complete response that answers or explains.
Lewis and Lisa both invested the same amount of money, $1000. They earned the same amount of interest because 0.04 \times 3 and 0.03 \times 4 both equal 0.12. They both earned 0.12 \times $1000, or $120.

Try This

Write a convincing argument or explanation.

1. What is the least whole number that is a solution of \(12x + 15.4 > 118.92\)? Explain.

2. Which equation has an error? Explain the error.
   - A. \(4(6 \cdot 5) = (4)6 \cdot (4)5\)
   - B. \(4(6 \cdot 5) = (4 \cdot 6)5\)
Solve Linear Equations by Using a Spreadsheet

You can use a spreadsheet to answer “What if...?” questions. By changing one or more values, you can quickly model different scenarios.

Activity

Company Z makes DVD players. The company’s costs are $400 per week plus $20 per DVD player. Each DVD player sells for $45. How many DVD players must company Z sell in one week to make a profit?

Let \( n \) represent the number of DVD players company Z sells in one week.

\[
\begin{align*}
    c &= 400 + 20n \quad \text{The total cost is $400 plus $20 times the number of DVD players made.} \\
    s &= 45n \quad \text{The total sales income is $45 times the number of DVD players sold.} \\
    p &= s - c \quad \text{The total profit is the sales income minus the total cost.}
\end{align*}
\]

1. Set up your spreadsheet with columns for number of DVD players, total cost, total income, and profit.

2. Under Number of DVD Players, enter 1 in cell A2.

3. Use the equations above to enter the formulas for total cost, total sales, and total profit in row 2.
   - In cell B2, enter the formula for total cost.
   - In cell C2, enter the formula for total sales income.
   - In cell D2, enter the formula for total profit.

4. Fill columns A, B, C, and D by selecting cells A1 through D1, clicking the small box at the bottom right corner of cell D2, and dragging the box down through several rows.

5. Find the point where the profit is $0. This is known as the breakeven point, where total cost and total income are the same.

Company Z must sell 17 DVD players to make a profit. The profit is $25.

Try This

For Exercises 1 and 2, use the spreadsheet from the activity.

1. If company Z sells 10 DVD players, will they make a profit? Explain. What if they sell 16?

2. Company Z makes a profit of $225 dollars. How many DVD players did they sell?

For Exercise 3, make a spreadsheet.

3. Company Y’s costs are $400 per week plus $20 per DVD player. They want the breakeven point to occur with sales of 8 DVD players. What should the sales price be?
Why learn this?
You can compare costs by graphing a system of linear equations. (See Example 3.)

Objectives
Identify solutions of systems of linear equations in two variables.
Solve systems of linear equations in two variables by graphing.

Vocabulary
system of linear equations
solution of a system of linear equations

Solving Systems by Graphing

Sometimes there are different charges for the same service or product at different places. For example, Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental while Bowling Pinz charges $2 per game plus $4 for shoe rental. A system of linear equations can be used to compare these charges.

A system of linear equations is a set of two or more linear equations containing two or more variables. A solution of a system of linear equations with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

Identifying Solutions of Systems

Tell whether the ordered pair is a solution of the given system.

A  (4, 1); \[
\begin{align*}
2x + y &= 6 \\
x - y &= 3
\end{align*}
\]

Substitute 4 for \(x\) and 1 for \(y\) in each equation in the system.

<table>
<thead>
<tr>
<th>(x + 2y = 6)</th>
<th>(x - y = 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 + 2(1) = 6</td>
<td>4 - 1 = 3</td>
</tr>
<tr>
<td>4 + 2 = 6</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3 ✔</td>
</tr>
</tbody>
</table>

The ordered pair \((4, 1)\) makes both equations true. \((4, 1)\) is a solution of the system.

B  \((-1, 2)\); \[
\begin{align*}
2x + 5y &= 8 \\
3x - 2y &= 5
\end{align*}
\]

Substitute \(-1\) for \(x\) and 2 for \(y\) in each equation in the system.

<table>
<thead>
<tr>
<th>(2x + 5y = 8)</th>
<th>(3x - 2y = 5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2(-1) + 5(2) = 8</td>
<td>3(-1) - 2(2) = 5</td>
</tr>
<tr>
<td>-2 + 10 = 8</td>
<td>-3 - 4 = 5</td>
</tr>
<tr>
<td>8</td>
<td>8 ✔</td>
</tr>
</tbody>
</table>

The ordered pair \((-1, 2)\) makes one equation true, but not the other. \((-1, 2)\) is not a solution of the system.

Check it out!
Tell whether the ordered pair is a solution of the given system.

1a. \((1, 3)\); \[
\begin{align*}
2x + y &= 5 \\
-2x + y &= 1
\end{align*}
\]

1b. \((2, -1)\); \[
\begin{align*}
x - 2y &= 4 \\
3x + y &= 6
\end{align*}
\]
All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their point of intersection.

\[
\begin{align*}
y &= 2x - 1 \\
y &= -x + 5
\end{align*}
\]

The point \((2, 3)\) is where the two lines intersect and is a solution of both equations, so \((2, 3)\) is the solution of the system.

**Example 2**

**Solving a System of Linear Equations by Graphing**

Solve each system by graphing. Check your answer.

**A**

\[
\begin{align*}
y &= x - 3 \\
y &= -x - 1
\end{align*}
\]

**Graph the system.**

The solution appears to be at \((1, -2)\).

**Check**

Substitute \((1, -2)\) into the system.

\[
\begin{array}{c|cc}
y = x - 3 & y = -x - 1 \\
\hline
-2 & 1 - 3 & -2 \\
-2 & -2 & -2 \\
\end{array}
\]

The solution is \((1, -2)\).

**B**

\[
\begin{align*}
x + y &= 0 \\
y &= \frac{1}{2}x + 1
\end{align*}
\]

**Rewrite the first equation in slope-intercept form.**

\[
x + y = 0 \\
-x \\
y = -x
\]

**Graph using a calculator and then use the intersection command.**

**Check**

Substitute \((-2, 2)\) into the system.

\[
\begin{array}{c|cc}
x + y &= 0 & y = \frac{1}{2}x + 1 \\
\hline
-2 + 2 & 0 & 2 - \frac{1}{2}(-2) + 1 \\
0 & 0 & 2 \\
2 & 1 + 1 & 2 \\
2 & 2 & 2 \\
\end{array}
\]

The solution is \((-2, 2)\).

**Check It Out!**

Solve each system by graphing. Check your answer.

2a. \[
\begin{align*}
y &= -2x - 1 \\
y &= x + 5
\end{align*}
\]

2b. \[
\begin{align*}
y &= \frac{1}{3}x - 3 \\
2x + y &= 4
\end{align*}
\]
Problem-Solving Application

Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental, and Bowling Pinz charges $2 per game plus $4 for shoe rental. For how many games will the cost to bowl be the same at both places? What is that cost?

1. Understand the Problem

The answer will be the number of games played for which the total cost is the same at both bowling alleys. List the important information:

- Game price: Bowl-o-Rama $2.50 Bowling Pinz: $2
- Shoe-rental fee: Bowl-o-Rama $2 Bowling Pinz: $4

2. Make a Plan

Write a system of equations, one equation to represent the price at each company. Let \( x \) be the number of games played and \( y \) be the total cost.

<table>
<thead>
<tr>
<th>Total cost</th>
<th>is</th>
<th>price per game</th>
<th>times</th>
<th>games</th>
<th>plus</th>
<th>shoe rental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bowl-o-Rama</td>
<td>( y ) = 2.5 ( x ) + 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bowling Pinz</td>
<td>( y ) = 2 ( x ) + 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Solve

Graph \( y = 2.5x + 2 \) and \( y = 2x + 4 \). The lines appear to intersect at \((4, 12)\). So, the cost at both places will be the same for 4 games bowled and that cost will be $12.

4. Look Back

Check \((4, 12)\) using both equations.

- Cost of bowling 4 games at Bowl-o-Rama:
  \[ \$2.5(4) + 2 = 10 + 2 = 12 \]
  \( \checkmark \)

- Cost of bowling 4 games at Bowling Pinz:
  \[ \$2(4) + 4 = 8 + 4 = 12 \]
  \( \checkmark \)

3. Video club A charges $10 for membership and $3 per movie rental. Video club B charges $15 for membership and $2 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost?

THINK AND DISCUSS

1. Explain how to use a graph to solve a system of linear equations.

2. Explain how to check a solution of a system of linear equations.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a step for solving a linear system by graphing. More boxes may be added.
GUIDED PRACTICE
1. **Vocabulary**  Describe a solution of a system of linear equations.

Tell whether the ordered pair is a solution of the given system.

2. $(2, -2)$; \[egin{cases} 3x + y = 4 \\ x - 3y = -4 \end{cases}\]

3. $(3, -1)$; \[egin{cases} x - 2y = 5 \\ 2x - y = 7 \end{cases}\]

4. $(-1, 5)$; \[egin{cases} -x + y = 6 \\ 2x + 3y = 13 \end{cases}\]

Solve each system by graphing. Check your answer.

5. \[egin{cases} y = \frac{1}{2}x \\ y = -x + 3 \end{cases}\]

6. \[egin{cases} y = x - 2 \\ 2x + y = 1 \end{cases}\]

7. \[egin{cases} -2x - 1 = y \\ x + y = 3 \end{cases}\]

8. To deliver mulch, Lawn and Garden charges $30 per cubic yard of mulch plus a $30 delivery fee. Yard Depot charges $25 per cubic yard of mulch plus a $55 delivery fee. For how many cubic yards will the cost be the same? What will that cost be?

PRACTICE AND PROBLEM SOLVING

Tell whether the ordered pair is a solution of the given system.

9. $(1, -4)$; \[egin{cases} x - 2y = 8 \\ 4x - y = 8 \end{cases}\]

10. $(-2, 1)$; \[egin{cases} 2x - 3y = -7 \\ 3x + y = -5 \end{cases}\]

11. $(5, 2)$; \[egin{cases} 2x + y = 12 \\ -3y - x = -11 \end{cases}\]

Solve each system by graphing. Check your answer.

12. \[egin{cases} y = \frac{1}{2}x + 2 \\ y = -x - 1 \end{cases}\]

13. \[egin{cases} y = x \\ y = -x + 6 \end{cases}\]

14. \[egin{cases} -2x - 1 = y \\ x = -y + 3 \end{cases}\]

15. \[egin{cases} x + y = 2 \\ y = x - 4 \end{cases}\]

16. **Multi-Step**  Angelo runs 7 miles per week and increases his distance by 1 mile each week. Marc runs 4 miles per week and increases his distance by 2 miles each week. In how many weeks will Angelo and Marc be running the same distance? What will that distance be?

17. **School**  The school band sells carnations on Valentine's Day for $2 each. They buy the carnations from a florist for $0.50 each, plus a $16 delivery charge.
   a. Write a system of equations to describe the situation.
   b. Graph the system. What does the solution represent?
   c. Explain whether the solution shown on the graph makes sense in this situation. If not, give a reasonable solution.

18. **Multi-Step Test Prep**  a. The Warrior baseball team is selling hats as a fund-raiser. They contacted two companies. Hats Off charges a $50 design fee and $5 per hat. Top Stuff charges a $25 design fee and $6 per hat. Write an equation for each company’s pricing.
   b. Graph the system of equations from part a. For how many hats will the cost be the same? What is that cost?
   c. Explain when it is cheaper for the baseball team to use Top Stuff and when it is cheaper to use Hats Off.
Middleton Place Gardens, South Carolina, are the United States’ oldest landscaped gardens. The gardens were established in 1741 and opened to the public in the 1920s.

Landscaping

The gardeners at Middleton Place Gardens want to plant a total of 45 white and pink hydrangeas in one flower bed. In another flower bed, they want to plant 120 hydrangeas. In this bed, they want 2 times the number of white hydrangeas and 3 times the number of pink hydrangeas as in the first bed. Use a system of equations to find how many white and how many pink hydrangeas the gardeners should buy altogether.

Fitness

Rusty burns 5 Calories per minute swimming and 11 Calories per minute jogging. In the morning, Rusty burns 200 Calories walking and swims for \( x \) minutes. In the afternoon, Rusty will jog for \( x \) minutes. How many minutes must he jog to burn at least as many Calories \( y \) in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes.

Critical Thinking

Write a real-world situation that could be represented by the system:

\[
\begin{align*}
\begin{cases}
y = 3x + 10 \\
y = 5x + 20
\end{cases}
\end{align*}
\]

Write About It

When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?

Test Prep

28. Taxi company A charges $4 plus $0.50 per mile. Taxi company B charges $5 plus $0.25 per mile. Which system best represents this problem?

\[
\begin{align*}
\begin{cases}
y = 4x + 0.5 \\
y = 5x + 0.25
\end{cases}
\end{align*}
\]

29. Which system of equations represents the given graph?

\[
\begin{align*}
\begin{cases}
y = 2x - 1 \\
y = \frac{1}{3}x + 3
\end{cases}
\end{align*}
\]

30. Gridded Response Which value of \( b \) will make the system \( y = 2x + 2 \) and \( y = 2.5x + b \) intersect at the point \( (2, 6) \)?

Graphing Calculator

Use a graphing calculator to graph and solve the systems of equations in Exercises 19–22. Round your answer to the nearest tenth.

19. \[
\begin{align*}
\begin{cases}
y = 4.7x + 2.1 \\
y = 1.6x - 5.4
\end{cases}
\end{align*}
\]

20. \[
\begin{align*}
\begin{cases}
4.8x + 0.6y = 4 \\
y = -3.2x + 2.7
\end{cases}
\end{align*}
\]

21. \[
\begin{align*}
\begin{cases}
y = \frac{5}{4}x - \frac{2}{3} \\
8x + y = \frac{5}{9}
\end{cases}
\end{align*}
\]

22. \[
\begin{align*}
\begin{cases}
y = 6.9x + 12.4 \\
y = -4.1x - 5.3
\end{cases}
\end{align*}
\]
CHALLENGE AND EXTEND

31. **Entertainment** If the pattern in the table continues, in what month will the number of sales of VCRs and DVD players be the same? What will that number be?

<table>
<thead>
<tr>
<th>Total Number Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Month</td>
</tr>
<tr>
<td>VCRs</td>
</tr>
<tr>
<td>DVD Players</td>
</tr>
</tbody>
</table>

32. Long Distance Inc. charges a $1.45 connection charge and $0.03 per minute. Far Away Calls charges a $1.52 connection charge and $0.02 per minute.

a. For how many minutes will a call cost the same from both companies? What is that cost?

b. When is it better to call using Long Distance Inc.? Far Away Calls? Explain.

c. **What if...?** Long Distance Inc. raised its connection charge to $1.50 and Far Away Calls decreased its connection charge by 2 cents. How will this affect the graphs? Now which company is better to use for calling long distance? Why?

---

**Career Path**

**Q:** What math classes did you take in high school?

**A:** Career Math, Algebra, and Geometry

**Q:** What are you studying and what math classes have you taken?

**A:** I am really interested in aviation. I am taking Statistics and Trigonometry. Next year I will take Calculus.

**Q:** How is math used in aviation?

**A:** I use math to interpret aeronautical charts. I also perform calculations involving wind movements, aircraft weight and balance, and fuel consumption. These skills are necessary for planning and executing safe air flights.

**Q:** What are your future plans?

**A:** I could work as a commercial or corporate pilot or even as a flight instructor. I could also work toward a bachelor’s degree in aviation management, air traffic control, aviation electronics, aviation maintenance, or aviation computer science.
Model Systems of Linear Equations

You can use algebra tiles to model and solve some systems of linear equations.

**Activity**

Use algebra tiles to model and solve \[
\begin{align*}
\begin{cases}
y &= 2x - 3 \\
x + y &= 9
\end{cases}
\end{align*}
\]

**Model**

The first equation is solved for \( y \).
Model the second equation, \( x + y = 9 \), by substituting \( 2x - 3 \) for \( y \).

- Add 3 yellow tiles on both sides of the mat. This represents adding 3 to both sides of the equation.
- Remove zero pairs.

**Algebra**

\[
\begin{align*}
x + y &= 9 \\
2x - 3 &= y \\
3x - 3 &= 9
\end{align*}
\]

- Divide each side into 3 equal groups.
  Align one x-tile with each group on the right side. One x-tile is equivalent to 4 yellow tiles. \( x = 4 \)

To solve for \( y \), substitute 4 for \( x \) in one of the equations:

\[
y = 2x - 3 = 2(4) - 3 = 5
\]

The solution is \( (4, 5) \).

**Try This**

Model and solve each system of equations.

1. \[
\begin{align*}
y &= x + 3 \\
2x + y &= 6
\end{align*}
\]
2. \[
\begin{align*}
2x + 3 &= y \\
x + y &= 6
\end{align*}
\]
3. \[
\begin{align*}
2x + 3y &= 1 \\
x &= -1 - y
\end{align*}
\]
4. \[
\begin{align*}
y &= x + 1 \\
2x - y &= -5
\end{align*}
\]
Solving Systems by Substitution

**Objective**
Solve systems of linear equations in two variables by substitution.

**Why learn this?**
You can solve systems of equations to help select the best value among high-speed Internet providers. (See Example 3.)

Sometimes it is difficult to identify the exact solution to a system by graphing. In this case, you can use a method called substitution.

The goal when using substitution is to reduce the system to one equation that has only one variable. Then you can solve this equation, and substitute into an original equation to find the value of the other variable.

### Solving Systems of Equations by Substitution

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solve for one variable in at least one equation, if necessary.</td>
</tr>
<tr>
<td>2</td>
<td>Substitute the resulting expression into the other equation.</td>
</tr>
<tr>
<td>3</td>
<td>Solve that equation to get the value of the first variable.</td>
</tr>
<tr>
<td>4</td>
<td>Substitute that value into one of the original equations and solve.</td>
</tr>
<tr>
<td>5</td>
<td>Write the values from Steps 3 and 4 as an ordered pair, ((x, y)), and check.</td>
</tr>
</tbody>
</table>

### Example 1

**Solving a System of Linear Equations by Substitution**

Solve each system by substitution.

**A**

\[
\begin{align*}
  y &= 2x \\
  y &= x + 5 
\end{align*}
\]

**Step 1**

Both equations are solved for \(y\).

\[
y = 2x \\
y = x + 5
\]

**Step 2**

Substitute \(2x\) for \(y\) in the second equation.

\[
y = x + 5 \\
2x = x + 5
\]

**Step 3**

Solve for \(x\).

\[
x = 5
\]

**Step 4**

Write one of the original equations.

\[
y = 2x \\
y = 2(5)
\]

Substitute 5 for \(x\).

\[
y = 10
\]

**Step 5**

Write the solution as an ordered pair.

\((5, 10)\)

**Check**

Substitute \((5, 10)\) into both equations in the system.

\[
\begin{array}{c|c|c}
  y &= 2x & y &= x + 5 \\
  10 & 2(5) & 10 & 5 + 5 \\
  10 & 10 & 10 & 10 
\end{array}
\]

You can substitute the value of one variable into either of the original equations to find the value of the other variable.

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Solve each system by substitution.

**B**

\[
\begin{align*}
2x + y &= 5 \\
y &= x - 4
\end{align*}
\]

**Step 1** \( y = x - 4 \)  
The second equation is solved for \( y \).

**Step 2** \( 2x + y = 5 \)

\( 2x + (x - 4) = 5 \)

**Step 3** \( 3x - 4 = 5 \)

\[
3x - 4 = 5 \\
+4 +4 \\
3x = 9 \\
\frac{3x}{3} = \frac{9}{3} \\
x = 3
\]

**Step 4** \( y = x - 4 \)

\( y = 3 - 4 \)

\( y = -1 \)

**Step 5** \((3, -1)\)

Write the solution as an ordered pair.

**C**

\[
\begin{align*}
x + 4y &= 6 \\
x + y &= 3
\end{align*}
\]

**Step 1** \( x + 4y = 6 \)

Solve the first equation for \( x \) by subtracting \( 4y \) from both sides.

\[
x = 6 - 4y
\]

**Step 2** \( x + y = 3 \)

\( (6 - 4y) + y = 3 \)

**Step 3** \( 6 - 3y = 3 \)

\[
6 - 3y = 3 \\
-3y = -3 \\
\frac{-3y}{-3} = \frac{-3}{-3} \\
y = 1
\]

**Step 4** \( x + y = 3 \)

\( x + 1 = 3 \)

\( x = 2 \)

**Step 5** \((2, 1)\)

Write the solution as an ordered pair.

**Helpful Hint**

Sometimes neither equation is solved for a variable. You can begin by solving either equation for either \( x \) or \( y \).

---

**Check it Out!**

Solve each system by substitution.

1a. \[
\begin{align*}
y &= x + 3 \\
y &= 2x + 5
\end{align*}
\]

1b. \[
\begin{align*}
x &= 2y - 4 \\
x + 8y &= 16
\end{align*}
\]

1c. \[
\begin{align*}
2x + y &= -4 \\
x + y &= -7
\end{align*}
\]

Sometimes you substitute an expression for a variable that has a coefficient. When solving for the second variable in this situation, you can use the Distributive Property.
Using the Distributive Property

Solve \( \begin{cases} 4y - 5x = 9 \\ x - 4y = 11 \end{cases} \) by substitution.

Step 1
\[
\begin{align*}
4y - 5x &= 9 \\
x - 4y &= 11 \\
+ 4y &+ 4y \\
\hline
x &= 4y + 11
\end{align*}
\]
Solve the second equation for \( x \) by adding 4\( y \) to each side.

Step 2
\[
4y - 5(4y + 11) = 9
\]
Substitute 4\( y \) + 11 for \( x \) in the first equation.

Step 3
\[
\begin{align*}
4y - 5(4y) - 5(11) &= 9 \\
4y - 20y - 55 &= 9 \\
-16y - 55 &= 9 \\
+ 55 &+ 55 \\
\hline
-16y &= 64 \\
-16y &= 64 \\
-16 &= -16 \\
y &= -4
\end{align*}
\]
Distribute –5 to the expression in parentheses. Simplify. Solve for \( y \).

Step 4
\[
\begin{align*}
x - 4y &= 11 \\
x - 4(-4) &= 11 \\
x + 16 &= 11 \\
\hline
x &= -5
\end{align*}
\]
Write one of the original equations. Subtract 16 from both sides.

Step 5
\( (-5, -4) \)
Write the solution as an ordered pair.

2. Solve \( \begin{cases} -2x + y = 8 \\ 3x + 2y = 9 \end{cases} \) by substitution.

**Caution!**
When you solve one equation for a variable, you must substitute the value or expression into the other original equation, not the one that has just been solved.

**Solving Systems by Substitution**
I always look for a variable with a coefficient of 1 or –1 when deciding which equation to solve for \( x \) or \( y \).

**Erika Chu**
Terrell High School

I would solve the first equation for \( y \) because it has a coefficient of 1.

\[
\begin{align*}
2x + y &= 14 \\
y &= -2x + 14
\end{align*}
\]
Then I use substitution to find the values of \( x \) and \( y \).

\[
\begin{align*}
-3x + 4y &= -10 \\
-3x + 4(-2x + 14) &= -10 \\
-3x + (-8x) + 56 &= -10 \\
-11x + 56 &= -10 \\
-11x &= -66 \\
x &= 6 \\
y &= -2x + 14 \\
y &= -2(6) + 14 = 2
\end{align*}
\]
The solution is (6, 2).
**Consumer Economics Application**

One high-speed Internet provider has a $50 setup fee and costs $30 per month. Another provider has no setup fee and costs $40 per month.

a. In how many months will both providers cost the same? What will that cost be?

Write an equation for each option. Let \( t \) represent the total amount paid and \( m \) represent the number of months.

<table>
<thead>
<tr>
<th>Total paid</th>
<th>is</th>
<th>setup fee</th>
<th>plus</th>
<th>cost per month</th>
<th>times</th>
<th>months.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Option 1</td>
<td>( t )</td>
<td>=</td>
<td>50</td>
<td>+</td>
<td>30</td>
<td>( \cdot )</td>
</tr>
<tr>
<td>Option 2</td>
<td>( t )</td>
<td>=</td>
<td>0</td>
<td>+</td>
<td>40</td>
<td>( \cdot )</td>
</tr>
</tbody>
</table>

**Step 1** \( t = 50 + 30m \)  
**Both equations are solved for \( t \).**

**Step 2** \( 50 + 30m = 40m \)  
**Substitute 50 + 30m for \( t \) in the second equation.**

**Step 3** \( \frac{-30m}{50} = \frac{-30m}{10} \)  
**Solve for \( m \). Subtract 30m from both sides.**

\[
\frac{50}{10} = \frac{10m}{10} \\
5 = m
\]

**Step 4** \( t = 40m \)  
**Write one of the original equations.**

\[
= 40(5) \\
= 200
\]

**Step 5** \( (5, 200) \)  
**Write the solution as an ordered pair.**

In 5 months, the total cost for each option will be the same—$200.

b. If you plan to cancel in 1 year, which is the cheaper provider? Explain.

Option 1: \( t = 50 + 30(12) = 410 \)  
Option 2: \( t = 40(12) = 480 \)

Option 1 is cheaper.

---

3. One cable television provider has a $60 setup fee and charges $80 per month, and another provider has a $160 equipment fee and charges $70 per month.

a. In how many months will the cost be the same? What will that cost be?

b. If you plan to move in 6 months, which is the cheaper option? Explain.

---

**THINK AND DISCUSS**

1. If you graphed the equations in Example 1A, where would the lines intersect?

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, solve the system by substitution using the first step given. Show that each method gives the same solution.
GUIDED PRACTICE
Solve each system by substitution.

1. \[\begin{align*}
    y &= 5x - 10 \\
    y &= 3x + 8 \\
    x - 2y &= 10 \\
    \frac{1}{2}x - 2y &= 4
\end{align*}\]

2. \[\begin{align*}
    3x + y &= 2 \\
    4x + y &= 20
\end{align*}\]

3. \[\begin{align*}
    y &= x + 5 \\
    4x + y &= 20
\end{align*}\]

4. \[\begin{align*}
    y &= 3x + 8 \\
    x - 2y &= 10 \\
    \frac{1}{2}x - 2y &= 4
\end{align*}\]

5. \[\begin{align*}
    y - 4x &= 3 \\
    2x - 3y &= 21
\end{align*}\]

6. \[\begin{align*}
    x &= y - 8 \\
    -x - y &= 0
\end{align*}\]

7. **Consumer Economics** The Strauss family is deciding between two lawn-care services. Green Lawn charges a $49 startup fee, plus $29 per month. Grass Team charges a $25 startup fee, plus $37 per month.
   a. In how many months will both lawn-care services cost the same? What will that cost be?
   b. If the family will use the service for only 6 months, which is the better option? Explain.

PRACTICE AND PROBLEM SOLVING
Solve each system by substitution.

8. \[\begin{align*}
    y &= x + 3 \\
    y &= 2x + 4
\end{align*}\]

9. \[\begin{align*}
    y &= 2x + 10 \\
    y &= -2x - 6
\end{align*}\]

10. \[\begin{align*}
    x + 2y &= 8 \\
    x + 3y &= 12
\end{align*}\]

11. \[\begin{align*}
    2x + 2y &= 2 \\
    -4x + 4y &= 12
\end{align*}\]

12. \[\begin{align*}
    y &= 0.5x + 2 \\
    -y &= -2x + 4
\end{align*}\]

13. \[\begin{align*}
    -x + y &= 4 \\
    3x - 2y &= -7
\end{align*}\]

14. \[\begin{align*}
    3x + y &= -8 \\
    -2x - y &= 6
\end{align*}\]

15. \[\begin{align*}
    x + 2y &= -1 \\
    4x - 4y &= 20
\end{align*}\]

16. \[\begin{align*}
    4x &= y - 1 \\
    6x - 2y &= -3
\end{align*}\]

17. **Recreation** Casey wants to buy a gym membership. One gym has a $150 joining fee and costs $35 per month. Another gym has no joining fee and costs $60 per month.
   a. In how many months will both gym memberships cost the same? What will that cost be?
   b. If Casey plans to cancel in 5 months, which is the better option for him? Explain.

Solve each system by substitution. Check your answer.

18. \[\begin{align*}
    x &= 5 \\
    x + y &= 8
\end{align*}\]

19. \[\begin{align*}
    y &= -3x + 4 \\
    x &= 2y + 6
\end{align*}\]

20. \[\begin{align*}
    3x - y &= 11 \\
    5y - 7x &= 1
\end{align*}\]

21. \[\begin{align*}
    \frac{1}{2}x + \frac{1}{3}y &= 6 \\
    x - y &= 2
\end{align*}\]

22. \[\begin{align*}
    x &= 7 - 2y \\
    2x + y &= 5
\end{align*}\]

23. \[\begin{align*}
    y &= 1.2x - 4 \\
    2.2x + 5 &= y
\end{align*}\]

24. The sum of two numbers is 50. The first number is 43 less than twice the second number. Write and solve a system of equations to find the two numbers.

25. **Money** A jar contains \(n\) nickels and \(d\) dimes. There are 20 coins in the jar, and the total value of the coins is $1.40. How many nickels and how many dimes are in the jar? (Hint: Nickels are worth $0.05 and dimes are worth $0.10.)
26. **Multi-Step** Use the receipts below to write and solve a system of equations to find the cost of a large popcorn and the cost of a small drink.

![Receipts](image)

27. **Finance** Helene invested a total of $1000 in two simple-interest bank accounts. One account paid 5% annual interest; the other paid 6% annual interest. The total amount of interest she earned after one year was $58. Write and solve a system of equations to find the amount invested in each account. (*Hint: Change the interest rates into decimals first.*)

**Geometry** Two angles whose measures have a sum of 90° are called complementary angles. For Exercises 28–30, *x* and *y* represent the measures of complementary angles. Use this information and the equation given in each exercise to find the measure of each angle.

28. \( y = 4x - 10 \)

29. \( x = 2y \)

30. \( y = 2(x - 15) \)

31. **Aviation** With a headwind, a small plane can fly 240 miles in 3 hours. With a tailwind, the plane can fly the same distance in 2 hours. Follow the steps below to find the rates of the plane and wind.

   a. Copy and complete the table. Let *p* be the rate of the plane and *w* be the rate of the wind.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Time</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p - w )</td>
<td>( _ )</td>
<td>( 240 )</td>
</tr>
<tr>
<td>( p + w )</td>
<td>( 2 )</td>
<td>( _ )</td>
</tr>
</tbody>
</table>

   b. Use the information in each row to write a system of equations.

   c. Solve the system of equations to find the rates of the plane and wind.

32. **Write About It** Explain how to solve a system of equations by substitution.

33. **Critical Thinking** Explain the connection between the solution of a system solved by graphing and the solution of the same system solved by substitution.

34. At the school store, Juanita bought 2 books and a backpack for a total of $26 before tax. Each book cost $8 less than the backpack.

   a. Write a system of equations that can be used to find the price of each book and the price of the backpack.

   b. Solve this system by substitution.

   c. Solve this system by graphing. Discuss advantages and disadvantages of solving by substitution and solving by graphing.
35. **Estimation** Use the graph to estimate the solution to
\[
\begin{align*}
2x - y &= 6 \\
x + y &= -0.6
\end{align*}
\]
Round your answer to the nearest tenth.
Then solve the system by substitution.

36. Elizabeth met 24 of her cousins at a family reunion. The number of male cousins \(m\) was 6 less than twice the number of female cousins \(f\). Which system can be used to find the number of male cousins and female cousins?

A) \[
\begin{align*}
m + f &= 24 \\
f &= 2m - 6
\end{align*}
\]

B) \[
\begin{align*}
m + f &= 24 \\
f &= 2m
\end{align*}
\]

C) \[
\begin{align*}
m &= 24 + f \\
m &= f - 6
\end{align*}
\]

D) \[
\begin{align*}
f &= 24 - m \\
m &= 2f - 6
\end{align*}
\]

37. Which problem is best represented by the system \[
\begin{align*}
d &= n + 5 \\
d + n &= 12
\end{align*}
\]

F) Roger has 12 coins in dimes and nickels. There are 5 more dimes than nickels.

G) Roger has 5 coins in dimes and nickels. There are 12 more dimes than nickels.

H) Roger has 12 coins in dimes and nickels. There are 5 more nickels than dimes.

J) Roger has 5 coins in dimes and nickels. There are 12 more nickels than dimes.

**CHALLENGE AND EXTEND**

38. A car dealership has 378 cars on its lot. The ratio of new cars to used cars is 5:4. Write and solve a system of equations to find the number of new and used cars on the lot.

Solve each system by substitution.

39. \[
\begin{align*}
2r - 3s - t &= 12 \\
s + 3t &= 10 \\
t &= 4
\end{align*}
\]

40. \[
\begin{align*}
x + y + z &= 7 \\
y + z &= 5 \\
2y - 4z &= -14
\end{align*}
\]

41. \[
\begin{align*}
a + 2b + c &= 19 \\
-b + c &= -5 \\
3b + 2c &= 15
\end{align*}
\]
Objectives
Solve systems of linear equations in two variables by elimination. Compare and choose an appropriate method for solving systems of linear equations.

Why learn this?
You can solve a system of linear equations to determine how many flowers of each type you can buy to make a bouquet. (See Example 4.)

Another method for solving systems of equations is elimination. Like substitution, the goal of elimination is to get one equation that has only one variable.

Remember that an equation stays balanced if you add equal amounts to both sides. Consider the system
\[
\begin{align*}
    x - 2y & = 19 \\
    5x + 2y & = 1 
\end{align*}
\]
Since \(5x + 2y = 1\), you can add \(5x + 2y\) to one side of the first equation and 1 to the other side and the balance is maintained.

\[
\begin{align*}
    x - 2y & + 5x + 2y \\
    6x + 0 & = -18
\end{align*}
\]

Since \(-2y\) and \(2y\) have opposite coefficients, you can eliminate the \(y\) by adding the two equations. The result is one equation that has only one variable: \(6x = -18\).

When you use the elimination method to solve a system of linear equations, align all like terms in the equations. Then determine whether any like terms can be eliminated because they have opposite coefficients.

Know it! Note

<table>
<thead>
<tr>
<th>Solving Systems of Equations by Elimination</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong> Write the system so that like terms are aligned.</td>
</tr>
<tr>
<td><strong>Step 2</strong> Eliminate one of the variables and solve for the other variable.</td>
</tr>
<tr>
<td><strong>Step 3</strong> Substitute the value of the variable into one of the original equations and solve for the other variable.</td>
</tr>
<tr>
<td><strong>Step 4</strong> Write the answers from Steps 2 and 3 as an ordered pair, ((x, y)), and check.</td>
</tr>
</tbody>
</table>

Later in this lesson you will learn how to multiply one or more equations by a number in order to produce opposites that can be eliminated.
**EXAMPLE 1**

**Elimination Using Addition**

Solve \( \begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases} \) by elimination.

**Step 1**

Write the system so that like terms are aligned.

\[ \begin{align*}
\begin{array}{r}
\text{Step 1} \\
 & x - 2y = -19 \\
& + 5x + 2y = 1 \\
\end{array}
\end{align*} \]

Notice that \(-2y\) and \(2y\) are opposites.

**Step 2**

Add the equations to eliminate \(y\).

\[ \begin{align*}
\begin{array}{r}
\text{Step 2} \\
6x + 0 = -18
\end{array}
\end{align*} \]

Simplify and solve for \(x\).

\[ \begin{align*}
6x &= -18 \\
x &= -3
\end{align*} \]

Divide both sides by 6.

**Step 3**

Write one of the original equations.

\[ \begin{align*}
\begin{array}{r}
\text{Step 3} \\
-3 - 2y = -19
\end{array}
\end{align*} \]

Substitute \(-3\) for \(x\).

\[ \begin{align*}
\begin{array}{r}
\text{Step 3} \\
+ 3
\end{array}
\end{align*} \]

Add 3 to both sides.

\[ \begin{align*}
-2y &= -16 \\
y &= 8
\end{align*} \]

Divide both sides by \(-2\).

**Step 4**

Write the solution as an ordered pair.

\[ (-3, 8) \]

**Check It Out!**

1. Solve \( \begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases} \) by elimination. Check your answer.

When two equations each contain the same term, you can subtract one equation from the other to solve the system. To subtract an equation, add the opposite of each term.

**EXAMPLE 2**

**Elimination Using Subtraction**

Solve \( \begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases} \) by elimination.

**Step 1**

Notice that both equations contain \(4y\).

\[ \begin{align*}
\begin{array}{r}
\text{Step 1} \\
3x + 4y = 18 \\
- ( -2x + 4y = 8 )
\end{array}
\end{align*} \]

Add the opposite of each term in the second equation.

\[ \begin{align*}
\begin{array}{r}
\text{Step 1} \\
3x + 4y = 18 \\
+ 2x - 4y = -8
\end{array}
\end{align*} \]

**Step 2**

Eliminate \(y\).

\[ \begin{align*}
\begin{array}{r}
\text{Step 2} \\
5x + 0 = 10
\end{array}
\end{align*} \]

Simplify and solve for \(x\).

\[ \begin{align*}
x &= 2
\end{align*} \]

**Step 3**

Write one of the original equations.

\[ \begin{align*}
\begin{array}{r}
\text{Step 3} \\
-2(2) + 4y = 8
\end{array}
\end{align*} \]

Substitute \(2\) for \(x\).

\[ \begin{align*}
\begin{array}{r}
\text{Step 3} \\
-4 + 4y = 8
\end{array}
\end{align*} \]

Add 4 to both sides.

\[ \begin{align*}
4y &= 12 \\
y &= 3
\end{align*} \]

Simplify and solve for \(y\).

**Step 4**

Write the solution as an ordered pair.

\[ (2, 3) \]

Remember to check by substituting your answer into both original equations.
2. Solve \[
\begin{align*}
3x + 3y &= 15 \\
-2x + 3y &= -5
\end{align*}
\] by elimination. Check your answer.

In some cases, you will first need to multiply one or both of the equations by a number so that one variable has opposite coefficients.

**Example 3**

**Elimination Using Multiplication First**

Solve each system by elimination.

**A**
\[
\begin{align*}
2x + y &= 3 \\
-x + 3y &= -12
\end{align*}
\]

**Step 1**
Multiply each term in the second equation by 2 to get opposite x-coefficients.

\[
\begin{align*}
2x + y &= 3 \\
+2(-x + 3y &= -12) \\
\rightarrow 2x + y &= 3 \\
+(-2x + 6y &= -24)
\end{align*}
\]

Add the new equation to the first equation to eliminate x.

\[
\begin{align*}
2x + y &= 3 \\
\rightarrow 7y &= -21 \\
\rightarrow y &= -3
\end{align*}
\]

**Step 2**
Solve for y.

**Step 3**
Write one of the original equations.

\[
\begin{align*}
2x + y &= 3 \\
2x + (-3) &= 3 \\
\rightarrow +3 + 3 \\
2x &= 6 \\
x &= 3
\end{align*}
\]

Add 3 to both sides.

**Step 4**
Write the solution as an ordered pair.

\((3, -3)\)

**B**
\[
\begin{align*}
7x - 12y &= -22 \\
5x - 8y &= -14
\end{align*}
\]

**Step 1**
Multiply the first equation by 2 and the second equation by -3 to get opposite y-coefficients.

\[
\begin{align*}
2(7x - 12y &= -22) \\
+(-3)(5x - 8y &= -14) \\
\rightarrow 14x - 24y &= -44 \\
+(-15x + 24y &= 42)
\end{align*}
\]

Add the new equations to eliminate y.

\[
\begin{align*}
-x &= -2 \\
x &= 2
\end{align*}
\]

**Step 2**
Solve for x.

**Step 3**
Write one of the original equations.

\[
\begin{align*}
7x - 12y &= -22 \\
7(2) - 12y &= -22 \\
14 - 12y &= -22 \\
-14 - 14 \\
-12y &= -36 \\
y &= 3
\end{align*}
\]

Subtract 14 from both sides.

**Step 4**
Write the solution as an ordered pair.

\((2, 3)\)

Solve each system by elimination. Check your answer.

3a. \[
\begin{align*}
3x + 2y &= 6 \\
-x + y &= -2
\end{align*}
\]

3b. \[
\begin{align*}
2x + 5y &= 26 \\
-3x - 4y &= -25
\end{align*}
\]
**Consumer Economics Application**

Sam spent $24.75 to buy 12 flowers for his mother. The bouquet contained roses and daisies. How many of each type of flower did Sam buy?

Write a system. Use \( r \) for the number of roses and \( d \) for the number of daisies.

\[
\begin{align*}
2.50r + 1.75d &= 24.75 & \text{The cost of roses and daisies totals$24.75.}\1.75 \times 12 &= 21.00 & \text{The total number of roses and daisies is 12.}\end{align*}
\]

**Step 1**

\[
\begin{align*}
2.50r + 1.75d &= 24.75 & \text{Multiply the second equation by } -2.50 \text{ to get opposite } r\text{-coefficients.}\1.75 \times 12 &= 21.00 & \text{Add this equation to the first equation to eliminate } r.\end{align*}
\]

**Step 2**

\[
\begin{align*}
-0.75d &= -5.25 & \text{Solve for } d.\end{align*}
\]

\[
d = 7
\]

**Step 3**

\[
\begin{align*}
r + d &= 12 & \text{Write one of the original equations.}\r + 7 &= 12 & \text{Substitute 7 for } d.\r - 7 &= -7 & \text{Subtract 7 from both sides.}\r \quad &= \quad 5
\end{align*}
\]

**Step 4**

\[
(5, 7) \quad \text{Write the solution as an ordered pair.}
\]

Sam can buy 5 roses and 7 daisies.

**4. What if...?** Sally spent $14.85 to buy 13 flowers. She bought lilies, which cost $1.25 each, and tulips, which cost $0.90 each. How many of each flower did Sally buy?

All systems can be solved in more than one way. For some systems, some methods may be better than others.

**Systems of Linear Equations**

<table>
<thead>
<tr>
<th>METHOD</th>
<th>USE WHEN...</th>
<th>EXAMPLE</th>
</tr>
</thead>
</table>
| Graphing  | • Both equations are solved for y.               | \( \begin{align*} y &= 3x + 2 \\
y &= -2x + 6 \end{align*} \) |
|           | • You want to estimate a solution.               |                                |
| Substitution | • A variable in either equation has a coefficient of 1 or -1. | \( \begin{align*} x + 2y &= 7 \\
x &= 10 - 5y \end{align*} \) or \( \begin{align*} x &= 2y + 10 \\
x &= 3y + 5 \end{align*} \) |
|           | • Both equations are solved for the same variable. |                                |
|           | • Either equation is solved for a variable.     |                                |
| Elimination | • Both equations have the same variable with the same or opposite coefficients. | \( \begin{align*} 3x + 2y &= 8 \\
5x + 2y &= 12 \end{align*} \) or \( \begin{align*} 6x + 5y &= 10 \\
3x + 2y &= 15 \end{align*} \) |
|           | • A variable term in one equation is a multiple of the corresponding variable term in the other equation. |                                |
THINK AND DISCUSS

1. Explain how multiplying the second equation in a system by −1 and eliminating by adding is the same as elimination by subtraction. Give an example of a system for which this applies.

2. Explain why it does not matter which variable you solve for first when solving a system by elimination.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example of a system of equations that you could solve using the given method.

GUIDED PRACTICE

Solve each system by elimination. Check your answer.

SEE EXAMPLE 1

1. \[
\begin{align*}
-x + y &= 5 \\
x - 5y &= -9
\end{align*}
\]

2. \[
\begin{align*}
x + y &= 12 \\
x - y &= 2
\end{align*}
\]

3. \[
\begin{align*}
2x + 5y &= -24 \\
3x - 5y &= 14
\end{align*}
\]

SEE EXAMPLE 2

4. \[
\begin{align*}
x - 10y &= 60 \\
x + 14y &= 12
\end{align*}
\]

5. \[
\begin{align*}
5x + y &= 0 \\
5x + 2y &= 30
\end{align*}
\]

6. \[
\begin{align*}
-5x + 7y &= 11 \\
-5x + 3y &= 19
\end{align*}
\]

SEE EXAMPLE 3

7. \[
\begin{align*}
2x + 3y &= 12 \\
5x - y &= 13
\end{align*}
\]

8. \[
\begin{align*}
-3x + 4y &= 12 \\
2x + y &= -8
\end{align*}
\]

9. \[
\begin{align*}
2x + 4y &= -4 \\
3x + 5y &= -3
\end{align*}
\]

SEE EXAMPLE 4

10. Consumer Economics Each family in a neighborhood is contributing $20 worth of food to the neighborhood picnic. The Harlin family is bringing 12 packages of buns. The hamburger buns cost $2.00 per package. The hot-dog buns cost $1.50 per package. How many packages of each type of bun did they buy?

PRACTICE AND PROBLEM SOLVING

Solve each system by elimination. Check your answer.

11. \[
\begin{align*}
-x + y &= -1 \\
2x - y &= 0
\end{align*}
\]

12. \[
\begin{align*}
-2x + y &= -20 \\
2x + y &= 48
\end{align*}
\]

13. \[
\begin{align*}
3x - y &= -2 \\
-2x + y &= 3
\end{align*}
\]

14. \[
\begin{align*}
x - y &= 4 \\
x - 2y &= 10
\end{align*}
\]

15. \[
\begin{align*}
x + 2y &= 5 \\
3x + 2y &= 17
\end{align*}
\]

16. \[
\begin{align*}
3x - 2y &= -1 \\
3x - 4y &= 9
\end{align*}
\]

17. \[
\begin{align*}
x - y &= -3 \\
5x + 3y &= 1
\end{align*}
\]

18. \[
\begin{align*}
9x - 3y &= 3 \\
3x + 8y &= -17
\end{align*}
\]

19. \[
\begin{align*}
5x + 2y &= -1 \\
3x + 7y &= 11
\end{align*}
\]

20. Multi-Step Mrs. Gonzalez bought centerpieces to put on each table at a graduation party. She spent $31.50. There are 8 tables each requiring either a candle or vase. Candles cost $3 and vases cost $4.25. How many of each type did she buy?
21. **Geometry** The difference between the length and width of a rectangle is 2 units. The perimeter is 40 units. Write and solve a system of equations to determine the length and width of the rectangle. (Hint: The perimeter of a rectangle is \(2\ell + 2w\).)

22. //ERROR ANALYSIS// Which is incorrect? Explain the error.

\[
\begin{align*}
A: & \quad \begin{cases} \ x + y = -3 \\ 3x + y = 3 \end{cases} \\
& \quad \begin{cases} \ x + y = -3 \\ -2x = 0 \end{cases}
\end{align*}
\]

\[
\begin{align*}
B: & \quad \begin{cases} \ x + y = -3 \\ 3x + y = 3 \end{cases} \\
& \quad \begin{cases} \ x + y = -3 \\ -2x = -6 \end{cases}
\end{align*}
\]

23. **Chemistry** A chemist has a bottle of a 1% acid solution and a bottle of a 5% acid solution. She wants to mix the two solutions to get 100 mL of a 4% acid solution. Follow the steps below to find how much of each solution she should use.

<table>
<thead>
<tr>
<th>(\text{Amount of Solution (mL)})</th>
<th>(\text{1% Solution})</th>
<th>(\text{5% Solution})</th>
<th>(\text{4% Solution})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>(0.01x)</td>
<td>(y)</td>
<td>(0.04(100))</td>
</tr>
</tbody>
</table>

a. Copy and complete the table.
b. Use the information in the table to write a system of equations.
c. Solve the system of equations to find how much she will use from each bottle to get 100 mL of a 4% acid solution.

**Critical Thinking** Which method would you use to solve each system? Explain.

24. \(\begin{cases} \frac{1}{2}x - 5y = 30 \\ \frac{1}{2}x + 7y = 6 \end{cases}\)

25. \(\begin{cases} -x + 2y = 3 \\ 4x - 5y = -3 \end{cases}\)

26. \(\begin{cases} 3x - y = 10 \\ 2x - y = 7 \end{cases}\)

27. \(\begin{cases} 3y + x = 10 \\ x = 4y + 2 \end{cases}\)

28. \(\begin{cases} y = -4x \\ y = 2x + 3 \end{cases}\)

29. \(\begin{cases} 2x + 6y = 12 \\ 4x + 5y = 15 \end{cases}\)

30. **Business** A local boys club sold 176 bags of mulch and made a total of $520. They did not sell any of the expensive cocoa mulch. Use the table to determine how many bags of each type of mulch they sold.

<table>
<thead>
<tr>
<th>(\text{Mulch Prices ($)})</th>
<th>(\text{Cocoa})</th>
<th>(\text{Hardwood})</th>
<th>(\text{Pine Bark})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{Cocoa})</td>
<td>4.75</td>
<td>3.50</td>
<td>2.75</td>
</tr>
</tbody>
</table>

31. a. The school store is running a promotion on school supplies. Different supplies are placed on two shelves. You can purchase 3 items from shelf A and 2 from shelf B for $16. Or you can purchase 2 items from shelf A and 3 from shelf B for $14. Write a system of equations that can be used to find the individual prices for the supplies on shelf A and on shelf B.

b. Solve the system of equations by elimination.

c. If the supplies on shelf A are normally $6 each and the supplies on shelf B are normally $3 each, how much will you save on each package plan from part a?
32. **Write About It** Solve the system \[\begin{align*}
3x + y &= 1 \\
2x + 4y &= -6
\end{align*}\]. Explain how you can check your solution algebraically and graphically.

33. **Test Prep**

A math test has 25 problems. Some are worth 2 points, and some are worth 3 points. The test is worth 60 points total. Which system can be used to determine the number of 2-point problems and the number of 3-point problems on the test?

A \[\begin{align*}
x + y &= 25 \\
2x + 3y &= 60
\end{align*}\]

B \[\begin{align*}
x + y &= 60 \\
2x + 3y &= 25
\end{align*}\]

C \[\begin{align*}
x - y &= 25 \\
2x + 3y &= 60
\end{align*}\]

D \[\begin{align*}
x - y &= 60 \\
2x - 3y &= 25
\end{align*}\]

34. An electrician charges $15 plus $11 per hour. Another electrician charges $10 plus $15 per hour. For what amount of time will the cost be the same? What is that cost?

A 1 hour; $25

B 1 \(\frac{3}{4}\) hours; $28.75

C 1 \(\frac{1}{2}\) hours; $30

D 1 \(\frac{3}{4}\) hours; $32.50

35. **Short Response**

Three hundred fifty-eight tickets to the school basketball game on Friday were sold. Student tickets were $1.50, and nonstudent tickets were $3.25. The school made $752.25.

a. Write a system of linear equations that could be used to determine how many student and how many nonstudent tickets were sold. Define the variables you use.

b. Solve the system you wrote in part a. How many student and how many nonstudent tickets were sold?

**CHALLENGE AND EXTEND**

Solve each system by any method.

36. \[\begin{align*}
x + 16 \frac{1}{2} &= - \frac{3}{4}y \\
y &= \frac{1}{2}x
\end{align*}\]

37. \[\begin{align*}
2x + y + z &= 17 \\
\frac{1}{2}z &= 5 \\
x - y &= 5
\end{align*}\]

38. \[\begin{align*}
x - 2y - z &= -1 \\
-x + 2y + 4z &= -11 \\
2x + y + z &= 1
\end{align*}\]

39. The sum of the digits of a two-digit number is 5. If the number is multiplied by 3, the result is 42. Write and solve a system of equations to find the number. *(Hint: One equation involves the digits in the number. The other equation involves the values of the digits.)*
Solving Classic Problems

You can use systems of linear equations to solve some “classic” math problems that are common in textbooks and puzzle books.

Example 1

Yuri is twice as old as Zack. Four years from now, the sum of their ages will be 23. How old is Yuri?

Step 1  Write a system. Let \( y \) represent Yuri’s age. Let \( z \) represent Zack’s age.

Yuri is twice as old as Zack. In 4 years, the sum of their ages will be 23.

\[
\begin{align*}
y &= 2z \\
y + z + 8 &= 23 \\
\end{align*}
\]

Step 2  Solve the equations for \( y \).

\[
\begin{cases}
y = 2z \\
y + z = 15
\end{cases} 
\Rightarrow
\begin{cases}
y = 2z \\
y = -z + 15
\end{cases}
\]

Step 3  Graph \( y = 2z \) and \( y = -z + 15 \).

The lines appear to intersect at \((5, 10)\). The solution \((5, 10)\) means that Yuri is 10 years old.

Example 2

Mandy has 11 coins in dimes and quarters. The value of her coins is $2.15. How many dimes does she have?

Step 1  Write a system. Let \( d \) be the number of dimes. Let \( q \) be the number of quarters.

The total number of coins is 11. The value of the coins is $2.15.

\[
\begin{align*}
q + d &= 11 \\
0.25q + 0.10d &= 2.15 \\
100(0.25q + 0.10d) &= 215 \\
25q + 10d &= 215 \\
5q + 2d &= 43
\end{align*}
\]

Step 2  Solve the first equation for \( d \).

\[
d = 11 - q
\]

Step 3  Substitute \( 11 - q \) for \( d \) in the second equation.

\[
5q + 2(11 - q) = 43
\]

Distribute 2 and then combine like terms.

\[
3q + 22 = 43
\]

\[
3q = 21
\]

\[
q = 7
\]

Solve for \( q \).

\[
Chapter 7 Systems of Equations and Inequalities
\]
Try This

1. The sum of the digits of a two-digit number is 17. When the digits are reversed, the new number is 9 more than the original number. What is the original number?

2. Vic has 14 coins in nickels and quarters. The value of his coins is $1.70. How many quarters does he have?

3. Grace is 8 years older than her brother Sam. The sum of their ages is 24. How old is Grace?
Solving Special Systems

When two lines intersect at a point, there is exactly one solution to the system. A system with at least one solution is a **consistent system**.

When the two lines in a system do not intersect, they are parallel lines. There are no ordered pairs that satisfy both equations, so there is no solution. A system that has no solution is an **inconsistent system**.

**Example 1**

**Systems with No Solution**

Show that \( \begin{cases} y = x - 1 \\ -x + y = 2 \end{cases} \) has no solution.

**Method 1** Compare slopes and \( y \)-intercepts.

\[
y = x - 1 \quad \rightarrow \quad y = 1x - 1 \\
-x + y = 2 \quad \rightarrow \quad y = 1x + 2
\]

Write both equations in slope-intercept form. The lines are parallel because they have the same slope and different \( y \)-intercepts.

This system has no solution.

**Method 2** Graph the system.

The lines are parallel.

This system has no solution.

**Method 3** Solve the system algebraically. Use the substitution method.

\[
-x + (x - 1) = 2 \\
-1 = 2x
\]

Substitute \( x - 1 \) for \( y \) in the second equation, and solve. False

This system has no solution.

1. Show that \( \begin{cases} y = -2x + 5 \\ 2x + y = 1 \end{cases} \) has no solution.
If two linear equations in a system have the same graph, the graphs are coincident lines, or the same line. There are infinitely many solutions of the system because every point on the line represents a solution of both equations.

**Example 2** Systems with Infinitely Many Solutions

Show that \( \begin{cases} y = 2x + 1 \\ 2x - y + 1 = 0 \end{cases} \) has infinitely many solutions.

**Method 1** Compare slopes and \( y \)-intercepts.

\[
\begin{align*}
y &= 2x + 1 \\
2x - y + 1 &= 0
\end{align*}
\]

Write both equations in slope-intercept form. The lines have the same slope and the same \( y \)-intercept.

If this system were graphed, the graphs would be the same line. There are infinitely many solutions.

**Method 2** Solve the system algebraically. Use the elimination method.

\[
\begin{align*}
y &= 2x + 1 \\
2x - y + 1 &= 0
\end{align*}
\]

Write equations to line up like terms. Add the equations.

\[
\begin{align*}
y &= 2x + 1 \\
2x - y + 1 &= 0 \\
\underline{+2x - y = -1}
\end{align*}
\]

\[
\begin{align*}
0 &= 0 \\. \checkmark
\end{align*}
\]

There are infinitely many solutions.

2. Show that \( \begin{cases} y = x - 3 \\ x - y - 3 = 0 \end{cases} \) has infinitely many solutions.

**Classification of Systems of Linear Equations**

<table>
<thead>
<tr>
<th>CLASSIFICATION</th>
<th>CONSISTENT AND INDEPENDENT</th>
<th>CONSISTENT AND DEPENDENT</th>
<th>INCONSISTENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Solutions</td>
<td>Exactly one</td>
<td>Infinitely many</td>
<td>None</td>
</tr>
<tr>
<td>Description</td>
<td>Different slopes</td>
<td>Same slope, same ( y )-intercept</td>
<td>Same slope, different ( y )-intercepts</td>
</tr>
<tr>
<td>Graph</td>
<td>Intersecting lines</td>
<td>Coincident lines</td>
<td>Parallel lines</td>
</tr>
</tbody>
</table>


### Example 3

**Classifying Systems of Linear Equations**

Classify each system. Give the number of solutions.

**A**

\[
\begin{align*}
2y &= x + 2 \\
-\frac{1}{2}x + y &= 1
\end{align*}
\]

Write both equations in slope-intercept form.

The lines have the same slope and the same y-intercepts. They are the same.

The system is consistent and dependent. It has infinitely many solutions.

**B**

\[
\begin{align*}
y &= 2(x - 1) \\
y &= x + 1
\end{align*}
\]

Write both equations in slope-intercept form.

The lines have different slopes. They intersect.

The system is consistent and independent. It has one solution.

### Check It Out

Classify each system. Give the number of solutions.

3a. \[ \begin{align*} x + 2y &= -4 \\
-2(y + 2) &= x \end{align*} \]

3b. \[ \begin{align*} y &= -2(x - 1) \\
y &= -x + 3 \end{align*} \]

3c. \[ \begin{align*} 2x - 3y &= 6 \\
y &= \frac{2}{3}x \end{align*} \]

### Example 4

**Business Application**

The sales manager at Comics Now is comparing its sales with the sales of its competitor, Dynamo Comics. If the sales patterns continue, will the sales for Comics Now ever equal the sales for Dynamo Comics? Explain.

Use the table to write a system of linear equations. Let \( y \) represent the sales total and \( x \) represent the number of years since 2005.

**Computer Books Sold per Year (thousands)**

<table>
<thead>
<tr>
<th>Year</th>
<th>Comics Now</th>
<th>Dynamo Comics</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>130</td>
<td>180</td>
</tr>
<tr>
<td>2006</td>
<td>170</td>
<td>220</td>
</tr>
<tr>
<td>2007</td>
<td>210</td>
<td>260</td>
</tr>
<tr>
<td>2008</td>
<td>250</td>
<td>300</td>
</tr>
</tbody>
</table>

Use the table to write a system of linear equations. Let \( y \) represent the sales total and \( x \) represent the number of years since 2005.

- **Sales total** equals increase in sales per year times years plus beginning sales.

<table>
<thead>
<tr>
<th>Sales total</th>
<th>equals</th>
<th>increase in sales per year</th>
<th>times years</th>
<th>plus beginning sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comics Now</td>
<td>( y )</td>
<td>40</td>
<td>( x )</td>
<td>130</td>
</tr>
<tr>
<td>Dynamo Comics</td>
<td>( y )</td>
<td>40</td>
<td>( x )</td>
<td>180</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
y &= 40x + 130 \\
y &= 40x + 180
\end{align*}
\]

Both equations are in slope-intercept form.

The lines have the same slope, but different y-intercepts.

The graphs of the two equations are parallel lines, so there is no solution. If the patterns continue, sales for the two companies will never be equal.

### Check It Out!

4. Matt has $100 in a checking account and deposits $20 per month. Ben has $80 in a checking account and deposits $30 per month. Will the accounts ever have the same balance? Explain.
GUIDED PRACTICE

1. Vocabulary A ______ system can be independent or dependent. (consistent or inconsistent)

Show that each system has no solution.

2. \[
\begin{align*}
  y &= x + 1 \\
  -x + y &= 3
\end{align*}
\]

3. \[
\begin{align*}
  3x + y &= 6 \\
  y &= -3x + 2
\end{align*}
\]

4. \[
\begin{align*}
  -y &= 4x + 1 \\
  4x + y &= 2
\end{align*}
\]

Show that each system has infinitely many solutions.

5. \[
\begin{align*}
  y &= -x + 3 \\
  x + y - 3 &= 0
\end{align*}
\]

6. \[
\begin{align*}
  y &= 2x - 4 \\
  2x - y - 4 &= 0
\end{align*}
\]

7. \[
\begin{align*}
  -7x + y &= -2 \\
  7x - y &= 2
\end{align*}
\]

Classify each system. Give the number of solutions.

8. \[
\begin{align*}
  y &= 2x + 3 \\
  -2y &= 2x + 6
\end{align*}
\]

9. \[
\begin{align*}
  y &= -3x - 1 \\
  3x + y &= 1
\end{align*}
\]

10. \[
\begin{align*}
  9y &= 3x + 18 \\
  \frac{1}{3}x - y &= -2
\end{align*}
\]

11. Athletics Micah walks on a treadmill at 4 miles per hour. He has walked 2 miles when Luke starts running at 6 miles per hour on the treadmill next to him. If their rates continue, will Luke’s distance ever equal Micah’s distance? Explain.

PRACTICE AND PROBLEM SOLVING

Show that each system has no solution.

12. \[
\begin{align*}
  y &= 2x - 2 \\
  -2x + y &= 1
\end{align*}
\]

13. \[
\begin{align*}
  x + y &= 3 \\
  y &= -x - 1
\end{align*}
\]

14. \[
\begin{align*}
  x + 2y &= -4 \\
  y &= -\frac{1}{2}x - 4
\end{align*}
\]

15. \[
\begin{align*}
  -6 + y &= 2x \\
  y &= 2x - 36
\end{align*}
\]

Show that each system has infinitely many solutions.

16. \[
\begin{align*}
  y &= -2x + 3 \\
  2x + y - 3 &= 0
\end{align*}
\]

17. \[
\begin{align*}
  y &= x - 2 \\
  x - y - 2 &= 0
\end{align*}
\]

18. \[
\begin{align*}
  x + y &= -4 \\
  y &= -x - 4
\end{align*}
\]

19. \[
\begin{align*}
  -9x - 3y &= -18 \\
  3x + y &= 6
\end{align*}
\]
Classify each system. Give the number of solutions.

20. \[
\begin{align*}
  y &= -x + 5 \\
  x + y &= 5
\end{align*}
\]

21. \[
\begin{align*}
  y &= -3x + 2 \\
  y &= 3x
\end{align*}
\]

22. \[
\begin{align*}
  y - 1 &= 2x \\
  y &= 2x - 1
\end{align*}
\]

23. **Sports** Mandy is skating at 5 miles per hour. Nikki is skating at 6 miles per hour and started 1 mile behind Mandy. If their rates stay the same, will Mandy catch up with Nikki? Explain.

24. **Multi-Step** Photocopier A can print 35 copies per minute. Photocopier B can print 35 copies per minute. Copier B is started and makes 10 copies. Copier A is then started. If the copiers continue, will the number of copies from machine A ever equal the number of copies from machine B? Explain.

25. **Entertainment** One week Trey rented 4 DVDs and 2 video games for $18. The next week he rented 2 DVDs and 1 video game for $9. Find the rental costs for each video game and DVD. Explain your answer.

26. Rosa bought 1 pound of cashews and 2 pounds of peanuts for $10. At the same store, Sabrina bought 2 pounds of cashews and 1 pound of peanuts for $11. Find the cost per pound for cashews and peanuts.

27. **Geology** Pam and Tommy collect geodes. Pam’s parents gave her 2 geodes to start her collection, and she buys 4 every year. Tommy has 2 geodes that were given to him for his birthday. He buys 4 every year. If Pam and Tommy continue to buy the same amount of geodes per year, when will Tommy have as many geodes as Pam? Explain your answer.

28. Use the data given in the tables.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>24</td>
<td>26</td>
<td>28</td>
<td>30</td>
</tr>
</tbody>
</table>

a. Write an equation to describe the data in each table.
b. Graph the system of equations from part a. Describe the graph.
c. How could you have predicted the graph by looking at the equations?
d. **What if…?** Each y-value in the second table increases by 1. How does this affect the graphs of the two equations? How can you tell how the graphs would be affected without actually graphing?

29. **Critical Thinking** Describe the graphs of two equations if the result of solving the system by substitution or elimination is the statement \(1 = 3\).

30. The Crusader pep club is selling team buttons that support the sports teams. They contacted Buttons, Etc. which charges $50 plus $1.10 per button, and Logos, which charges $40 plus $1.10 per button.
a. Write an equation for each company’s cost.
b. Use the system from part a to find when the price for both companies is the same. Explain.
c. What part of the equation should the pep club negotiate to change so that the cost of Buttons, Etc. is the same as Logos? What part of the equation should change in order to get a better price?
31. **ERROR ANALYSIS**  
Student A says there is no solution to the graphed system of equations. Student B says there is one solution. Which student is incorrect? Explain the error.

32. **Write About It**  
Compare the graph of a system that is consistent and independent with the graph of a system that is consistent and dependent.

33. **Test Prep**  
Which of the following classifications fit the following system?

\[
\begin{align*}
2x - y &= 3 \\
6x - 3y &= 9
\end{align*}
\]

A. Inconsistent and independent  
B. Consistent and independent  
C. Inconsistent and dependent  
D. Consistent and dependent

34. Which of the following would be enough information to classify a system of two linear equations?

F. The graphs have the same slope.
G. The y-intercepts are the same.
H. The graphs have different slopes.
I. The y-intercepts are different.

35. **CHALLENGE AND EXTEND**  
What conditions are necessary for the system \(\begin{align*}
y &= 2x + p \\
y &= 2x + q
\end{align*}\) to have infinitely many solutions? no solution?

36. Solve the systems in parts a and b. Use this information to make a conjecture about all solutions that exist for the system in part c.

a. \(\begin{align*}
3x + 4y &= 0 \\
4x + 3y &= 0
\end{align*}\)

b. \(\begin{align*}
2x + 5y &= 0 \\
5x + 2y &= 0
\end{align*}\)

c. \(\begin{align*}
ax + by &= 0 \\
bx + ay &= 0, \text{ for } a > 0, b > 0, a \neq b
\end{align*}\)
Example 1
Solve for $x$ and $y$.

Since the lines are perpendicular, all of the angles are right angles. To write two equations, you can set each expression equal to $90^\circ$.

$$(3x + 2y)^\circ = 90^\circ, \quad (6x - 2y)^\circ = 90^\circ$$

Step 1  $3x + 2y = 90$
          $6x - 2y = 90$

Write the system so that like terms are under one another.

Step 2  $9x + 0 = 180$

Add like terms on each side of the equations.
The $y$-term has been eliminated.

$x = 20$

Divide both sides by 9 to solve for $x$.

Step 3  $3x + 2y = 90$

Write one of the original equations.

$3(20) + 2y = 90$

Substitute 20 for $x$.

$60 + 2y = 90$

Simplify.

$2y = 30$

Subtract 60 from both sides.

$y = 15$

Divide by 2 on both sides.

Step 4  $(20, 15)$

Write the solution as an ordered pair.

Step 5  Check the solution by substituting 20 for $x$ and 15 for $y$ in the original equations.

$$
\begin{array}{c}
3x + 2y = 90 \\
3(20) + 2(15) = 90 \\
60 + 30 = 90 \\
90 = 90 \checkmark
\end{array}
\quad
\begin{array}{c}
6x - 2y = 90 \\
6(20) - 2(15) = 90 \\
120 - 30 = 90 \\
90 = 90 \checkmark
\end{array}
$$

In some cases, before you can do Step 1 you will need to multiply one or both of the equations by a number so that you can eliminate a variable.
Example 2

Solve for \( x \) and \( y \).

\[(2x + 4y)^\circ = 72^\circ \quad \text{Vertical Angles Theorem}\]
\[(5x + 2y)^\circ = 108^\circ \quad \text{Linear Pair Theorem}\]

The equations cannot be added or subtracted to eliminate a variable. Multiply the second equation by \(-2\) to get opposite \( y \)-coefficients.

\[5x + 2y = 108 \rightarrow -2(5x + 2y) = -2(108) \rightarrow -10x - 4y = -216\]

**Step 1**
\[
\begin{align*}
2x + 4y &= 72 \\
-10x - 4y &= -216
\end{align*}
\]
Write the system so that like terms are under one another.

**Step 2**
\[
-8x = -144
\]
Add like terms on both sides of the equations.
The \( y \)-term has been eliminated.

\[x = 18 \quad \text{Divide both sides by } -8 \text{ to solve for } x.\]

**Step 3**
\[
2x + 4y = 72
\]
Write one of the original equations.
\[2(18) + 4y = 72 \quad \text{Substitute 18 for } x.\]
\[36 + 4y = 72 \quad \text{Simplify.}\]
\[4y = 36 \quad \text{Subtract 36 from both sides.}\]
\[y = 9 \quad \text{Divide by 4 on both sides.}\]

**Step 4**
\[(18, 9) \quad \text{Write the solution as an ordered pair.}\]

**Step 5**
Check the solution by substituting 18 for \( x \) and 9 for \( y \) in the original equations.

\[
\begin{array}{c|c|c}
2x + 4y &= 72 & 5x + 2y &= 108 \\ 
3(18) + 4(9) &= 72 & 5(18) + 2(9) &= 108 \\ 
36 + 36 &= 72 & 90 + 18 &= 108 \\ 
72 & 72 & 108 & 108 & \checkmark
\end{array}
\]

Try This

Solve for \( x \) and \( y \).

1.
\[(10x + 4y)^\circ \quad (26x - 4y)^\circ \]

2.
\[(3x + 3y)^\circ \quad (3x + 3y)^\circ \quad 45^\circ \quad (3x + 3y)^\circ \quad (-3x + 17y)^\circ\]

3.
\[(18x + 6y)^\circ \quad (6x + 10y)^\circ \quad 36^\circ \]

4.
\[(32x + 2y)^\circ \quad (32x + 2y)^\circ \quad (19x + 4y)^\circ \quad (19x + 4y)^\circ \]
Objective
Graph and solve linear inequalities in two variables.

Vocabulary
linear inequality
solution of a linear inequality

Who uses this?
Consumers can use linear inequalities to determine how much food they can buy for an event. (See Example 3.)

A linear inequality is similar to a linear equation, but the equal sign is replaced with an inequality symbol. A solution of a linear inequality is any ordered pair that makes the inequality true.

**EXAMPLE 1**
Identifying Solutions of Inequalities
Tell whether the ordered pair is a solution of the inequality.

A \[(7, 3); y < x - 1\]

\[
\begin{align*}
y &< x - 1 \\
3 &< 7 - 1 \\
3 &< 6 \checkmark
\end{align*}
\]

\[(7, 3)\] is a solution.

B \[(4, 5); y > 3x + 2\]

\[
\begin{align*}
y &> 3x + 2 \\
5 &> 3(4) + 2 \\
5 &> 14 \times
\end{align*}
\]

\[(4, 5)\] is not a solution.

Tell whether the ordered pair is a solution of the inequality.

1a. \[(4, 5); y < x + 1\]  
1b. \[(1, 1); y > x - 7\]

A linear inequality describes a region of a coordinate plane called a half-plane. All points in the region are solutions of the linear inequality. The boundary line of the region is the graph of the related equation.

When the inequality is written as \(y \leq \) or \(y \geq\), the points on the boundary line are solutions of the inequality, and the line is solid.

When the inequality is written as \(y > \) or \(y \geq\), the points above the boundary line are solutions of the inequality.

When the inequality is written as \(y < \) or \(y >\), the points below the boundary line are solutions of the inequality.
Graphing Linear Inequalities

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solve the inequality for $y$.</td>
</tr>
<tr>
<td>2</td>
<td>Graph the boundary line. Use a solid line for $\leq$ or $\geq$. Use a dashed line for $&lt;$ or $&gt;$.</td>
</tr>
<tr>
<td>3</td>
<td>Shade the half-plane above the line for $y &gt;$ or $y \geq$. Shade the half-plane below the line for $y &lt;$ or $y \leq$. Check your answer.</td>
</tr>
</tbody>
</table>

**EXAMPLE 2**

Graphing Linear Inequalities in Two Variables

Graph the solutions of each linear inequality.

**A** $y < 3x + 4$

**Step 1** The inequality is already solved for $y$.

**Step 2** Graph the boundary line $y = 3x + 4$. Use a dashed line for $<$. 

**Step 3** The inequality is $<$, so shade below the line.

**Check**

<table>
<thead>
<tr>
<th>$y$</th>
<th>$3(x) + 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$3(0) + 4$</td>
</tr>
<tr>
<td>0</td>
<td>$0 + 4$</td>
</tr>
<tr>
<td>0</td>
<td>$&lt; 4 \checkmark$</td>
</tr>
</tbody>
</table>

Substitute $(0, 0)$ for $(x, y)$ because it is not on the boundary line. The point $(0, 0)$ satisfies the inequality, so the graph is shaded correctly.

**B** $3x + 2y \geq 6$

**Step 1** Solve the inequality for $y$.

\[
3x + 2y \geq 6 \\
3x - \frac{3}{2}x + 3 \\
y \geq \frac{3}{2}x + 3
\]

**Step 2** Graph the boundary line $y = -\frac{3}{2}x + 3$. Use a solid line for $\geq$.

**Step 3** The inequality is $\geq$, so shade above the line.

**Check**

<table>
<thead>
<tr>
<th>$y$</th>
<th>$\frac{3}{2}(x) + 3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{3}{2}(0) + 3$</td>
</tr>
<tr>
<td>0</td>
<td>$0 + 3$</td>
</tr>
<tr>
<td>0</td>
<td>$\geq 3 \times$</td>
</tr>
</tbody>
</table>

A false statement means that the half-plane containing $(0, 0)$ should NOT be shaded. $(0, 0)$ is not one of the solutions, so the graph is shaded correctly.

Graph the solutions of each linear inequality.

2a. $4x - 3y > 12$
2b. $2x - y - 4 > 0$
2c. $y \geq -\frac{2}{3}x + 1$

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Example 3

Consumer Economics Application

Sarah can spend at most $7.50 on vegetables for a party. Broccoli costs $1.25 per bunch and carrots cost $0.75 per package.

a. Write a linear inequality to describe the situation.

Let \( x \) represent the number of bunches of broccoli and let \( y \) represent the number of packages of carrots.

Write an inequality. Use \( \leq \) for “at most.”

\[
\text{Cost of broccoli} + \text{cost of carrots} \leq \text{at most} \ 7.50.
\]

\[
1.25x + 0.75y \leq 7.50
\]

Solve the inequality for \( y \).

\[
1.25x + 0.75y \leq 7.50
\]

\[
100(1.25x + 0.75y) \leq 100(7.50)
\]

\[
125x + 75y \leq 750
\]

\[
-125x
\]

\[
75y \leq 750 - 125x
\]

\[
75
\]

\[
75
\]

\[
y \leq 10 - \frac{5}{3}x
\]

b. Graph the solutions.

Step 1 Since Sarah cannot buy a negative amount of vegetables, the system is graphed only in Quadrant I. Graph the boundary line \( y = -\frac{5}{3}x + 10 \). Use a solid line for \( \leq \).

Step 2 Shade below the line. Sarah must buy whole numbers of bunches or packages. All points on or below the line with whole-number coordinates represent combinations of broccoli and carrots that Sarah can buy.

c. Give two combinations of vegetables that Sarah can buy.

Two different combinations that Sarah could buy for $7.50 or less are 2 bunches of broccoli and 4 packages of carrots, or 3 bunches of broccoli and 5 packages of carrots.

Check It Out!

3. Dirk is going to bring two types of olives to the Honor Society induction and can spend no more than $6. Green olives cost $2 per pound and black olives cost $2.50 per pound.

a. Write a linear inequality to describe the situation.

b. Graph the solutions.

c. Give two combinations of olives that Dirk could buy.
EXAMPLE 4 Writing an Inequality from a Graph

Write an inequality to represent each graph.

A

y-intercept: 2; slope: $-\frac{1}{3}$
Write an equation in slope-intercept form.

\[ y = mx + b \rightarrow y = -\frac{1}{3}x + 2 \]

The graph is shaded below a dashed boundary line.

Replace $=$ with $<$ to write the inequality $y < -\frac{1}{3}x + 2$.

B

y-intercept: $-2$; slope: 5
Write an equation in slope-intercept form.

\[ y = mx + b \rightarrow y = 5x + (-2) \]

The graph is shaded above a solid boundary line.

Replace $=$ with $\geq$ to write the inequality $y \geq 5x - 2$.

C

y-intercept: none; slope: undefined
The graph is a vertical line at $x = -2$.
The graph is shaded on the right side of a solid boundary line.

Replace $=$ with $\geq$ to write the inequality $x \geq -2$.

THINK AND DISCUSS

1. Tell how graphing a linear inequality is the same as graphing a linear equation. Tell how it is different.
2. Explain how you would write a linear inequality from a graph.
3. GET ORGANIZED Copy and complete the graphic organizer.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>$y &lt; 5x + 2$</th>
<th>$y &gt; 7x - 3$</th>
<th>$y \leq 9x + 1$</th>
<th>$y \geq -3x - 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>&lt;</td>
<td>&gt;</td>
<td>≤</td>
<td>≥</td>
</tr>
<tr>
<td>Boundary Line</td>
<td>Dashed</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shading</td>
<td>Below</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

1. **Vocabulary** Can a solution of a linear inequality lie on a dashed boundary line? Explain.

2. Tell whether the ordered pair is a solution of the given inequality.
   2. \((0, 3); y \leq -x + 3\)
   3. \((2, 0); y > -2x - 2\)
   4. \((-2, 1); y < 2x + 4\)

3. Graph the solutions of each linear inequality.
   5. \(y \leq -x\)
   6. \(y > 3x + 1\)
   7. \(-y < -x + 4\)
   8. \(-y \geq x + 1\)

9. **Multi-Step** Jack is making punch with orange juice and pineapple juice. He can make at most 16 cups of punch.
   a. Write an inequality to describe the situation.
   b. Graph the solutions.
   c. Give two combinations of cups of orange juice and pineapple juice that Jack can use in his punch.

10. Write an inequality to represent each graph.
    10. [Graph]
    11. [Graph]

PRACTICE AND PROBLEM SOLVING

Tell whether the ordered pair is a solution of the given inequality.
12. \((2, 3); y \geq 2x + 3\)
13. \((1, -1); y < 3x - 3\)
14. \((0, 7); y > 4x + 7\)

Graph the solutions of each linear inequality.
15. \(y > -2x + 6\)
16. \(-y \geq 2x\)
17. \(x + y \leq 2\)
18. \(x - y \geq 0\)

19. **Multi-Step** Beverly is serving hamburgers and hot dogs at her cookout. Hamburger meat costs $3 per pound, and hot dogs cost $2 per pound. She wants to spend no more than $30.
   a. Write an inequality to describe the situation.
   b. Graph the solutions.
   c. Give two combinations of pounds of hamburger and hot dogs that Beverly can buy.

Write an inequality to represent each graph.
20. [Graph]
21. [Graph]
22. **Business** An electronics store makes $125 profit on every DVD player it sells and $100 on every CD player it sells. The store owner wants to make a profit of at least $500 a day selling DVD players and CD players.

   a. Write a linear inequality to determine the number of DVD players \( x \) and the number of CD players \( y \) that the owner needs to sell to meet his goal.

   b. Graph the linear inequality.

   c. Describe the possible values of \( x \). Describe the possible values of \( y \).

   d. List three combinations of DVD players and CD players that the owner could sell to meet his goal.

   **Graph the solutions of each linear inequality.**

23. \( y \leq 2 - 3x \)

24. \( -y < 7 + x \)

25. \( 2x - y \leq 4 \)

26. \( 3x - 2y > 6 \)

27. **Geometry** Marvin has 18 yards of fencing that he can use to put around a rectangular garden.

   a. Write an inequality to describe the possible lengths and widths of the garden.

   b. Graph the inequality and list three possible solutions to the problem.

   c. What are the dimensions of the largest square garden that can be fenced in with whole-number dimensions?

28. **Hobbies** Stephen wants to buy yellow tangs and clown fish for his saltwater aquarium. He wants to spend no more than $77 on fish. At the store, yellow tangs cost $15 each and clown fish cost $11 each. Write and graph a linear inequality to find the number of yellow tangs \( x \) and the number of clown fish \( y \) that Stephen could purchase. Name a solution of your inequality that is not reasonable for the situation. Explain.

   **Graph each inequality on a coordinate plane.**

29. \( y > 1 \)

30. \( -2 < x \)

31. \( x \geq -3 \)

32. \( y \leq 0 \)

33. \( 0 \geq x \)

34. \( -12 + y > 0 \)

35. \( x + 7 < 7 \)

36. \( -4 \geq x - y \)

37. **School** At a high school football game, tickets at the gate cost $7 per adult and $4 per student. Write a linear inequality to determine the number of adult and student tickets that need to be sold so that the amount of money taken in at the gate is at least $280. Graph the inequality and list three possible solutions.

38. **Critical Thinking** Why must a region of a coordinate plane be shaded to show all solutions of a linear inequality?

39. **Write About It** Give a real-world situation that can be described by a linear inequality. Then graph the inequality and give two solutions.

40. Gloria is making teddy bears. She is making boy and girl bears. She has enough stuffing to create 50 bears. Let \( x \) represent the number of girl bears and \( y \) represent the number of boy bears.

   a. Write an inequality that shows the possible number of boy and girl bears Gloria can make.

   b. Graph the inequality.

   c. Give three possible solutions for the numbers of boy and girl bears that can be made.
41. ***ERROR ANALYSIS*** Student A wrote $y < 2x - 1$ as the inequality represented by the graph. Student B wrote $y \leq 2x - 1$ as the inequality represented by the graph. Which student is incorrect? Explain the error.

42. **Write About It** How do you decide to shade above or below a boundary line? What does this shading represent?

43. Which point is a solution of the inequality $y > -x + 3$?
   - (A) $(0, 3)$
   - (B) $(1, 4)$
   - (C) $(-1, 4)$
   - (D) $(0, -3)$

44. Which inequality is represented by the graph at right?
   - (F) $2x + y \geq 3$
   - (G) $2x + y > 3$
   - (H) $2x + y \leq 3$
   - (I) $2x + y < 3$

45. Which of the following describes the graph of $3 \leq x$?
   - (A) The boundary line is dashed, and the shading is to the right.
   - (B) The boundary line is dashed, and the shading is to the left.
   - (C) The boundary line is solid, and the shading is to the right.
   - (D) The boundary line is solid, and the shading is to the left.

**CHALLENGE AND EXTEND**

Graph each inequality.

46. $0 \geq -6 - 2x - 5y$
47. $y > |x|$
48. $y \geq |x - 3|$

49. A linear inequality has the points $(0, 3)$ and $(-3, 1.5)$ as solutions on the boundary line. Also, the point $(1, 1)$ is not a solution. Write the linear inequality.

50. Two linear inequalities are graphed on the same coordinate plane. The point $(0, 0)$ is a solution of both inequalities. The entire coordinate plane is shaded except for Quadrant I. What are the two inequalities?
Solving Systems of Linear Inequalities

Objective
Graph and solve systems of linear inequalities in two variables.

Vocabulary
system of linear inequalities
solutions of a system of linear inequalities

Who uses this?
The owner of a surf shop can use systems of linear inequalities to determine how many surfboards and wakeboards need to be sold to make a certain profit. (See Example 4.)

A system of linear inequalities is a set of two or more linear inequalities containing two or more variables. The solutions of a system of linear inequalities are all of the ordered pairs that satisfy all the linear inequalities in the system.

Example 1
Identifying Solutions of Systems of Linear Inequalities

Tell whether the ordered pair is a solution of the given system.

A \((2, 1)\);
\[
\begin{align*}
y &< -x + 4 \\
y &\leq x + 1
\end{align*}
\]

\((2, 1)\)
\[
\begin{array}{c|c}
1 & < 2 \checkmark \\
\end{array}
\]

(2, 1) is a solution to the system because it satisfies both inequalities.

B \((2, 0)\);
\[
\begin{align*}
y &\geq 2x \\
y &< x + 1
\end{align*}
\]

\((2, 0)\)
\[
\begin{array}{c|c|c}
0 & \geq 4 \times \checkmark \\
0 & < 3 \checkmark \\
\end{array}
\]

(2, 0) is not a solution to the system because it does not satisfy both inequalities.

Remember!
An ordered pair must be a solution of all inequalities to be a solution of the system.

Tell whether the ordered pair is a solution of the given system.

1a. \((0, 1)\);
\[
\begin{align*}
y &< -3x + 2 \\
y &\geq x - 1
\end{align*}
\]

1b. \((0, 0)\);
\[
\begin{align*}
y &> -x + 1 \\
y &> x - 1
\end{align*}
\]

To show all the solutions of a system of linear inequalities, graph the solutions of each inequality. The solutions of the system are represented by the overlapping shaded regions. Below are graphs of Examples 1A and 1B.

Example 1A

(2, 1) is in the overlapping shaded regions, so it is a solution.

Example 1B

(2, 0) is not in the overlapping shaded regions, so it is not a solution.
**Example 2**

**Solving a System of Linear Inequalities by Graphing**

Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

\[
\begin{align*}
8x + 4y &\leq 12 \\
y &> \frac{1}{2}x - 2
\end{align*}
\]

- Solve the first inequality for \( y \).
- \( 4y \leq -8x + 12 \)
- \( y \leq -2x + 3 \)

**Graph the system.**

\[
\begin{align*}
y &\leq -2x + 3 \\
y &> \frac{1}{2}x - 2
\end{align*}
\]

\((-1, 1)\) and \((-3, 4)\) are solutions.

\((2, -1)\) and \((2, -4)\) are not solutions.

**Example 3**

**Graphing Systems with Parallel Boundary Lines**

Graph each system of linear inequalities. Describe the solutions.

**A** \[
\begin{align*}
y &< 2x - 3 \\
y &> 2x + 2
\end{align*}
\]

This system has no solution.

**B** \[
\begin{align*}
y &> x - 3 \\
y &\leq x + 1
\end{align*}
\]

The solutions are all points between the parallel lines and on the solid line.

**C** \[
\begin{align*}
y &\leq -3x - 2 \\
y &\leq -3x + 4
\end{align*}
\]

The solutions are the same as the solutions of \( y \leq -3x - 2 \).
Graph each system of linear inequalities. Describe the solutions.

3a. \[ \begin{align*}
    y &> x + 1 \\
    y &\leq x - 3
\end{align*} \]

3b. \[ \begin{align*}
    y &\geq 4x - 2 \\
    y &\leq 4x + 2
\end{align*} \]

3c. \[ \begin{align*}
    y &> -2x + 3 \\
    y &\geq -2x
\end{align*} \]

EXAMPLE 4

Business Application

A surf shop makes the profits given in the table. The shop owner sells at least 10 surfboards and at least 20 wakeboards per month. He wants to earn at least $2000 a month. Show and describe all possible combinations of surfboards and wakeboards that the store owner needs to sell to meet his goals. List two possible combinations.

Step 1 Write a system of inequalities.

Let \( x \) represent the number of surfboards and \( y \) represent the number of wakeboards.

\[ x \geq 10 \] He sells at least 10 surfboards.

\[ y \geq 20 \] He sells at least 20 wakeboards.

\[ 150x + 100y \geq 2000 \] He wants to earn a total of at least $2000.

Step 2 Graph the system.

The graph should be in only the first quadrant because sales are not negative.

Step 3 Describe all possible combinations.

To meet the sales goals, the shop could sell any combination represented by an ordered pair of whole numbers in the solution region. Answers must be whole numbers because the shop cannot sell part of a surfboard or wakeboard.

Step 4 List two possible combinations.

Two possible combinations are:

- 15 surfboards and 25 wakeboards
- 25 surfboards and 20 wakeboards

THINK AND DISCUSS

1. How would you write a system of linear inequalities from a graph?

2. GET ORGANIZED Copy and complete each part of the graphic organizer. In each box, draw a graph and list one solution.

<table>
<thead>
<tr>
<th>Profit per Board Sold ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surfboard</td>
</tr>
<tr>
<td>150</td>
</tr>
<tr>
<td>Wakeboard</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price per Pound ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pepper Jack</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>Cheddar</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

An ordered pair solution of the system need not have whole numbers, but answers to many application problems may be restricted to whole numbers.
GUIDED PRACTICE

1. **Vocabulary** A solution of a system of inequalities is a solution of _____ of the inequalities in the system. (at least one or all)

Tell whether the ordered pair is a solution of the given system.

2. \((0, 0); \begin{cases} y < -x + 3 \\ y < x + 2 \end{cases}\)
3. \((0, 0); \begin{cases} y < 3 \\ y > x - 2 \end{cases}\)
4. \((1, 0); \begin{cases} y > 3x \\ y \leq x + 1 \end{cases}\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

5. \(\begin{cases} y < 2x - 1 \\ y > 2 \end{cases}\)
6. \(\begin{cases} x < 3 \\ y > x - 2 \end{cases}\)
7. \(\begin{cases} y \geq 3x \\ 3x + y \geq 3 \end{cases}\)
8. \(\begin{cases} 2x - 4y \leq 8 \\ y > x - 2 \end{cases}\)

Graph each system of linear inequalities. Describe the solutions.

9. \(\begin{cases} y > 2x + 3 \\ y < 2x \end{cases}\)
10. \(\begin{cases} y \leq -3x - 1 \\ y \geq -3x + 1 \end{cases}\)
11. \(\begin{cases} y > 4x - 1 \\ y \leq 4x + 1 \end{cases}\)
12. \(\begin{cases} y < -x + 3 \\ y > -x + 2 \end{cases}\)
13. \(\begin{cases} y > 2x - 1 \\ y > 2x - 4 \end{cases}\)
14. \(\begin{cases} y \leq -3x + 4 \\ y \leq -3x - 3 \end{cases}\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

15. **Business** Sandy makes $2 profit on every cup of lemonade that she sells and $1 on every cupcake that she sells. Sandy wants to sell at least 5 cups of lemonade and at least 5 cupcakes per day. She wants to earn at least $25 per day. Show and describe all the possible combinations of lemonade and cupcakes that Sandy needs to sell to meet her goals. List two possible combinations.

PRACTICE AND PROBLEM SOLVING

Tell whether the ordered pair is a solution of the given system.

16. \((0, 0); \begin{cases} y > -x - 1 \\ y < 2x + 4 \end{cases}\)
17. \((0, 0); \begin{cases} x + y < 3 \\ y > 3x - 4 \end{cases}\)
18. \((1, 0); \begin{cases} y > 3x \\ y \geq 3x + 1 \end{cases}\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

19. \(\begin{cases} y < -3x - 3 \\ y \geq 0 \end{cases}\)
20. \(\begin{cases} y < -1 \\ y > 2x - 1 \end{cases}\)
21. \(\begin{cases} y > 2x + 4 \\ 6x + 2y \geq -2 \end{cases}\)
22. \(\begin{cases} 9x + 3y \leq 6 \\ y > x \end{cases}\)

Graph each system of linear inequalities. Describe the solutions.

23. \(\begin{cases} y < 3 \\ y > 5 \end{cases}\)
24. \(\begin{cases} y < x - 1 \\ y \geq x - 2 \end{cases}\)
25. \(\begin{cases} x \geq 2 \\ x \leq 2 \end{cases}\)
26. \(\begin{cases} y > -4x - 3 \\ y < -4x + 2 \end{cases}\)
27. \(\begin{cases} y > -1 \\ y > 2 \end{cases}\)
28. \(\begin{cases} y \leq 2x + 1 \\ y \leq 2x - 4 \end{cases}\)
29. **Multi-Step** Linda works at a pharmacy for $15 an hour. She also baby-sits for $10 an hour. Linda needs to earn at least $90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.

30. **Farming** Tony wants to plant at least 40 acres of corn and at least 50 acres of soybeans. He wants no more than 200 acres of corn and soybeans. Show and describe all the possible combinations of the number of acres of corn and of soybeans Tony could plant. List two possible combinations.

Graph each system of linear inequalities.

31. \[
\begin{align*}
  y &\geq -3 \\
  y &\geq 2
\end{align*}
\]

32. \[
\begin{align*}
  y &> -2x - 1 \\
  y &> -2x - 3
\end{align*}
\]

33. \[
\begin{align*}
  x &\leq -3 \\
  x &\geq 1
\end{align*}
\]

34. \[
\begin{align*}
  y &< 4 \\
  y &> 0
\end{align*}
\]

Write a system of linear inequalities to represent each graph.

35. [Graph Not Provided]

36. [Graph Not Provided]

37. [Graph Not Provided]

38. **Military** For males to enter the United States Air Force Academy, located in Colorado Springs, CO, they must be at least 17 but less than 23 years of age. Their standing height must be not less than 60 inches and not greater than 80 inches. Graph all possible heights and ages for eligible male candidates. Give three possible combinations.

39. //**ERROR ANALYSIS**// Two students wrote a system of linear inequalities to describe the graph. Which student is incorrect? Explain the error.

\[
\begin{align*}
  A: & \begin{align*}
  y &< x - 3 \\
  y &> x - 1
\end{align*} \\
  B: & \begin{align*}
  y &> x - 3 \\
  y &< x - 1
\end{align*}
\]

40. **Recreation** Vance wants to fence in a rectangular area for his dog. He wants the length of the rectangle to be at least 30 feet and the perimeter to be no more than 150 feet. Graph all possible dimensions of the rectangle.

41. **Critical Thinking** Can the solutions of a system of linear inequalities be the points on a line? Explain.

42. Gloria is starting her own company making teddy bears. She has enough bear bodies to create 40 bears. She will make girl bears and boy bears.

   a. Write an inequality to show this situation.

   b. Gloria will charge $15 for girl bears and $12 for boy bears. She wants to earn at least $540 a week. Write an inequality to describe this situation.

   c. Graph this situation and locate the solution region.
43. Write About It  What must be true of the boundary lines in a system of two linear inequalities if there is no solution of the system? Explain.

44. Which point is a solution of \[\begin{cases} 2x + y \geq 3 \\ y \geq -2x + 1 \end{cases}\]
A) (0, 0)  B) (0, 1)  C) (1, 0)  D) (1, 1)

45. Which system of inequalities best describes the graph?
F) \[\begin{cases} y < 2x - 3 \\ y > 2x + 1 \end{cases}\]  G) \[\begin{cases} y > 2x - 3 \\ y < 2x + 1 \end{cases}\]  H) \[\begin{cases} y < 2x - 3 \\ y < 2x + 1 \end{cases}\]  I) \[\begin{cases} y > 2x - 3 \\ y > 2x + 1 \end{cases}\]

46. Short Response  Graph and describe \[\begin{cases} y + x > 2 \\ y \leq -3x + 4 \end{cases}\]. Give two possible solutions of the system.

CHALLENGE AND EXTEND

47. Estimation  Graph the given system of inequalities. Estimate the area of the overlapping solution regions.
\[\begin{cases} y \geq 0 \\ y \leq x + 3.5 \\ y \leq -x + 3.5 \end{cases}\]

48. Write a system of linear inequalities for which \((-1, 1)\) and \((1, 4)\) are solutions and \((0, 0)\) and \((2, -1)\) are not solutions.

49. Graph \(|y| < 1|\).

50. Write a system of linear inequalities for which the solutions are all the points in the third quadrant.
Solve Systems of Linear Inequalities

A graphing calculator gives a visual solution to a system of linear inequalities.

Activity

Graph the system \[
\begin{cases}
y > 2x - 4 \\
2.75y - x < 6
\end{cases}
\]
. Give two ordered pairs that are solutions.

1. The first inequality is solved for \( y \).

2. Graph the first inequality. First graph the boundary line \( y = 2x - 4 \). Press \( \text{Y=} \) and enter \( 2x - 4 \) for \( Y1 \).

   The inequality contains the symbol \( > \). The solution region is above the boundary line. Press \( \text{ENTER} \) to move the cursor to the left of \( Y1 \). Press \( \text{GRAPH} \) until the icon that looks like a region above a line appears. Press \( \text{GRAPH} \).

3. Solve the second inequality for \( y \).

   \[
   2.75y - x < 6 \\
   2.75y < x + 6 \\
   y < \frac{x + 6}{2.75}
   \]

4. Graph the second inequality. First graph the boundary line \( y = \frac{x + 6}{2.75} \). Press \( \text{Y=} \) and enter \( (x + 6)/2.75 \) for \( Y2 \).

   The inequality contains the symbol \( < \). The solution region is below the boundary line. Press \( \text{ENTER} \) to move the cursor to the left of \( Y2 \). Press \( \text{GRAPH} \) until the icon that looks like a region below a line appears. Press \( \text{GRAPH} \).

5. The solutions of the system are represented by the overlapping shaded regions. The points \((0, 0)\) and \((-1, 0)\) are in the shaded region.

   \[
   \begin{array}{lll}
   \text{Check} & \text{Test \((0, 0)\) in both inequalities.} & \text{Test \((-1, 0)\) in both inequalities.} \\
   y > 2x - 4 & 2.75y - x < 6 & y > 2x - 4 \\
   0 > 2(0) - 4 & 2.75(0) - 0 < 6 & 0 > 2(-1) - 4 \\
   0 > -4 & 0 < 6 & 0 > -6 \\
   \end{array}
   \]

Try This

Graph each system. Give two ordered pairs that are solutions.

1. \[
\begin{cases}
x + 5y > -10 \\
x - y < 4
\end{cases}
\]

2. \[
\begin{cases}
y > x - 2 \\
y \leq x + 2
\end{cases}
\]

3. \[
\begin{cases}
y > x - 2 \\
y \leq 3
\end{cases}
\]

4. \[
\begin{cases}
y < x - 3 \\
y - 3 > x
\end{cases}
\]
**Vocabulary**

consistent system  dependent system  inconsistent system  independent system  linear inequality  solution of a linear inequality

Complete the sentences below with vocabulary words from the list above.

1. A(n) __________ is a system that has exactly one solution.
2. A set of two or more linear equations that contain the same variable(s) is a(n) __________.
3. The __________ consists of all the ordered pairs that satisfy all the inequalities in the system.
4. A system consisting of equations of parallel lines with different y-intercepts is a(n) __________.
5. A(n) __________ consists of two intersecting lines.

**EXERCISES**

Tell whether the ordered pair is a solution of the given system.

6. (0, −5); \( \begin{cases} y = -6x + 5 \\ x - y = 5 \end{cases} \)
7. (4, 3); \( \begin{cases} x - 2y = -2 \\ y = \frac{1}{2}x + 1 \end{cases} \)
8. \( \left( \frac{3}{4}, \frac{7}{4} \right) \); \( \begin{cases} x + y = 9 \\ 2y = 6x + 4 \end{cases} \)
9. (−1, −1); \( \begin{cases} y = -2x + 5 \\ 3y = 6x + 3 \end{cases} \)

Solve each system by graphing. Check your answer.

10. \( \begin{cases} y = 3x + 2 \\ y = -2x - 3 \end{cases} \)
11. \( \begin{cases} y = -\frac{1}{3}x + 5 \\ 2x - 2y = -2 \end{cases} \)

12. Raheel is comparing the cost of two parking garages. Garage A charges a flat fee of $6 per car plus $0.50 per hour. Garage B charges a flat fee of $2 per car plus $1 per hour. After how many hours will the cost at garage A be the same as the cost at garage B? What will that cost be?
7-2 Solving Systems by Substitution

**EXAMPLE**

Solve \( \begin{align*} 2x - 3y &= -2 \\ y - 3x &= 10 \end{align*} \) by substitution.

**Step 1**

\( y - 3x = 10 \)
\( y = 3x + 10 \)

**Step 2**

\( 2x - 3y = -2 \)
\( 2x - 3(3x + 10) = -2 \)
\( 2x - 3y = -2 \)
\( 2x - 9x - 30 = -2 \)
\( -7x - 30 = -2 \)
\( -7x = 28 \)
\( x = -4 \)

**Step 3**

\( y - 3x = 10 \)
\( y - 3(-4) = 10 \)
\( y + 12 = 10 \)
\( y = -2 \)

**Step 4**

\( y - 3x = 10 \)
\( y - 3(-4) = 10 \)
\( y + 12 = 10 \)
\( y = -2 \)

\( (-4, -2) \)

**Step 5**

\( (-4, -2) \)

Write the solution as an ordered pair.

To check the solution, substitute \( (-4, -2) \) into both equations in the system.

**EXERCISES**

Solve each system by substitution.

13. \( \begin{align*} y &= x + 3 \\ y &= 2x + 12 \end{align*} \)

14. \( \begin{align*} y &= -4x \\ y &= 2x - 3 \end{align*} \)

15. \( \begin{align*} 2x + y &= 4 \\ 3x + y &= 3 \end{align*} \)

16. \( \begin{align*} x + y &= -1 \\ y &= -2x + 3 \end{align*} \)

17. \( \begin{align*} x &= y - 7 \\ -y - 2x &= 8 \end{align*} \)

18. \( \begin{align*} \frac{1}{2} x + y &= 9 \\ 3x - 4y &= -6 \end{align*} \)

19. The Nash family’s car needs repairs. Estimates for parts and labor from two garages are shown below.

<table>
<thead>
<tr>
<th>Garage</th>
<th>Parts ($)</th>
<th>Labor ($ per hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor Works</td>
<td>650</td>
<td>70</td>
</tr>
<tr>
<td>Jim’s Car Care</td>
<td>800</td>
<td>55</td>
</tr>
</tbody>
</table>

For how many hours of labor will the total cost of fixing the car be the same at both garages? What will that cost be? Which garage will be cheaper if the repairs require 8 hours of labor? Explain.

7-3 Solving Systems by Elimination

**EXAMPLE**

Solve \( \begin{align*} 2x - 3y &= -8 \\ x + 4y &= 7 \end{align*} \) by elimination.

**Step 1**

\( \begin{align*} 2x - 3y &= -8 \\ +\left(-2\right)(x + 4y) &= +7 \end{align*} \)
\( 2x - 3y = -8 \)
\( 2x - 8y = -14 \)
\( -11y = -22 \)
\( y = 2 \)

**Step 2**

\( 2x - 3y = -8 \)
\( 2x - 3(2) = -8 \)
\( 2x - 6 = -8 \)
\( 2x = -2 \)
\( x = -1 \)

**Step 3**

\( 2x - 3y = -8 \)
\( 2x - 3(2) = -8 \)
\( 2x - 6 = -8 \)
\( 2x = -2 \)
\( x = -1 \)

\( (-1, 2) \)

**Step 4**

\( (-1, 2) \)

Write the solution as an ordered pair.

To check the solution, substitute \( (-1, 2) \) into both equations in the system.

**EXERCISES**

Solve each system by elimination.

20. \( \begin{align*} 4x + y &= -1 \\ 2x - y &= -5 \end{align*} \)

21. \( \begin{align*} x + 2y &= -1 \\ x + y &= 2 \end{align*} \)

22. \( \begin{align*} x + y &= 12 \\ 2x + 5y &= 27 \end{align*} \)

23. \( \begin{align*} 3x - 2y &= -6 \\ \frac{1}{3} x + 3y &= 9 \end{align*} \)

Solve each system by any method. Explain why you chose each method. Check your answer.

24. \( \begin{align*} 3x + y &= 2 \\ y &= -4x \end{align*} \)

25. \( \begin{align*} y &= \frac{1}{3} x - 6 \\ y &= -2x + 1 \end{align*} \)

26. \( \begin{align*} 2y &= -3x \\ y &= -2x + 2 \end{align*} \)

27. \( \begin{align*} x - y &= 0 \\ 3x + y &= 8 \end{align*} \)
Classify each system. Give the number of solutions.

1. \[ \begin{align*}
    y &= 3x + 4 \\
    6x - 2y &= -8
\end{align*} \]

Use the substitution method because the first equation is solved for \( y \).

\[
6x - 2(3x + 4) = -8
\]

Substitute 3x + 4 for \( y \) in the second equation.

\[
6x - 6x - 8 = -8
\]

\(-8 = -8 \checkmark \) True

The equation is an identity. There are infinitely many solutions.

This system is consistent and dependent. When graphed, the two lines are coincident (the same line)—they have identical slopes and \( y \)-intercepts.

2. \[ \begin{align*}
    y &= 2x - 1 \\
    2x - y &= -2
\end{align*} \]

Compare slopes and \( y \)-intercepts. Write both equations in slope-intercept form.

\[
\begin{align*}
    y &= 2x - 1 \\
    2x - y &= -2
\end{align*} \Rightarrow
\begin{align*}
    y &= 2x - 1 \\
    y &= 2x + 2
\end{align*}
\]

The lines have the same slope and different \( y \)-intercepts. The lines are parallel.

The lines never intersect, so this system is inconsistent. It has no solution.

3. \[ \begin{align*}
    2x - y &= 6 \\
    y &= x - 1
\end{align*} \]

Write both equations in slope-intercept form.

\[
\begin{align*}
    2x - y &= 6 \\
    y &= 1x - 1
\end{align*} \Rightarrow
\begin{align*}
    y &= 2x - 6 \\
    y &= 1x - 1
\end{align*}
\]

The lines intersect because they have different slopes.

The system is consistent and independent. There is one solution: \((5, 4)\).
7-5 Solving Linear Inequalities

**EXAMPLE**

Graph the solutions of $x - 2y < 6$.

**Step 1** Solve the inequality for $y$.

$x - 2y < 6$

$-2y < -x + 6$

$y > \frac{1}{2}x - 3$

**Step 2** Graph $y = \frac{1}{2}x - 3$.

Use a dashed line for $>$.  

**Step 3** The inequality is $>$, so shade above the boundary line.

**Check** Substitute $(0, 0)$ for $(x, y)$.

$x - 2y < 6$

$0 - 2(0) < 6$

$(0, 0)$ satisfies the inequality, so the graph is shaded correctly.

---

7-6 Solving Systems of Linear Inequalities

**EXAMPLES**

Graph $\begin{cases} y < -x + 5 \\ y \geq 2x - 3 \end{cases}$. Give two ordered pairs that are solutions and two that are not solutions.

Graph both inequalities.

The solutions of the system are represented by the overlapping shaded regions.

The points $(0, 0)$ and $(-2, 2)$ are solutions of the system.

The points $(3, -2)$ and $(4, 4)$ are not solutions.

---

**EXERCISES**

Tell whether the ordered pair is a solution of the given system.

53. $(3, 3)$; $\begin{cases} y > -2x + 9 \\ y \geq x \end{cases}$

54. $(-1, 0)$; $\begin{cases} 2x - y > -5 \\ y \leq -3x - 3 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

55. $\begin{cases} y \geq x + 4 \\ y > 6x - 3 \end{cases}$

56. $\begin{cases} y \leq -2x + 8 \\ y > 3x - 5 \end{cases}$

57. $\begin{cases} -x + 2y > 6 \\ x + y < 4 \end{cases}$

58. $\begin{cases} x - y > 7 \\ x + 3y \leq 15 \end{cases}$

Graph each system of linear inequalities.

59. $\begin{cases} y > -x - 6 \\ y < -x + 5 \end{cases}$

60. $\begin{cases} 4x + 2y \geq 10 \\ 6x + 3y < -9 \end{cases}$
Tell whether the ordered pair is a solution of the given system.

1. \((1, -4)\); \[
\begin{align*}
y &= -4x \\
y &= 2x - 2
\end{align*}
\]

2. \((0, -1)\); \[
\begin{align*}
3x - y &= 1 \\
x + 5y &= -5
\end{align*}
\]

3. \((3, 2)\); \[
\begin{align*}
x - 2y &= -1 \\
-3x + 2y &= 5
\end{align*}
\]

Solve each system by graphing.

4. \[
\begin{align*}
y &= x - 3 \\
y &= -2x - 3
\end{align*}
\]

5. \[
\begin{align*}
2x + y &= -8 \\
y &= \frac{1}{3}x - 1
\end{align*}
\]

6. \[
\begin{align*}
y &= -x + 4 \\
x &= y + 2
\end{align*}
\]

Solve each system by substitution.

7. \[
\begin{align*}
y &= -6 \\
y &= -2x - 2
\end{align*}
\]

8. \[
\begin{align*}
-x + y &= -4 \\
y &= 2x - 11
\end{align*}
\]

9. \[
\begin{align*}
x - 3y &= 3 \\
2x &= 3y
\end{align*}
\]

10. The costs for services at two kennels are shown in the table. Joslyn plans to board her dog and have him bathed once during his stay. For what number of days will the cost for boarding and bathing her dog at each kennel be the same? What will that cost be? If Joslyn plans a week-long vacation, which is the cheaper service? Explain.

<table>
<thead>
<tr>
<th>Kennel Costs</th>
<th>Boarding ($ per day)</th>
<th>Bathing ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pet Care</td>
<td>30</td>
<td>15</td>
</tr>
<tr>
<td>Fido’s</td>
<td>28</td>
<td>27</td>
</tr>
</tbody>
</table>

Solve each system by elimination.

11. \[
\begin{align*}
3x - y &= 7 \\
2x + y &= 3
\end{align*}
\]

12. \[
\begin{align*}
4x + y &= 0 \\
x + y &= -3
\end{align*}
\]

13. \[
\begin{align*}
2x + y &= 3 \\
x - 2y &= -1
\end{align*}
\]

Classify each system. Give the number of solutions.

14. \[
\begin{align*}
y &= 6x - 1 \\
6x - y &= 1
\end{align*}
\]

15. \[
\begin{align*}
y &= -3x - 3 \\
3x + y &= 3
\end{align*}
\]

16. \[
\begin{align*}
2x - y &= 1 \\
-4x + y &= 1
\end{align*}
\]

Graph the solutions of each linear inequality.

17. \(y < 2x - 5\)

18. \(-y \geq 8\)

19. \(y > \frac{1}{3}x\)

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

20. \[
\begin{align*}
y &> \frac{1}{2}x - 5 \\
y &\leq 4x - 1
\end{align*}
\]

21. \[
\begin{align*}
y &> -x + 4 \\
3x - y &> 3
\end{align*}
\]

22. \[
\begin{align*}
y &\geq 2x \\
y - 2x &< 6
\end{align*}
\]

23. Ezra and Tava sold at least 150 coupon books. Ezra sold at most 30 books more than twice the number Tava sold. Show and describe all possible combinations of the numbers of coupon books Ezra and Tava sold. List two possible combinations.
Exponential Functions

Population Explosion

The concepts in this chapter are used to model many real-world phenomena, such as changes in wildlife populations.

- Graph and use exponential functions to model real-world problems.
- Compare linear, quadratic, and exponential functions.

Exponential Functions

LAB  Model Growth and Decay
8-2  Exponential Growth and Decay
EXT  Patterns and Recursion
8-3  Comparing Functions

Chapter Project Online
Study Strategy: Remember Properties

In math, there are many formulas, properties, and rules that you should commit to memory.

To memorize a property, create flash cards. Write the name of the property on one side of a card. Write the property on the other side of the card. You might also include a diagram or an example if helpful. Study your flash cards on a regular basis.

Sample Flash Card

Knowing when and how to apply a mathematical property is as important as memorizing the property itself.

To know what property to apply, read the problem carefully and look for key words.

A professional baseball player earns an annual salary of $5.7 \times 10^6. There are 162 games in a baseball season. How much does the player earn per game?

The key words have been highlighted. You are asked for an amount per game so you need to find a quotient. The annual salary is given in exponential form so you will need to use the quotient of powers property.

Try This

Read each problem. Then write the property needed to solve it. What key words helped you identify the property?

1. Light travels at about $1.86 \times 10^5$ miles per second. Find the approximate distance that light travels in one year if there are about $3.15 \times 10^7$ seconds in a year.

2. The area of a rectangular pool is 120 square feet. The length is 1 foot less than twice the width. What is the perimeter of the pool?
The table and the graph show an insect population that increases over time.

<table>
<thead>
<tr>
<th>Time (days)</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>54</td>
</tr>
</tbody>
</table>

A function rule that describes the pattern above is \( f(x) = 2(3)^x \). This type of function, in which the independent variable appears in an exponent, is an **exponential function**. Notice that 2 is the starting population and 3 is the amount by which the population is multiplied each day.

**Example 1**

**A** The function \( f(x) = 2(3)^x \) models an insect population after \( x \) days. What will the population be on the 5th day?

\[
\begin{align*}
  f(x) &= 2(3)^x & \text{Write the function.} \\
  f(5) &= 2(3)^5 & \text{Substitute 5 for } x. \\
  &= 2(243) & \text{Evaluate } 3^5. \\
  &= 486 & \text{Multiply.}
\end{align*}
\]

There will be 486 insects on the 5th day.

**B** The function \( f(x) = 1500(0.995)^x \), where \( x \) is the time in years, models a prairie dog population. How many prairie dogs will there be in 8 years?

\[
\begin{align*}
  f(x) &= 1500(0.995)^x \\
  f(8) &= 1500(0.995)^8 \\
  &\approx 1441 & \text{Use a calculator. Round to the nearest whole number.}
\end{align*}
\]

There will be about 1441 prairie dogs in 8 years.

1. The function \( f(x) = 8(0.75)^x \) models the width of a photograph in inches after it has been reduced by 25% \( x \) times. What is the width of the photograph after it has been reduced 3 times?
Remember that linear functions have constant first differences and quadratic functions have constant second differences. Exponential functions do not have constant differences, but they do have constant ratios.

As the $x$-values increase by a constant amount, the $y$-values are multiplied by a constant amount. This amount is the constant ratio and is the value of $b$ in $f(x) = ab^x$.

**Example 2**

**Identifying an Exponential Function**

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

**A** \[ \{(-1, 1.5), (0, 3), (1, 6), (2, 12)\} \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x) = 2(3)^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td>162</td>
</tr>
</tbody>
</table>

This is an exponential function. As the $x$-values increase by a constant amount, the $y$-values are multiplied by a constant amount.

**B** \[ \{(-1, -9), (1, 9), (3, 27), (5, 45)\} \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-9</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
</tr>
</tbody>
</table>

This is not an exponential function. As the $x$-values increase by a constant amount, the $y$-values are not multiplied by a constant amount.

**Check It Out**

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

2a. \[ \{(-1, 1), (0, 0), (1, 1), (2, 4)\} \]

2b. \[ \{(-2, 4), (-1, 2), (0, 1), (1, 0.5)\} \]

To graph an exponential function, choose several values of $x$ (positive, negative, and 0) and generate ordered pairs. Plot the points and connect them with a smooth curve.

**Example 3**

**Graphing** \[ y = ab^x \] **with** \[ a > 0 \] **and** \[ b > 1 \]

Graph $y = 3(4)^x$.

Choose several values of $x$ and generate ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y = 3(4)^x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0.75</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>48</td>
</tr>
</tbody>
</table>

Graph the ordered pairs and connect with a smooth curve.

3a. Graph $y = 2^x$.

3b. Graph $y = 0.2(5)^x$. 

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**Example 4**

Graphing \( y = ab^x \) with \( a < 0 \) and \( b > 1 \)

Graph \( y = -5(2)^x \).

Choose several values of \( x \) and generate ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -5(2)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2.5</td>
</tr>
<tr>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>1</td>
<td>-10</td>
</tr>
<tr>
<td>2</td>
<td>-20</td>
</tr>
</tbody>
</table>

Graph the ordered pairs and connect with a smooth curve.

4a. Graph \( y = -6^x \).

4b. Graph \( y = -3(3)^x \).

**Example 5**

Graphing \( y = ab^x \) with \( 0 < b < 1 \)

Graph each exponential function.

**A** \( y = 3 \left( \frac{1}{2} \right)^x \)

Choose several values of \( x \) and generate ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = 3 \left( \frac{1}{2} \right)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Graph the ordered pairs and connect with a smooth curve.

**B** \( y = -2(0.4)^x \)

Choose several values of \( x \) and generate ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = -2(0.4)^x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-12.5</td>
</tr>
<tr>
<td>-1</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

Graph the ordered pairs and connect with a smooth curve.

**Check It Out**

Graph each exponential function.

5a. \( y = 4 \left( \frac{1}{4} \right)^x \)

5b. \( y = -2(0.1)^x \)
The box summarizes the general shapes of exponential function graphs.

### Example 6

**Statistics Application**

In the year 2000, the world population was about 6 billion, and it was growing by 1.21% each year. At this growth rate, the function \( f(x) = 6(1.0121)^x \) gives the population, in billions, \( x \) years after 2000. Using this model, in about what year does the population reach 7 billion?

**Enter the function into the Y= editor of a graphing calculator.**

**Press 2nd TABLE. Use the arrow keys to find a y-value as close to 7 as possible. The corresponding x-value is 13.**

The world population reaches 7 billion in about 2013.

### Think and Discuss

1. How can you find the constant ratio of a set of exponential data?

2. **Get Organized** Copy and complete the graphic organizer. In each box, give an example of an appropriate exponential function and sketch its graph.

   **Exponential Functions**: \( y = ab^x \)

   - \( a > 0, b > 1 \)
   - \( a < 0, b > 1 \)
   - \( a > 0, 0 < b < 1 \)
   - \( a < 0, 0 < b < 1 \)
1. **Vocabulary**  Tell whether \( y = 3x^4 \) is an exponential function. Explain your answer.

2. **Physics**  The function \( f(x) = 50,000(0.975)^x \), where \( x \) represents the underwater depth in meters, models the intensity of light below the water's surface in lumens per square meter. What is the intensity of light 200 meters below the surface? Round your answer to the nearest whole number.

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

3. \( \{(−1, −1), (0, 0), (1, −1), (2, −4)\} \)

4. \( \{(0, 1), (1, 4), (2, 16), (3, 64)\} \)

Graph each exponential function.

5. \( y = 3^x \)

6. \( y = 5^x \)

7. \( y = 10(3)^x \)

8. \( y = 5(2)^x \)

9. \( y = −2(3)^x \)

10. \( y = −4(2)^x \)

11. \( y = −3(2)^x \)

12. \( y = 2(3)^x \)

13. \( y = −\left(\frac{1}{4}\right)^x \)

14. \( y = \left(\frac{1}{3}\right)^x \)

15. \( y = 2\left(\frac{1}{4}\right)^x \)

16. \( y = −2(0.25)^x \)

17. The function \( f(x) = 57.8(1.02)^x \) gives the number of passenger cars, in millions, in the United States \( x \) years after 1960. Using this model, in about what year does the number of passenger cars reach 200 million?

PRACTICE AND PROBLEM SOLVING

18. **Sports**  If a golf ball is dropped from a height of 27 feet, the function \( f(x) = 27\left(\frac{2}{3}\right)^x \) gives the height in feet of each bounce, where \( x \) is the bounce number. What will be the height of the 4th bounce?

19. Suppose the depth of a lake can be described by the function \( y = 334(0.976)^x \), where \( x \) represents the number of weeks from today. Today, the depth of the lake is 334 ft. What will the depth be in 6 weeks? Round your answer to the nearest whole number.

20. **Physics**  A ball rolling down a slope travels continuously faster. Suppose the function \( y = 1.3(1.41)^x \) describes the speed of the ball in inches per minute. How fast will the ball be rolling in 15 minutes? Round your answer to the nearest hundredth.

Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

21. \( \{(-2, 9), (-1, 3), (0, 1), (1, \frac{1}{3})\} \)

22. \( \{(-1, 0), (0, 1), (1, 4), (2, 9)\} \)

23. \( \{(-1, -5), (0, -3), (1, -1), (2, 1)\} \)

24. \( \{(-3, 6.25), (-2, 12.5), (-1, 25), (0, 50)\} \)
Graph each exponential function.

25. \( y = 1.5^x \)  
26. \( y = \frac{1}{3}(3)^x \)  
27. \( y = 100(0.7)^x \)  
28. \( y = -2(4)^x \)  
29. \( y = -1(5)^x \)  
30. \( y = -\frac{1}{2}(4)^x \)  
31. \( y = 4\left(\frac{1}{2}\right)^x \)  
32. \( y = -2\left(\frac{1}{3}\right)^x \)  
33. \( y = 0.5(0.25)^x \)

34. **Technology** Moore's law states that the maximum number of transistors that can fit on a silicon chip doubles every two years. The function \( f(x) = 42(1.41)^x \) models the number of transistors, in millions, that can fit on a chip, where \( x \) is the number of years since 2000. Using this model, in what year can a chip hold 1 billion transistors?

35. **Multi-Step** A computer randomly creates three different functions. The functions are \( y = (3.1x + 7)^2 \), \( y = 4.8(2)^x \), and \( y = \frac{1}{5}(6)^x \). The computer then generates the \( y \) value 38.4. Given the three different functions, determine which one is exponential and produces the generated number.

36. **Contests** As a promotion, a clothing store draws the name of one of its customers each week. The prize is a coupon for the store. If the winner is not present at the drawing, he or she cannot claim the prize, and the amount of the coupon increases for the following week's drawing. The function \( f(x) = 20(1.2)^x \) gives the amount of the coupon in dollars after \( x \) weeks of the prize going unclaimed.
   
a. What is the amount of the coupon after 2 weeks of the prize going unclaimed?
   
b. After how many weeks of the prize going unclaimed will the amount of the coupon be greater than $100?
   
c. What is the original amount of the coupon?
   
d. Find the percent increase each week.

37. **Critical Thinking** In the definition of exponential function, the value of \( b \) cannot be 1, and the value of \( a \) cannot be 0. Why?

38. Graphing Calculator Graph each group of functions on the same screen. How are their graphs alike? How are they different?

\[
38. \quad y = 2^x, \quad y = 3^x, \quad y = 4^x \\
39. \quad y = \left(\frac{1}{2}\right)^x, \quad y = \left(\frac{1}{3}\right)^x, \quad y = \left(\frac{1}{4}\right)^x
\]

Evaluate each of the following for the given value of \( x \).

40. \( f(x) = 4^x; \ x = 3 \)  
41. \( f(x) = -(0.25)^x; \ x = 1.5 \)  
42. \( f(x) = 0.4(10)^x; \ x = -3 \)

43. a. The annual tuition at a community college since 2001 is modeled by the equation \( C = 2000(1.08)^n \), where \( C \) is the tuition cost and \( n \) is the number of years since 2001. What was the tuition cost in 2001?
   b. What is the annual percentage of tuition increase?
   c. Find the tuition cost in 2006.
44. **Write About It** Your employer offers two salary plans. With plan A, your salary is $f(x) = 10,000(2x)$, where $x$ is the number of years you have worked for the company. With plan B, your salary is $g(x) = 10,000(2)^x$. Which plan would you choose? Why?

45. **Which graph shows an exponential function?**

![Graphs A, B, C, D](Images)

46. The function $f(x) = 15(1.4)^x$ represents the area in square inches of a photograph after it has been enlarged $x$ times by a factor of 140%. What is the area of the photograph after it has been enlarged 4 times?

- [ ] F 5.6 square inches
- [ ] G 57.624 square inches
- [ ] H 41.16 square inches
- [ ] J 560 square inches

47. **Look at the pattern. How many squares will there be in the $n$th stage?**

- [ ] A $5n$
- [ ] B $2.5 \cdot 2^n$
- [ ] C $25^{n-1}$
- [ ] D $5^n$

**CHALLENGE AND EXTEND**

Solve each equation.

48. $4^x = 64$

49. $\left(\frac{1}{3}\right)^x = \frac{1}{27}$

50. $2^x = \frac{1}{16}$

51. Graph the following functions: $y = 2(2)^x$, $y = 3(2)^x$, $y = -2(2)^x$. Then make a conjecture about the relationship between the value of $a$ and the $y$-intercept of $y = ab^x$. 

420 **Chapter 8 Exponential Functions**
Changing Dimensions

What happens to the volume of a three-dimensional figure when you repeatedly double the dimensions?

Recall these formulas for the volumes of common three-dimensional figures.

- **Cube** \( V = s^3 \)
- **Rectangular Prism** \( V = \ell wh \)
- **Pyramid** \( V = \frac{1}{3} (\text{area of base}) \cdot h \)

Changing the dimensions of three-dimensional figures results in geometric sequences.

**Example**

Find the volume of a cube with a side length of 3 cm. Double the side length and find the new volume. Repeat two more times. Show the patterns for the side lengths and volumes as geometric sequences. Identify the common ratios.

<table>
<thead>
<tr>
<th>Cube</th>
<th>Side Length (cm)</th>
<th>Volume (cm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>216</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>1,728</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>13,824</td>
</tr>
</tbody>
</table>

The side lengths and the volumes form geometric sequences. The sequence of the side lengths has a common ratio of 2. The sequence of the volumes has a common ratio of \(2^3\), or 8.

The patterns in the example above are a specific instance of a general rule.

When the dimensions of a solid figure are multiplied by \(x\), the volume of the figure is multiplied by \(x^3\).

**Try This**

1. The large rectangular prism at right is 8 in. wide, 16 in. long, and 32 in. tall. The dimensions are multiplied by \(\frac{1}{2}\) to create each next smaller prism. Show the patterns for the dimensions and the volumes as geometric sequences. Identify the common ratios.

2. A pyramid has a height of 8 cm and a square base of 3 cm on each edge. Triple the dimensions two times. Show the patterns for the dimensions and the volumes as geometric sequences. Identify the common ratios.
Model Growth and Decay

You can fold and cut paper to model quantities that increase or decrease exponentially.

Use with Exponential Growth and Decay

**Activity 1**

1. Copy the table at right.

<table>
<thead>
<tr>
<th>Folds</th>
<th>Regions</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

2. Fold a piece of notebook paper in half. Then open it back up. Count the number of regions created by the fold. Record your answer in the table.

3. Now fold the paper in half twice. Record the number of regions created by the folds in the table.

4. Repeat this process for 3, 4, and 5 folds.

**Try This**

1. When the number of folds increases by 1, the number of regions ?

2. For each row of the table, write the number of regions as a power of 2.

3. Write an exponential expression for the number of regions formed by \( n \) folds.

4. If you could fold the paper 8 times, how many regions would be formed?

5. How many times would you have to fold the paper to make 512 regions?

**Activity 2**

1. Copy the table at right.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

2. Begin with a square piece of paper. The area of the paper is 1 square unit. Cut the paper in half. Each piece has an area of \( \frac{1}{2} \) square unit. Record the result in the table.

3. Cut one of those pieces in half again, and record the area of one of the new, smaller pieces in the table.

4. Repeat this process for 3, 4, and 5 cuts.

**Try This**

6. When the number of cuts increases by 1, the area ?

7. For each row of the table, write the area as a power of 2.

8. Write an exponential expression for the area after \( n \) cuts.

9. What would be the area after 7 cuts?

10. How many cuts would you have to make to get an area of \( \frac{1}{256} \) square unit?
Exponential Growth and Decay

**Objective**
Solve problems involving exponential growth and decay.

**Vocabulary**
- exponential growth
- compound interest
- exponential decay
- half-life

---

### Why learn this?
Exponential growth and decay describe many real-world situations, such as the value of artwork. (See Example 1.)

**Exponential growth** occurs when a quantity increases by the same rate \( r \) in each time period \( t \). When this happens, the value of the quantity at any given time can be calculated as a function of the rate and the original amount.

### Exponential Growth
An exponential growth function has the form \( y = a(1 + r)^t \), where \( a > 0 \).
- \( y \) represents the final amount.
- \( a \) represents the original amount.
- \( r \) represents the rate of growth expressed as a decimal.
- \( t \) represents time.

---

### Example 1
The original value of a painting is $1400, and the value increases by 9% each year. Write an exponential growth function to model this situation. Then find the value of the painting in 25 years.

**Step 1** Write the exponential growth function for this situation.

\[
y = a(1 + r)^t
\]

\[
y = 1400(1 + 0.09)^t
\]

\[
y = 1400(1.09)^t
\]

**Step 2** Find the value in 25 years.

\[
y = 1400(1.09)^{25}
\]

\[
y \approx 12,072.31
\]

The value of the painting in 25 years is $12,072.31.

---

**Check It Out!**
1. A sculpture is increasing in value at a rate of 8% per year, and its value in 2000 was $1200. Write an exponential growth function to model this situation. Then find the sculpture’s value in 2006.
A common application of exponential growth is **compound interest**. Recall that simple interest is earned or paid only on the principal. **Compound interest** is interest earned or paid on *both* the principal and previously earned interest.

### Compound Interest

Given the formula:

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

- **A** represents the balance after *t* years.
- **P** represents the principal, or original amount.
- **r** represents the annual interest rate expressed as a decimal.
- **n** represents the number of times interest is compounded per year.
- **t** represents time in years.

#### Example 2: Finance Application

Write a compound interest function to model each situation. Then find the balance after the given number of years.

**A** $1000 invested at a rate of 3% compounded quarterly; 5 years

**Step 1** Write the compound interest function for this situation.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

Write the formula.

\[ = 1000 \left(1 + \frac{0.03}{4}\right)^{4t} \]

Substitute $1000$ for *P*, $0.03$ for *r*, and $4$ for *n*.

\[ = 1000(1.0075)^{4t} \]

Simplify.

**Step 2** Find the balance after 5 years.

\[ A = 1000(1.0075)^{4(5)} \]

Substitute $5$ for *t*.

\[ \approx 1161.18 \]

Use a calculator and round to the nearest hundredth.

The balance after 5 years is $1161.18$.

**B** $18,000 invested at a rate of 4.5% compounded annually; 6 years

**Step 1** Write the compound interest function for this situation.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

Write the formula.

\[ = 18,000 \left(1 + \frac{0.045}{1}\right)^{1t} \]

Substitute $18,000$ for *P*, $0.045$ for *r*, and $1$ for *n*.

\[ = 18,000(1.045)^{t} \]

Simplify.

**Step 2** Find the balance after 6 years.

\[ A = 18,000(1.045)^{6} \]

Substitute $6$ for *t*.

\[ \approx 23,440.68 \]

Use a calculator and round to the nearest hundredth.

The balance after 6 years is $23,440.68$.

#### Check It Out!

Write a compound interest function to model each situation. Then find the balance after the given number of years.

**2a.** $1200 invested at a rate of 3.5% compounded quarterly; 4 years

**2b.** $4000 invested at a rate of 3% compounded monthly; 8 years
Exponential decay occurs when a quantity decreases by the same rate \( r \) in each time period \( t \). Just like exponential growth, the value of the quantity at any given time can be calculated by using the rate and the original amount.

An exponential decay function has the form \( y = a(1 - r)^t \), where \( a > 0 \).
- \( y \) represents the final amount.
- \( a \) represents the original amount.
- \( r \) represents the rate of decay as a decimal.
- \( t \) represents time.

Notice an important difference between exponential growth functions and exponential decay functions. For exponential growth, the value inside the parentheses will be greater than 1 because \( r \) is added to 1. For exponential decay, the value inside the parentheses will be less than 1 because \( r \) is subtracted from 1.

**Example 3**

The population of a town is decreasing at a rate of 1% per year. In 2000 there were 1300 people. Write an exponential decay function to model this situation. Then find the population in 2008.

**Step 1** Write the exponential decay function for this situation.

\[
y = a(1 - r)^t
\]

- \( y \) represents the final amount.
- \( a \) represents the original amount.
- \( r \) represents the rate of decay as a decimal.
- \( t \) represents time.

\[
y = 1300(1 - 0.01)^t
\]

- \( 1300 \) for \( a \) and \( 0.01 \) for \( r \).

\[
y = 1300(0.99)^t
\]

- Simplify.

**Step 2** Find the population in 2008.

\[
y = 1300(0.99)^8
\]

- Substitute 8 for \( t \).

\[
y \approx 1200
\]

- Use a calculator and round to the nearest whole number.

The population in 2008 is approximately 1200 people.

**Check It Out!**

3. The fish population in a local stream is decreasing at a rate of 3% per year. The original population was 48,000. Write an exponential decay function to model this situation. Then find the population after 7 years.

A common application of exponential decay is *half-life*. The \( \text{half-life} \) of a substance is the time it takes for one-half of the substance to decay into another substance.

**Half-life**

\[
A = P(0.5)^t
\]

- \( A \) represents the final amount.
- \( P \) represents the original amount.
- \( t \) represents the number of half-lives in a given time period.
**Science Application**

Fluorine-20 has a half-life of 11 seconds.

**A** Find the amount of fluorine-20 left from a 40-gram sample after 44 seconds.

**Step 1** Find \( t \), the number of half-lives in the given time period.

\[
\frac{44 \text{ s}}{11 \text{ s}} = 4 \quad \text{Divide the time period by the half-life.}
\]

The value of \( t \) is 4.

**Step 2** \[ A = P (0.5)^t \]

\[ = 40 (0.5)^4 \quad \text{Substitute 40 for} \ P \ \text{and} \ 4 \ \text{for} \ \ t. \]

\[ = 2.5 \quad \text{Use a calculator.} \]

There are 2.5 grams of fluorine-20 remaining after 44 seconds.

**B** Find the amount of fluorine-20 left from a 40-gram sample after 2.2 minutes. Round your answer to the nearest hundredth.

**Step 1** Find \( t \), the number of half-lives in the given time period.

\[
\frac{2.2(60)}{11 \text{ s}} = 132 \quad \text{Divide the time period by the half-life.}
\]

The value of \( t \) is \( \frac{132}{11} = 12 \).

**Step 2** \[ A = P (0.5)^t \]

\[ = 40 (0.5)^{12} \quad \text{Substitute 40 for} \ P \ \text{and} \ 12 \ \text{for} \ \ t. \]

\[ \approx 0.01 \quad \text{Use a calculator. Round to the nearest hundredth.} \]

There is about 0.01 gram of fluorine-20 remaining after 2.2 minutes.

**CHECK IT OUT!**

4a. Cesium-137 has a half-life of 30 years. Find the amount of cesium-137 left from a 100-milligram sample after 180 years.

4b. Bismuth-210 has a half-life of 5 days. Find the amount of bismuth-210 left from a 100-gram sample after 5 weeks. *(Hint: Change 5 weeks to days.)*

**THINK AND DISCUSS**

1. Describe three real-world situations that can be described by exponential growth or exponential decay functions.

2. The population of a town after \( t \) years can be modeled by \( P = 1000(1.02)^t \). Is the population increasing or decreasing? By what percentage rate?

3. An exponential function is a function of the form \( y = ab^x \). Explain why both exponential growth functions and exponential decay functions are exponential functions.

4. **GET ORGANIZED** Copy and complete the graphic organizer.

![Exponential Growth vs. Exponential Decay](image)
1. **Vocabulary** The function $y = 0.68(2)^x$ is an example of ______? ______.
   *(exponential growth or exponential decay)*

2. Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.
   2. The cost of tuition at a college is $12,000 and is increasing at a rate of 6% per year; 4 years.
   3. The number of student-athletes at a local high school is 300 and is increasing at a rate of 8% per year; 5 years.

3. Write a compound interest function to model each situation. Then find the balance after the given number of years.
   4. $1500 invested at a rate of 3.5% compounded annually; 4 years
   5. $4200 invested at a rate of 2.8% compounded quarterly; 6 years

4. Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.
   6. The value of a car is $18,000 and is depreciating at a rate of 12% per year; 10 years.
   7. The amount (to the nearest hundredth) of a 10-mg dose of a certain antibiotic decreases in your bloodstream at a rate of 16% per hour; 4 hours.

5. Bismuth-214 has a half-life of approximately 20 minutes. Find the amount of bismuth-214 left from a 30-gram sample after 1 hour.
   8. Mendelevium-258 has a half-life of approximately 52 days. Find the amount of mendelevium-258 left from a 44-gram sample after 156 days.

**PRACTICE AND PROBLEM SOLVING**

Write an exponential growth function to model each situation. Then find the value of the function after the given amount of time.

10. Annual sales for a company are $149,000 and are increasing at a rate of 6% per year; 7 years.
11. The population of a small town is 1600 and is increasing at a rate of 3% per year; 10 years.
12. A new savings account starts at $700 and increases at 1.2% yearly; 8 years.
13. Membership of a local club grows at a rate of 7.8% yearly and currently has 30 members; 6 years.

Write a compound interest function to model each situation. Then find the balance after the given number of years.

14. $28,000 invested at a rate of 4% compounded annually; 5 years
15. $7000 invested at a rate of 3% compounded quarterly; 10 years
16. $3500 invested at a rate of 1.8% compounded monthly; 4 years
17. $12,000 invested at a rate of 2.6% compounded annually; 15 years
Write an exponential decay function to model each situation. Then find the value of the function after the given amount of time.

18. The population of a town is 18,000 and is decreasing at a rate of 2% per year; 6 years.
19. The value of a book is $58 and decreases at a rate of 10% per year; 8 years.
20. The half-life of bromine-82 is approximately 36 hours. Find the amount of bromine-82 left from an 80-gram sample after 6 days.

Identify each of the following functions as exponential growth or decay. Then give the rate of growth or decay as a percent.

21. \(y = 3(1.61)^t\)
22. \(y = 39(0.098)^t\)
23. \(y = a\left(\frac{2}{3}\right)^t\)
24. \(y = a\left(\frac{3}{2}\right)^t\)
25. \(y = a(1.1)^t\)
26. \(y = a(0.8)^t\)
27. \(y = a\left(\frac{5}{4}\right)^t\)
28. \(y = a\left(\frac{1}{2}\right)^t\)

Write an exponential growth or decay function to model each situation. Then find the value of the function after the given amount of time.

29. The population of a country is 58,000,000 and grows by 0.1% per year; 3 years.
30. An antique car is worth $32,000, and its value grows by 7% per year; 5 years.
31. An investment of $8200 loses value at a rate of 2% per year; 7 years.
32. A new car is worth $25,000, and its value decreases by 15% each year; 6 years.
33. The student enrollment in a local high school is 970 students and increases by 1.2% per year; 5 years.
34. **Archaeology** Carbon-14 dating is a way to determine the age of very old organic objects. Carbon-14 has a half-life of about 5700 years. An organic object with \(\frac{1}{2}\) as much carbon-14 as its living counterpart died 5700 years ago. In 1999, archaeologists discovered the oldest bridge in England near Testwood, Hampshire. Carbon dating of the wood revealed that the bridge was 3500 years old. Suppose that when the bridge was built, the wood contained 15 grams of carbon-14. How much carbon-14 would it have contained when it was found by the archaeologists? Round to the nearest hundredth.

35. **ERROR ANALYSIS** Two students were asked to find the value of a $1000-item after 3 years. The item was depreciating (losing value) at a rate of 40% per year. Which is incorrect? Explain the error.

36. **Critical Thinking** The value of a certain car can be modeled by the function \(y = 20,000(0.84)^t\), where \(t\) is time in years. Will the value ever be zero? Explain.

37. The value of a rare baseball card increases every year at a rate of 4%. Today, the card is worth $300. The owner expects to sell the card as soon as the value is over $600. How many years will the owner wait before selling the card? Round your answer to the nearest whole number.
38. **Multi-Step Test Prep**

a. The annual tuition at a prestigious university was $20,000 in 2002. It generally increases at a rate of 9% each year. Write a function to describe the cost as a function of the number of years since 2002. Use 2002 as year zero when writing the function rule.

b. What do you predict the cost of tuition will be in 2008?

c. Use a table of values to find the first year that the cost of the tuition is more than twice the cost in 2002.

---

39. **Multi-Step**

At bank A, $600 is invested with an interest rate of 5% compounded annually. At bank B, $500 is invested with an interest rate of 6% compounded quarterly. Which account will have a larger balance after 10 years? 20 years?

40. **Estimation**

The graph shows the decay of 100 grams of sodium-24. Use the graph to estimate the number of hours it will take the sample to decay to 10 grams. Then estimate the half-life of sodium-24.

41. **Graphing Calculator**

Use a graphing calculator to graph $y = 10(1 + r)^x$ for $r = 10\%$ and $r = 20\%$. Compare the two graphs. How does the value of $r$ affect the graphs?

42. **Write About It**

Write a real-world situation that could be modeled by $y = 400(1.08)^t$.

43. **Write About It**

Write a real-world situation that could be modeled by $y = 800(0.96)^t$.

44. **Critical Thinking**

The amount of water in a container doubles every minute. After 6 minutes, the container is full. Your friend says it was half full after 3 minutes. Do you agree? Why or why not?

45. **Test Prep**

A population of 500 is decreasing by 1% per year. Which function models this situation?

- \( y = 500(0.01)^t \)
- \( y = 500(0.1)^t \)
- \( y = 500(0.9)^t \)
- \( y = 500(0.99)^t \)

46. Which function is NOT an exponential decay model?

- \( y = 5\left(\frac{1}{3}\right)^x \)
- \( y = -5\left(\frac{1}{3}\right)^x \)
- \( y = 5(3)^{-x} \)
- \( y = 5(3^{-1})^x \)

47. **Stephanie**

Stephanie wants to save $1000 for a down payment on a car that she wants to buy in 3 years. She opens a savings account that pays 5% interest compounded annually. About how much should Stephanie deposit now to have enough money for the down payment in 3 years?

- \( $295 \)
- \( $333 \)
- \( $500 \)
- \( $865 \)

48. **Short Response**

In 2000, the population of a town was 1000 and was growing at a rate of 5% per year.

a. Write an exponential growth function to model this situation.

b. In what year is the population 1300? Show how you found your answer.
49. You invest $700 at a rate of 6% compounded quarterly. Use a graph to estimate the number of years it will take for your investment to increase to $2300.

50. Omar invested $500 at a rate of 4% compounded annually. How long will it take for Omar's money to double? How long would it take if the interest were 8% compounded annually?

51. An 80-gram sample of a radioactive substance decayed to 10 grams after 300 minutes. Find the half-life of the substance.

52. Praseodymium-143 has a half-life of 2 weeks. The original measurement for the mass of a sample was lost. After 6 weeks, 15 grams of praseodymium-143 remain. How many grams was the original sample?

53. Phillip invested some money in a business 8 years ago. Since then, his investment has grown at an average rate of 1.3% compounded quarterly. Phillip's investment is now worth $250,000. How much was his original investment? Round your answer to the nearest dollar.

54. **Personal Finance** Anna has a balance of $200 that she owes on her credit card. She plans to make a $30 payment each month. There is also a 1.5% finance charge (interest) on the remaining balance each month. Copy and complete the table to answer the questions below. You may add more rows to the table as necessary.

<table>
<thead>
<tr>
<th>Month</th>
<th>Balance ($)</th>
<th>Monthly Payment ($)</th>
<th>Remaining Balance ($)</th>
<th>1.5% Finance Charge ($)</th>
<th>New Balance ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>30</td>
<td>170</td>
<td>2.55</td>
<td>172.55</td>
</tr>
<tr>
<td>2</td>
<td>172.55</td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>30</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. How many months will it take Anna to pay the entire balance?

b. By the time Anna pays the entire balance, how much total interest will she have paid?
Objective

Identify and extend patterns using recursion.

Vocabulary

recursive pattern

In a recursive pattern or recursive sequence, each term is defined using one or more previous terms. For example, the sequence 1, 4, 7, 10, 13, ... can be defined recursively as follows: The first term is 1 and each term after the first is equal to the preceding term plus 3.

You can use recursive techniques to identify patterns. The table summarizes the characteristics of four types of patterns.

<table>
<thead>
<tr>
<th>Type of Pattern</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>First differences are constant.</td>
</tr>
<tr>
<td>Quadratic</td>
<td>Second differences are constant.</td>
</tr>
<tr>
<td>Cubic</td>
<td>Third differences are constant.</td>
</tr>
<tr>
<td>Exponential</td>
<td>Ratios between successive terms are constant.</td>
</tr>
</tbody>
</table>

**Example 1**

**Identifying and Extending a Pattern**

Identify the type of pattern. Then find the next three numbers in the pattern.

**A**

4, 6, 10, 16, 24, ...

Find first, second, and, if necessary, third differences.

\[
\begin{array}{cccccc}
4 & 6 & 10 & 16 & 24 \\
+2 & +4 & +6 & +8 \\
+2 & +2 & +2 \\
\end{array}
\]

Second differences are constant, so the pattern is quadratic.

Extend the pattern by continuing the sequence of first and second differences.

\[
\begin{array}{ccccccccc}
4 & 6 & 10 & 16 & 24 & 34 & 46 & 60 \\
+2 & +4 & +6 & +8 & +10 & +12 & +14 \\
+2 & +2 & +2 \\
\end{array}
\]

The next three numbers in the pattern are 34, 46, and 60.

**B**

\[\frac{1}{8}, \frac{1}{2}, 2, 8, 32\]

Find the ratio between successive terms.

\[
\begin{array}{cccc}
\frac{1}{8} & \frac{1}{2} & 2 & 8 \\
\times 4 & \times 4 & \times 4 & \times 4 \\
\end{array}
\]

Ratios between terms are constant, so the pattern is exponential.

Extend the pattern by continuing the sequence of ratios.

\[
\begin{array}{cccccccc}
\frac{1}{8} & \frac{1}{2} & 2 & 8 & 32 & 128 & 512 & 2048 \\
\times 4 & \times 4 & \times 4 & \times 4 & \times 4 & \times 4 & \times 4 \\
\end{array}
\]

The next three numbers in the pattern are 128, 512, and 2048.
Identify the type of pattern. Then find the next three numbers in the pattern.

1a. 56, 47, 38, 29, 20, ...

1b. 1, 8, 27, 64, 125, ...

You can use a similar process to determine whether a function is linear, quadratic, cubic, or exponential. Note that before comparing $y$-values, you must first make sure there is a constant change in the corresponding $x$-values.

### Using Recursive Techniques to Identify Functions

<table>
<thead>
<tr>
<th>Type of Function</th>
<th>Characteristics (Given a Constant Change in $x$-values)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>First differences of $y$-values are constant.</td>
</tr>
<tr>
<td>Quadratic</td>
<td>Second differences of $y$-values are constant.</td>
</tr>
<tr>
<td>Cubic</td>
<td>Third differences of $y$-values are constant.</td>
</tr>
<tr>
<td>Exponential</td>
<td>Ratios between successive $y$-values are constant.</td>
</tr>
</tbody>
</table>

### Example 2

#### Identifying a Function

The ordered pairs {($-4$, $-4$), (0, 0), (4, 4), (8, 32), (12, 108)} satisfy a function. Determine whether the function is linear, quadratic, cubic, or exponential. Then find three additional ordered pairs that satisfy the function.

Make a table. Check for a constant change in the $x$-values. Then find first, second, and third differences of $y$-values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-4$</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-4$</td>
<td>0</td>
<td>4</td>
<td>32</td>
<td>108</td>
</tr>
</tbody>
</table>

There is a constant change in the $x$-values. Third differences are constant. The function is a cubic function.

To find additional ordered pairs, extend the pattern by working backward from the constant third differences.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-4$</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-4$</td>
<td>0</td>
<td>4</td>
<td>32</td>
<td>108</td>
<td>256</td>
<td>500</td>
<td>864</td>
</tr>
</tbody>
</table>

Three additional ordered pairs that satisfy this function are (16, 256), (20, 500), and (24, 864).
Several ordered pairs that satisfy a function are given. Determine whether the function is linear, quadratic, cubic, or exponential. Then find three additional ordered pairs that satisfy the function.

2a. \{(0, 1), (1, 3), (2, 9), (3, 19), (4, 33)\}

2b. \[\left\{ \left( \frac{1}{2}, \frac{1}{2}\right), \left( \frac{3}{6}, \frac{1}{18}\right), \left( \frac{5}{10}, \frac{1}{54}\right), \left( \frac{7}{14}, \frac{1}{162}\right) \right\}\]

Identify the type of pattern. Then find the next three numbers in the pattern.

1. 25, 28, 31, 34, 37, ...
2. 20, 45, 80, 125, 180, ...
3. 128, 64, 32, 16, 8, ...
4. 4, 32, 108, 256, 500, ...
5. \[\frac{1}{2}, \frac{3}{4}, 1, \frac{1}{4}, \frac{1}{2}, \ldots\]
6. \[0.3, 0.03, 0.003, 0.0003, 0.00003, \ldots\]
7. 127, 66, 29, 10, 3, ...
8. 2, 8, 18, 32, 50, ...

Several ordered pairs that satisfy a function are given. Determine whether the function is linear, quadratic, cubic, or exponential. Then find three additional ordered pairs that satisfy the function.

9. \{(3, 1), (5, –3), (7, –7), (9, –11), (11, –15)\}
10. \{(-1, –2), (2, 7), (5, 124), (8, 511), (11, 1330)\}
11. \[\left\{ \left( \frac{2}{4}, \frac{3}{8}\right), \left( \frac{4}{16}, \frac{1}{32}\right), \left( \frac{5}{64}, \frac{1}{64}\right) \right\}\]
12. \{(-3, –7), (0, 2), (3, –7), (6, –34), (9, –79)\}
13. \{(0, 600), (10, 480), (20, 384), (30, 307.2), (40, 245.76)\}
14. \{(-8, 2), (-5, 7), (-2, 12), (1, 17), (4, 22)\}

15. Entertainment The table shows the cost of using an online DVD rental service for different numbers of months.

<table>
<thead>
<tr>
<th>Months</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>92</td>
</tr>
<tr>
<td>9</td>
<td>134</td>
</tr>
<tr>
<td>12</td>
<td>176</td>
</tr>
<tr>
<td>15</td>
<td>218</td>
</tr>
</tbody>
</table>

a. Determine whether the function that models the data is linear, quadratic, cubic, or exponential. Explain.

b. Graph the data in the table.

c. What do you notice about your graph? Why does this make sense?

d. Predict the cost of the service for 18 months.

16. A student claimed that the function shown in the table is a quadratic function. Do you agree or disagree? Explain.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>7</th>
<th>10</th>
<th>14</th>
<th>17</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

\[\begin{array}{c}
+4 \\
+4 \\
+2 \\
+2 \\
+2 \\
+2 \\
+10 \\
\end{array}\]
17. **Business** The table shows the annual sales for a small company.

a. Determine whether the function that models the data is linear, quadratic, cubic, or exponential. Explain.

b. Suppose sales continue to grow according to the pattern in the table. Predict the annual sales for 2011, 2012, and 2013.

c. If the pattern continues, in what year will annual sales be $17,000 greater than the previous year’s sales?

<table>
<thead>
<tr>
<th>Year</th>
<th>Sales ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>513,000</td>
</tr>
<tr>
<td>2007</td>
<td>516,000</td>
</tr>
<tr>
<td>2008</td>
<td>521,000</td>
</tr>
<tr>
<td>2009</td>
<td>528,000</td>
</tr>
<tr>
<td>2010</td>
<td>537,000</td>
</tr>
</tbody>
</table>

18. **Critical Thinking** Use the table for the following problems.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Copy and complete the table so that the function is a linear function.

b. Copy and complete the table so that the function is a quadratic function.

c. Copy and complete the table so that the function is an exponential function.

d. For which of these three types of functions is there more than one correct way to complete the table? Explain.

Use the description to write the first five terms in each numerical pattern.

19. The first term is 8. Each following term is 11 less than the term before it.

20. The first term is 1000. Each following term is 40% of the term before it.

21. The first two terms are 1 and 2. Each following term is the sum of the two terms before it.

Make a table for a function that has the given characteristics. Include at least five ordered pairs.

22. The function is linear. The first differences are –3.

23. The function is quadratic. The second differences are 6.

24. The function is cubic. The third differences are 1.

A **recursive formula** for a sequence shows how to find the value of a term from one or more terms that come before it. For example, the recursive formula $a_n = a_{n-1} + 3$ tells you that each term is equal to the preceding term plus 3. Given that $a_1 = 5$, you can use the formula to generate the sequence 5, 8, 11, 14, ….

Write the first four terms of each sequence.

25. $a_n = a_{n-1} + 2; a_1 = 12$

26. $a_n = a_{n-1} - 7; a_1 = 16$

27. $a_n = 2a_{n-1}; a_1 = 4$

28. $a_n = 0.6a_{n-1}; a_1 = 100$

29. $a_n = 5a_{n-1} - 2; a_1 = 0$

30. $a_n = (a_{n-1})^2; a_1 = -2$

31. A **recursive function** defines a function for whole numbers by referring to the value of the function at previous whole numbers. Consider the recursive function $f(n) = f(n - 1) + 5$ with $f(0) = 1$.

a. According to the formula, $f(1) = f(0) + 5$. What is the value of $f(1)$?

b. Use the formula to find $f(2), f(3), f(4)$, and $f(5)$.

c. Graph $f(n)$ by plotting points at $x = 0, x = 1, x = 2, x = 3, x = 4,$ and $x = 5$.

d. What do you notice about your graph? What does this tell you about $f(n)$?
Comparing Functions

Objectives
Compare functions in different representations.
Estimate and compare rates of change.

Who uses this?
Investment analysts can use different function representations to compare investments. (See Example 2.)

You have studied different types of functions and how they can be represented as equations, graphs, and tables. Below is a review of three types of functions and some of their key properties.

<table>
<thead>
<tr>
<th>Linear</th>
<th>Quadratic</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>$y = mx + b$</td>
<td>$y = ax^2 + bx + c$, $a \neq 0$</td>
</tr>
<tr>
<td>Example</td>
<td>$y = 2x + 1$</td>
<td>$y = ab^x$, $a \neq 0$, $b \neq 1$, $b &gt; 0$</td>
</tr>
<tr>
<td>Graph</td>
<td><img src="image" alt="Graph" /></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>Table</td>
<td><img src="image" alt="Table" /></td>
<td><img src="image" alt="Table" /></td>
</tr>
<tr>
<td>$x$</td>
<td>$y$</td>
<td>$x$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>Constant first differences</td>
<td></td>
<td>Constant second differences</td>
</tr>
</tbody>
</table>

Example 1
Comparing Linear Functions

Deirdre and Beth each deposit money into their checking accounts weekly. Their account information for the past several weeks is shown below.

<table>
<thead>
<tr>
<th>Deirdre’s Account</th>
<th>Beth’s Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weeks</td>
<td>Account Balance ($)</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>1</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>105</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
</tr>
</tbody>
</table>
Compare the accounts by finding slopes and y-intercepts and interpreting those values in the context of the situation.

<table>
<thead>
<tr>
<th>Deirdre</th>
<th>Beth</th>
<th>Interpret and Compare.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>Slope Use (1, 75) and (2, 90).</td>
<td>The slope is the rate of change. Beth is saving at a higher rate.</td>
</tr>
<tr>
<td>y-intercept (0, 60)</td>
<td>y-intercept (0, 40)</td>
<td>The y-intercept is the beginning account balance. Deirdre started with more money.</td>
</tr>
</tbody>
</table>

1. Dave and Arturo each deposit money into their checking accounts weekly. Their account information for the past several weeks is shown. Compare the accounts by finding and interpreting slopes and y-intercepts.

Dave’s Account

<table>
<thead>
<tr>
<th>Weeks</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance ($)</td>
<td>30</td>
<td>42</td>
<td>54</td>
<td>66</td>
</tr>
</tbody>
</table>

Remember that nonlinear functions do not have a constant rate of change. One way to compare two nonlinear functions is to calculate their average rates of change over a certain interval. For a function \( f(x) \) whose graph contains the points \((x_1, y_1)\) and \((x_2, y_2)\), the average rate of change over the interval \([x_1, x_2]\) is the slope of the line through \((x_1, y_1)\) and \((x_2, y_2)\).

**EXAMPLE 2**

Comparing Exponential Functions

An investment analyst offers two different investment options for her customers. Compare the investments by finding and interpreting the average rates of change from year 0 to year 20.

Investment A

<table>
<thead>
<tr>
<th>Years</th>
<th>Value ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10.00</td>
</tr>
<tr>
<td>5</td>
<td>13.38</td>
</tr>
<tr>
<td>10</td>
<td>17.91</td>
</tr>
<tr>
<td>15</td>
<td>23.97</td>
</tr>
<tr>
<td>20</td>
<td>32.07</td>
</tr>
<tr>
<td>25</td>
<td>42.92</td>
</tr>
</tbody>
</table>

**EXAMPLE 2 Checking It Out**

Arturo’s Account

The notation \([0, 20]\) means all x-values from 0 to 20, including 0 and 20.
Calculate the average rates of change over [0, 20] by using the points whose
x-coordinates are 0 and 20.

Investment A
\[
\frac{32.07 - 10.00}{20 - 0} = \frac{22.07}{20} \approx 1.10 \quad \text{Use (0, 10.00) and (20, 32.07).}
\]

Investment B
\[
\frac{27 - 10}{20 - 0} = \frac{17}{20} = 0.85 \quad \text{Use the graph to estimate. When } x = 20, \quad y \approx 27. \text{ Use (0, 10) and (20, 27).}
\]

From year 0 to year 20, investment A increased at an average rate of $1.10
per year, while investment B increased at an average rate of $0.85 per year.

2. Compare the same investments’ average rates of change from
year 10 to year 25.

3. **EXAMPLE**

**Comparing Quadratic Functions**

Students in an engineering class were given an
assignment to design a parabola-shaped bridge.
Two students’ models are shown at right. Compare
the models by finding and interpreting maximums,
x-intercepts, and average rates of change over the
x-interval [0, 20].

- The maximum is the height of the bridge at its
  highest point.
- The difference of the x-intercepts is the length of
  the bridge.
- The average rate of change over [0, 20] indicates
  the steepness of the bridge over the first 20 horizontal feet.

For Rosetta’s plan, use the function.

**Step 1** Find the maximum.
\[
x = -\frac{b}{2a} = -\frac{1.2}{2(-0.01)} = \frac{1.2}{0.02} = 60
\]
Find the x-value of the vertex.
\[
y = -0.01(60)^2 + 1.2(60)
\]
= -36 + 72 = 36

The vertex is (60, 36) and the maximum is 36.

**Step 2** Find the x-intercepts.
\[
-0.01x^2 + 1.2x = 0
\]
\[-x(0.01x - 1.2) = 0
\]
\[x = 0 \text{ or } 0.01x - 1.2 = 0
\]
\[0.01x = 1.2
\]
\[x = 120
\]

The x-intercepts are 0 and 120.

**Step 3** Find the average rate of change over [0, 20].
At \(x = 0, \ y = -0.01(0)^2 + 1.2(0) = 0
\] 
Find the points whose x-coordinates
are 0 and 20.
At \(x = 20, \ y = -0.01(20)^2 + 1.2(20)
\]
= -4 + 24 = 20
Use (0, 0) and (20, 20): \[\frac{20 - 0}{20 - 0} = 1\]
For Marco’s plan, use the graph.

**Step 1** Find the maximum.

The maximum is slightly greater than 24, between 24 and 32.

**Step 2** Find the $x$-intercepts.

The $x$-intercepts are 0 and 100.

**Step 3** Find the average rate of change over $[0, 20]$.

Use $(0, 0)$ and $(20, 16)$: 

\[
\frac{16 - 0}{20 - 0} = \frac{16}{20} = 0.8
\]

---

### Maximum Height

<table>
<thead>
<tr>
<th></th>
<th>Rosetta's Bridge</th>
<th>Marco's Bridge</th>
<th>Interpret and Compare.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Height</td>
<td>36 ft</td>
<td>just over 24 ft</td>
<td>Rosetta’s bridge is taller.</td>
</tr>
<tr>
<td>Length</td>
<td>$120 - 0 = 120$ ft</td>
<td>$100 - 0 = 100$ ft</td>
<td>Rosetta’s bridge is longer.</td>
</tr>
<tr>
<td>Average Steepness</td>
<td>1</td>
<td>0.8</td>
<td>Rosetta’s bridge is steeper over $[0, 20]$.</td>
</tr>
<tr>
<td>Over $[0, 20]$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

### What if…?

Suppose Rosetta uses $y = -0.01x^2 + 1.1x$ and Marco uses the same plan as above. Compare Marco’s model with Rosetta’s new model.

---

### Example 4

#### Comparing Different Types of Functions

A town has approximately 1000 homes. The town council is considering plans for future development. Plan A calls for an increase of 200 homes per year. Plan B calls for a 10% increase each year. Compare the plans.

Let $x$ be the number of years. Let $y$ be the number of homes. Write functions to model each plan.

- **Plan A:** $y = 200x + 1000$
- **Plan B:** $y = 1000(1.10)^x$

Use your calculator to graph both functions.

The graphs show that under plan A, there will be more homes built than under plan B in early years.

But by the end of the 15th year, the number of homes built under plan B exceeds the number of homes built under plan A. From that point on, plan B results in more homes than plan A by ever-increasing amounts every year.

---

### Example 4 (continued)

- **Two neighboring schools use different models for anticipated growth in enrollment:** School A has 850 students and predicts an increase of 100 students per year. School B also has 850 students, but predicts an increase of 8% per year. Compare the models.
THINK AND DISCUSS

1. Explain why you need to use the word *average* when comparing rates of change for quadratic or exponential functions, but not for linear functions.

2. A function can be represented by an equation or a graph. Describe a possible advantage of each representation.

3. GET ORGANIZED Copy and complete the graphic organizer. Complete the sentence in each column by writing important values to compare.

<table>
<thead>
<tr>
<th>Comparing Functions</th>
<th>Linear to Linear</th>
<th>Exponential to Exponential</th>
<th>Quadratic to Quadratic</th>
<th>Linear to Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare...</td>
<td>Compare...</td>
<td>Compare...</td>
<td>Compare...</td>
<td>Compare...</td>
</tr>
</tbody>
</table>

**GUIDED PRACTICE**

**SEE EXAMPLE 1**

1. **Personal Finance** Fay and Kara each withdraw money from their savings accounts weekly, as shown. Compare the accounts by finding and interpreting slopes and *y*-intercepts.

   **Fay’s Account**
   - **Weeks**: 0, 1, 2, 3
   - **Account Balance ($)**: 425, 375, 325, 255

   **Kara’s Account**
   - **Account balance ($)**
   - **Weeks**: 0, 1, 2, 3, 4

2. **Biology** A biologist tracked the hourly growth of two different strains of bacteria in the lab. Her data are shown below. Compare the number of bacteria by finding and interpreting the average rates of change from hour 0 to hour 4.

   **Bacteria A**
   - **Hours**: 0, 1, 2, 3, 4
   - **Number of Bacteria**: 5, 15, 45, 135, 405

   **Bacteria B**
   - **Number of bacteria**
   - **Hours**: 0, 1, 2, 3, 4
3. **Architecture**  An architect designs arch-shaped passageways in the form of a parabola. Models for two of his designs are shown at right. Compare the designs by finding and interpreting maximums, \( x \)-intercepts, and average rates of change over the interval \([0, 2]\).

![Design A and Design B graphs]

4. **Business**  A bicycle store has approximately 200 bicycles in stock. The store owner is considering plans for expanding his inventory. Plan A calls for an increase of 30 bicycles per year. Plan B calls for a 10% increase each year. Compare the plans.

### PRACTICE AND PROBLEM SOLVING

5. **Recreation**  Kevin and Darius each hiked a mountain trail at different rates, as shown below. Compare the hikes by finding and interpreting slopes and \( y \)-intercepts.

<table>
<thead>
<tr>
<th>Kevin’s Hike</th>
<th>Darius’s Hike</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>Time (s)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Distance from Camp (mi)</td>
<td>Height (ft)</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>3.5</td>
<td>48</td>
</tr>
<tr>
<td>5.5</td>
<td>64</td>
</tr>
<tr>
<td>7.5</td>
<td>48</td>
</tr>
</tbody>
</table>

### Anthropology

6. **Anthropology**  An archeologist used these functions to model the changing populations of two ancient cities as they grew in size. Compare the populations by finding and interpreting the average rates of change over the interval \([0, 40]\).

<table>
<thead>
<tr>
<th>City A</th>
<th>City B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (mi²)</td>
<td>Population</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>31</td>
</tr>
<tr>
<td>20</td>
<td>89</td>
</tr>
<tr>
<td>30</td>
<td>164</td>
</tr>
<tr>
<td>40</td>
<td>252</td>
</tr>
<tr>
<td>50</td>
<td>353</td>
</tr>
</tbody>
</table>

### Physics

7. **Physics**  Rhea’s science teacher launched two model rockets straight up into the air. The functions that model the heights of the rockets are shown below. Compare the functions by finding and interpreting maximums, \( x \)-intercepts, and average rates of change over the \( x \)-interval \([0, 2]\).

<table>
<thead>
<tr>
<th>Rocket A</th>
<th>Rocket B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -16x^2 + 80x )</td>
<td>( y = \frac{-16}{2}x + 80x )</td>
</tr>
<tr>
<td>Time (s)</td>
<td>Time (s)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Height (ft)</td>
<td>Height (ft)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>48</td>
<td>48</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
8. **Recreation** A summer boating camp has 75 boats. The camp director is considering two proposals for increasing the number of boats to match the increase in the number of campers. Proposal A recommends increasing the number of boats by 5 boats per year. Proposal B recommends a 5% increase each year. Compare the proposals.

9. Three functions are given below. Make a table of values for each function using nonnegative coordinates. Then graph all 3 functions on the same coordinate plane. Compare and interpret the tables, graphs, and rates of change over [0, 4].
   \[ y = 5x + 30 \quad y = 3 + 5^x \quad y = 3x^2 + 5x \]

10. **Business** The revenue of a company based on the price of its product is modeled by the function below.

   ![Product Price and Revenue graph](image)

   a. Estimate the average rate of change over [0, 4].
   b. What price will yield the maximum revenue?

11. **Critical Thinking** A karate center has 120 students. The director wants to set a goal to motivate her instructors to increase student enrollment. Under plan A, the goal is to increase the number of students by 12% each year. Under plan B, the goal is to increase the number of students by 20 each year.
   a. Compare the plans.
   b. Which plan should the director choose to double the enrollment in the shortest amount of time? Explain.
   c. Which plan should she use to triple the enrollment in the shortest amount of time? Explain.

12. **Write About It** Compare the characteristics of linear, quadratic, and exponential functions, including any of the following that are applicable: rate of change, \(x\)-intercept, and maximum/minimum. Explain how to decide which type of function is being shown on a graph and in a table.

13. Tanya has $2000 in her savings account. She wants to save more money. She is considering two savings plans. Under plan A, she will increase her account balance by $1000 per year. Under plan B, she will increase her account balance by 20% each year. How much more will she save with plan B after 10 years? Round your answer to the nearest dollar.
   - A $383
   - B $9562
   - C $12000
   - D $12383
14. The equation for the motion of a model rocket fired straight up with an initial velocity of 64 feet per second is \( h = 64t - 16t^2 \). Which could be the graph of this function?

[Graphs F, G, H, J showing different functions]

**CHALLENGE AND EXTEND**

15. Two functions are shown below.

**Function A**  
\[ y = 12 - 2x \]

**Function B**  
\[ y = 12 - 2^x \]

a. Estimate the average rate of change over the intervals [0, 4], [0, 6], [4, 6], [7, 10], and [8, 12] for both functions.

b. Explain how the average rate of change varies for each function.

c. **What if...?** For function B, if \( y \) is doubled while \( x \) remains the same, what is the effect on the rate of change over [0, 4]?
Vocabulary

common ratio  
exponential growth
compound interest  
geometric sequence
exponential decay  
half-life
exponential function

Complete the sentences below with vocabulary words from the list above.
1. 3, 6, 12, 24, ... is an example of a(n) ___?___.
2. A(n) ___?_____ function has the form \( y = a(1 - r)^t \), where \( a > 0 \).
3. In the formula \( a_n = a_1r^{n-1} \), the variable \( r \) represents the ___?_____.
4. \( f(x) = 2^x \) is an example of a(n) ___?_____.

8-1 Geometric Sequences

**Example**

What is the 10th term of the geometric sequence -6400, 3200, -1600, 800, ...?

*Find the common ratio by dividing consecutive terms.*

\[
\begin{align*}
\frac{3200}{-6400} &= -0.5 \\
\frac{-1600}{3200} &= -0.5
\end{align*}
\]

\( a_n = a_1r^{n-1} \)  \( \rightarrow \) Write the formula.

\[
\begin{align*}
\quad a_{10} &= -6400( -0.5)^{10-1} \\
\quad &= -6400( -0.5)^9 \\
\quad &= 12.5
\end{align*}
\]

**Exercises**

Find the next three terms in each geometric sequence.

5. 1, 3, 9, 27, ...  
6. 3, -6, 12, -24, ...  
7. 80, 40, 20, 10, ...  
8. -1, -4, -16, -64, ...  
9. The first term of a geometric sequence is 4 and the common ratio is 5. What is the 10th term?  
10. What is the 15th term of the geometric sequence 4, 12, 36, 108, ...?
8-2 Exponential Functions

**Example**

Tell whether the ordered pairs \{(1, 4), (2, 16), (3, 36), (4, 64)\} satisfy an exponential function. Explain.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
</tr>
</tbody>
</table>

As the x-values increase by a constant amount, the y-values are not multiplied by a constant amount. This function is not exponential.

**Exercises**

Tell whether each set of ordered pairs satisfies an exponential function. Explain.

11. \{(0, 1), (2, 9), (4, 81), (6, 729)\}
12. \{(-2, -8), (-1, -4), (0, 0), (1, 4)\}

Graph each exponential function.

13. \(y = 4^x\)
14. \(y = \left(\frac{1}{4}\right)^x\)

8-3 Exponential Growth and Decay

**Example**

The value of a piece of antique furniture has been increasing at a rate of 2% per year. In 1990, its value was $800. Write an exponential growth function to model the situation. Then find the value of the furniture in the year 2010.

Step 1 \(y = a(1 + r)^t\) Write the formula.
\(y = 800(1 + 0.02)^t\) Substitute.
\(y = 800(1.02)^t\) Simplify.

Step 2 \(y = 800(1.02)^{20}\) Substitute 20 for t.
\(\approx 1188.76\) Simplify and round.

The furniture’s value will be $1188.76.

**Exercises**

15. The number of students in the book club is increasing at a rate of 15% per year. In 2001, there were 9 students in the book club. Write an exponential growth function to model the situation. Then find the number of students in the book club in the year 2008.

16. The population of a small town is decreasing at a rate of 4% per year. In 1970, the population was 24,500. Write an exponential decay function to model the situation. Then find the population in the year 2020.
Graph each data set. Which kind of model best describes the data?

17. \[ \{(-2, -12), (-1, -3), (0, 0), (1, -3), (2, -12)\} \]

18. \[ \{(-2, -2), (-1, 2), (0, 6), (1, 10), (2, 14)\} \]

19. \[ \{(-2, -\frac{1}{4}), (-1, -\frac{1}{2}), (0, -1), (1, -2), (2, -4)\} \]

Look for a pattern in each data set to determine which kind of model best describes the data.

20. \[ \{(0, 2), (1, 6), (2, 18), (3, 54), (4, 162)\} \]

21. \[ \{(0, 0), (2, -20), (4, -80), (6, -180), (8, -320)\} \]

22. \[ \{(-8, 5), (-4, 3), (0, 1), (4, -1), (8, -3)\} \]

23. Write a function that models the data. Then use your function to predict how long the humidifier will produce steam with 10 quarts of water.

### Input and Output of a Humidifier

<table>
<thead>
<tr>
<th>Water Volume (qt)</th>
<th>Steam Time (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.5</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>7.5</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Jasmin’s debt doubles every year.

For a constant change in time (+1), there is a constant ratio of 2, so the data is exponential.

\[
\begin{align*}
    y &= ab^x \\
    y &= a(2)^x \\
    130 &= a(2)^1 \\
    a &= 65 \\
    y &= 65(2)^x \\
    y &= 65(2)^8 \\
    y &= 16,640
\end{align*}
\]

Jasmin’s debt in 8 years will be $16,640.
Decide which linear function is increasing at a greater rate.

- Function 1 has $x$-intercept $4$ and $y$-intercept $-2$.
- Function 2 includes the points in the table below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-2$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-11$</td>
<td>$-6$</td>
<td>$-1$</td>
<td>$4$</td>
</tr>
</tbody>
</table>

The points $(4, 0)$ and $(0, -2)$ are on the graph of Function 1. Its slope is $\frac{0 - (-2)}{4 - 0} = \frac{1}{2}$. The table for Function 2 shows that for each increase of 1 in the value of $x$ there is an increase of 5 in the value of $y$. Its slope is 5. So, Function 2 is increasing more rapidly.

Use the given information to decide which quadratic function has the lesser minimum value.

- Function 1: The function whose equation is $y = 3x^2 - 12x + 1$.
- Function 2: The function whose graph is shown.

The minimum value of Function 1 is $-11$. It can be found by identifying the $y$-value of the vertex of its parabola. The minimum value of Function 2 can be seen on the graph of the function; it is $-9$. So, Function 1 has the lesser minimum value.

23. Decide which linear function is increasing at a greater rate.

- Function 1 has $x$-intercept $-7$ and $y$-intercept $4$.
- Function 2 includes the points in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$-8$</td>
<td>$-3$</td>
<td>$2$</td>
<td>$7$</td>
</tr>
</tbody>
</table>

24. Use the given information to decide which quadratic function has the lesser minimum value.

- Function 1: The function whose equation is $y = -x^2 + 4x + 2$.
- Function 2: The function whose graph is shown.

25. Michael is studying population changes in two types of birds living on an island. Compare the populations by finding and interpreting the average rates of change over the interval $[0, 18]$.

**Bird A**

<table>
<thead>
<tr>
<th>Time (months)</th>
<th>0</th>
<th>6</th>
<th>12</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (thousands)</td>
<td>8.3</td>
<td>8.6</td>
<td>8.8</td>
<td>9.1</td>
</tr>
</tbody>
</table>

**Bird B**

$y = 3.6(1.06)x$
Find the next three terms in each geometric sequence.

1. 2, 6, 18, 54, …  
2. 4800, 2400, 1200, 600, …  
3. −4, 20, −100, 500, …

4. **Communication** If school is cancelled, the school secretary calls 2 families. Each of those families calls 2 other families. In the third round of calls, each of the 4 families calls 2 more families. If this pattern continues, how many families are called in the seventh round of calls?

Graph each exponential function.

5. \( y = -2(4)^x \)  
6. \( y = 3(2)^x \)  
7. \( y = 4\left(\frac{1}{2}\right)^x \)  
8. \( y = -\left(\frac{1}{3}\right)^x \)

9. A teacher is repeatedly enlarging a diagram on a photocopier. The function \( f(x) = 3(1.25)^x \) represents the length of the diagram, in centimeters, after \( x \) enlargements. What is the length after 5 enlargements? Round to the nearest centimeter.

10. Chelsea invested $5600 at a rate of 3.6% compounded quarterly. Write a compound interest function to model the situation. Then find the balance after 6 years.

11. The number of trees in a forest is decreasing at a rate of 5% per year. The forest had 24,000 trees 15 years ago. Write an exponential decay function to model the situation. Then find the number of trees now.

Look for a pattern in each data set to determine which kind of model best describes the data.

12. \( \{(-10, -17), (-5, -7), (0, 3), (5, 13), (10, 23)\} \)

13. \( \{(1, 3), (2, 9), (3, 27), (4, 81), (5, 243)\} \)

14. Use the data in the table to describe how the bacteria population is changing. Then write a function that models the data. Use your function to predict the bacteria population after 10 hours.

<table>
<thead>
<tr>
<th>Time (h)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacteria</td>
<td>6</td>
<td>18</td>
<td>54</td>
<td>162</td>
</tr>
</tbody>
</table>

15. Greta is measuring how much two different candles burn over time. Compare the two candles by finding and interpreting slopes and y-intercepts.

- Candle 1 was 6 inches tall and burned for 1.5 hours.
- Candle 2 reached the heights at certain times shown in the table.

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (inches)</td>
<td>9</td>
<td>7.5</td>
<td>6</td>
<td>4.5</td>
<td>3</td>
</tr>
</tbody>
</table>

16. The information below models the population of fish living in two different lakes. Compare the populations by finding and interpreting the average rates of change over the interval \([0, 50]\).

**Fish in Lake A**

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>0</th>
<th>10</th>
<th>30</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population (thousands)</td>
<td>32</td>
<td>33.3</td>
<td>36.1</td>
<td>39.1</td>
</tr>
</tbody>
</table>

**Fish in Lake B**

\( y = 45(1.007)^x \)
Chapter Project Online

• Apply transformations to different families of functions.
• Fit data to linear models.

**Big as a Whale**

Humpback whales are among the world’s largest animals. You can use expressions and functions to compare the sizes of whales to various objects.
Study Strategy: Use Your Book for Success

Understanding how your textbook is organized will help you locate and use helpful information.

Pay attention to the margin notes. Know-It Note icons point out key information. Helpful Hints, Remember notes, and Caution notes help you understand concepts and avoid common mistakes.

The Glossary is found in the back of your textbook. Use it as a resource when you need the definition of an unfamiliar word or property.

The Index is located at the end of your textbook. Use it to locate the page where a particular concept is taught.

The Problem Solving Handbook is found in the back of your textbook. These pages review strategies that can help you solve real-world problems.

Try This

Use your textbook for the following problems.

1. Use the index to find the page where a term from this chapter is defined.

2. Describe how a strategy from the Problem Solving Workbook can be used in this chapter.

3. Use the glossary to find the definition of a term from this chapter.
In this chapter, you have examined relationships between sets of ordered pairs, or data. Displaying data visually can help you see relationships. A **scatter plot** is a graph with points plotted to show a possible relationship between two sets of data. A scatter plot is an effective way to display some types of data.

**Example 1**

**Graphing a Scatter Plot from Given Data**

The table shows the number of species added to the list of endangered and threatened species in the United States during the given years. Graph a scatter plot using the given data.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Species</td>
<td>91</td>
<td>79</td>
<td>62</td>
<td>11</td>
<td>39</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

*Source: U.S. Fish and Wildlife Service*

Use the table to make ordered pairs for the scatter plot.

The x-value represents the calendar year and the y-value represents the number of species added.

Plot the ordered pairs.

**Helpful Hint**

The point (2000, 39) tells you that in the year 2000, the list increased by 39 species.

1. The table shows the number of points scored by a high school football team in the first four games of a season. Graph a scatter plot using the given data.

<table>
<thead>
<tr>
<th>Game</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>6</td>
<td>21</td>
<td>46</td>
<td>34</td>
</tr>
</tbody>
</table>

A **correlation** describes a relationship between two data sets. A graph may show the correlation between data. The correlation can help you analyze trends and make predictions. There are three types of correlations between data.
In the endangered species graph, as time increases, the number of new species added decreases. So the correlation between the data is negative.

**Example 2**

Describe the correlation illustrated by the scatter plot.

**TV Watching and Test Scores**

As the number of hours spent watching TV increased, test scores decreased.

There is a negative correlation between the two data sets.

**Example 3**

Identify the correlation you would expect to see between each pair of data sets. Explain.

**A** the number of empty seats in a classroom and the number of students seated in the class

You would expect to see a negative correlation. As the number of students increases, the number of empty seats decreases.

**B** the number of pets a person owns and the number of books that person read last year

You would expect to see no correlation. The number of pets a person owns has nothing to do with how many books the person has read.
Identify the correlation you would expect to see between each pair of data sets. Explain.

- the monthly rainfall and the depth of water in a reservoir
  You would expect to see a positive correlation. As more rain falls, there is more water in the reservoir.

Identify the correlation you would expect to see between each pair of data sets. Explain.

3a. the temperature in Houston and the number of cars sold in Boston
3b. the number of members in a family and the size of the family’s grocery bill
3c. the number of times you sharpen your pencil and the length of your pencil

**Example 4**  
**Matching Scatter Plots to Situations**

Choose the scatter plot that best represents the relationship between the number of days since a sunflower seed was planted and the height of the plant. Explain.

Graph A

There will be a positive correlation between the number of days and the height because the plant will grow each day.

Graph B

Neither the number of days nor the plant heights can be negative.

Graph C

This graph shows all positive coordinates and a positive correlation, so it could represent the data sets.

Graph C is the correct scatter plot.

4. Choose the scatter plot that best represents the relationship between the number of minutes since a pie has been taken out of the oven and the temperature of the oven. Explain.

Graph A

Neither the number of minutes nor the temperature can be negative.

Graph B

This graph shows a negative correlation, so it is incorrect.

Graph C

This graph shows all positive coordinates and a negative correlation, so it is incorrect.

Graph B is the correct scatter plot.
You can graph a line on a scatter plot to help show a relationship in the data. This line, called a trend line, helps show the correlation between data sets more clearly. It can also be helpful when making predictions based on the data.

**Fund-raising Application**

The scatter plot shows a relationship between the total amount of money collected and the total number of rolls of wrapping paper sold as a school fund-raiser. Based on this relationship, predict how much money will be collected when 175 rolls have been sold.

Draw a trend line and use it to make a prediction.

Based on the data, $1200 is a reasonable prediction of how much money will be collected when 175 rolls have been sold.

**Check It Out!**

5. Based on the trend line above, predict how many wrapping paper rolls need to be sold to raise $500.

**Think and Discuss**

1. Is it possible to make a prediction based on a scatter plot with no correlation? Explain your answer.

2. GET ORGANIZED Copy and complete the graphic organizer with either a scatter plot, a real-world example, or both.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td><img src="positive_correlation.png" alt="" /></td>
</tr>
<tr>
<td>Negative</td>
<td><img src="negative_correlation.png" alt="" /></td>
</tr>
<tr>
<td>No</td>
<td>The amount of water in a watering can and the number of flowers watered</td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

**Vocabulary** Apply the vocabulary from this lesson to answer each question.
1. Give an example of a graph that is not a scatter plot.
2. How is a scatter plot that shows no correlation different from a scatter plot that shows a negative correlation?
3. Does a trend line always pass through every point on a scatter plot? Explain.
4. Graph a scatter plot using the given data.

<table>
<thead>
<tr>
<th>Garden Statue</th>
<th>Cupid</th>
<th>Gnome</th>
<th>Lion</th>
<th>Flamingo</th>
<th>Wishing well</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.)</td>
<td>32</td>
<td>18</td>
<td>35</td>
<td>28</td>
<td>40</td>
</tr>
<tr>
<td>Price ($)</td>
<td>50</td>
<td>25</td>
<td>80</td>
<td>15</td>
<td>75</td>
</tr>
</tbody>
</table>

Describe the correlation illustrated by each scatter plot.
5. Turnpike Tolls

![Graph A](image)

6. Movie Circulation

![Graph B](image)

Identify the correlation you would expect to see between each pair of data sets. Explain.
7. the volume of water poured into a container and the amount of empty space left in the container
8. a person’s shoe size and the length of the person’s hair
9. the outside temperature and the number of people at the beach

Choose the scatter plot that best represents the described relationship. Explain.
10. age of car and number of miles traveled
11. age of car and sales price of car
12. age of car and number of states traveled to

![Graph C](image)
13. **Transportation** The scatter plot shows the total number of miles passengers flew on U.S. domestic flights in the month of April for the years 1997–2004. Based on this relationship, predict how many miles passengers flew in April 2008.

![U.S. Domestic Air Travel in April](image)

**PRACTICE AND PROBLEM SOLVING**

14. Graph a scatter plot using the given data.

<table>
<thead>
<tr>
<th>Train Arrival Time</th>
<th>Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:45 A.M.</td>
<td>160</td>
</tr>
<tr>
<td>7:30 A.M.</td>
<td>148</td>
</tr>
<tr>
<td>8:15 A.M.</td>
<td>194</td>
</tr>
<tr>
<td>9:45 A.M.</td>
<td>152</td>
</tr>
<tr>
<td>10:30 A.M.</td>
<td>64</td>
</tr>
</tbody>
</table>

Describe the correlation illustrated by each scatter plot.

15. **Nascar**

![Nascar Scatter Plot](image)

16. **Concert Ticket Costs**

![Concert Ticket Costs Scatter Plot](image)

Identify the correlation you would expect to see between each pair of data sets. Explain.

17. the speed of a runner and the distance she can cover in 10 minutes

18. the year a car was made and the total mileage

Choose the scatter plot that best represents the described relationship. Explain.

19. the number of college classes taken and the number of roommates

20. the number of college classes taken and the hours of free time.

21. **Ecology** The scatter plot shows a projection of the average ocelot population living in Laguna Atascosa National Wildlife Refuge near Brownsville, Texas. Based on this relationship, predict the number of ocelots living at the wildlife refuge in 2014 if nothing is done to help manage the ocelot population.

![Ocelot Population Scatter Plot](image)

Ecology

The ocelot population in Texas is dwindling due in part to their habitat being destroyed. The ocelot population at Laguna Atascosa National Wildlife Refuge is monitored by following 5–10 ocelots yearly by radio telemetry.
22. **Estimation** Angie enjoys putting jigsaw puzzles together. The scatter plot shows the number of puzzle pieces and the time in minutes it took her to complete each of her last six puzzles. Use the trend line to estimate the time in minutes it will take Angie to complete a 1200-piece puzzle.

23. **Critical Thinking** Describe the correlation between the number of left shoes sold and the number of right shoes sold.

24. Roma had guests for dinner at her house eight times and has recorded the number of guests and the total cost for each meal in the table.

<table>
<thead>
<tr>
<th>Guests</th>
<th>3</th>
<th>4</th>
<th>4</th>
<th>6</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>30</td>
<td>65</td>
<td>88</td>
<td>90</td>
<td>115</td>
<td>160</td>
<td>150</td>
<td>162</td>
</tr>
</tbody>
</table>

a. Graph a scatter plot of the data.
b. Describe the correlation.
c. Draw a trend line.
d. Based on the trend line you drew, predict the cost of dinner for 11 guests.
e. **What if...?** Suppose that each cost in the table increased by $5. How will this affect the cost of dinner for 11 guests?

25. **ERROR ANALYSIS** Students graphed a scatter plot for the temperature of hot bath water and time if no new water is added. Which graph is incorrect? Explain the error.

26. **Critical Thinking** Will more people or fewer people buy an item if the price goes up? Explain the relationship and describe the correlation.

27. Juan and his parents are visiting a university 205 miles from their home. As they travel, Juan uses the car odometer and his watch to keep track of the distance.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>28</td>
</tr>
<tr>
<td>60</td>
<td>58</td>
</tr>
<tr>
<td>90</td>
<td>87</td>
</tr>
<tr>
<td>120</td>
<td>117</td>
</tr>
<tr>
<td>150</td>
<td>148</td>
</tr>
<tr>
<td>180</td>
<td>178</td>
</tr>
<tr>
<td>210</td>
<td>205</td>
</tr>
</tbody>
</table>

a. Make a scatter plot for this data set.
b. Describe the correlation. Explain.
c. Draw a trend line for the data and predict the distance Juan would have traveled going to a university 4 hours away.
28. Write About It Conduct a survey of your classmates to find the number of siblings they have and the number of pets they have. Predict whether there will be a positive, negative, or no correlation. Then graph the data in a scatter plot. What is the relationship between the two data sets? Was your prediction correct?

29. Which graph is the best example of a negative correlation?

30. Which situation best describes a positive correlation?
   - The amount of rainfall on Fridays
   - The height of a candle and the amount of time it stays lit
   - The price of a pizza and the number of toppings added
   - The temperature of a cup of hot chocolate and the length of time it sits

31. Short Response Write a real-world situation for the graph. Explain your answer.

CHALLENGE AND EXTEND

32. Describe a situation that involves a positive correlation. Gather data on the situation. Make a scatter plot showing the correlation. Use the scatter plot to make a prediction. Repeat for a negative correlation and for no correlation.

33. Research an endangered or threatened species in your state. Gather information on its population for several years. Make a scatter plot using the data you gather. Is there a positive or negative correlation? Explain. Draw a trend line and make a prediction about the species population over the next 5 years.
**Interpret Scatter Plots and Trend Lines**

You can use a graphing calculator to graph a trend line on a scatter plot.

**Activity**

The table shows the recommended dosage of a particular medicine as related to a person’s weight. Graph a scatter plot of the given data. Draw the trend line. Then predict the dosage for a person weighing 240 pounds.

<table>
<thead>
<tr>
<th>Weight (lb)</th>
<th>90</th>
<th>100</th>
<th>110</th>
<th>125</th>
<th>140</th>
<th>155</th>
<th>170</th>
<th>180</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dosage (mg)</td>
<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
<td>40</td>
<td>53</td>
<td>60</td>
<td>66</td>
<td>75</td>
</tr>
</tbody>
</table>

1. First enter the data. Press \( \text{STAT} \) and select 1: Edit. In L1, enter the first weight. Press \( \text{ENTER} \). Continue entering all weights. Use \( \text{C-} \) to move to L2. Enter the first dosage. Press \( \text{ENTER} \). Continue entering all dosages.

2. To view the scatter plot, press \( \text{2nd} \ Y= \). Select Plot 1. Select On, the first plot type, and the plot mark +. Press \( \text{ZOOM} \). Select 9: ZoomStat. You should see a scatter plot of the data.

3. To find the trend line, press \( \text{STAT} \) and select the CALC menu. Select \( \text{LinReg} (ax+b) \). Press \( \text{ENTER} \). This gives you the values of \( a \) and \( b \) in the trend line.

4. To enter the equation for the trend line, press \( Y= \) and then input \( 0.5079441502x - 26.78767453 \). Press \( \text{GRAPH} \).

5. Now predict the dosage for a weight of 240 pounds. Press \( \text{VARS} \). Select \( \text{Y-VARS} \) menu and select 1:Function. Select 1:Y1. Enter (240). Press \( \text{ENTER} \). The dosage is about 95 milligrams.

**Try This**

1. The table shows the price of a stock over an 8-month period. Graph a scatter plot of the given data. Draw the trend line. Then predict what the price of one share of stock will be in the twelfth month.

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ($)</td>
<td>32</td>
<td>35</td>
<td>37</td>
<td>41</td>
<td>46</td>
<td>50</td>
<td>54</td>
<td>59</td>
</tr>
</tbody>
</table>
Median-Fit Line

You have learned about trend lines. Now you will learn about another line of fit called the *median-fit line*.

**Example**

At a water raft rental shop, a group of up to four people can rent a single raft. The table shows the number of rafts rented to different groups of people one morning. Graph the median-fit line for the data.

<table>
<thead>
<tr>
<th>People x</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rafts Rented y</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

1. Plot the points on a coordinate plane.
2. Divide the data into three sections of equal size. Find the medians of the *x*-values and the *y*-values for each section. Plot the three median points with an X.

<table>
<thead>
<tr>
<th>People x</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rafts Rented y</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

   Median point: (3, 1)   Median point: (6, 4.5)   Median point: (11.5, 4.5)

3. Connect the outside, or first and third, median points with a line.
4. Lightly draw a dashed line straight down from the middle median point to the line just drawn. Mark the dashed line to create three equal segments.
5. Keeping your ruler parallel to the first line you drew, move your ruler to the mark closest to the line. Draw the line. This is the median-fit line.

**Try This**

1. A manager at a restaurant kept track one afternoon of the number of people in a party and the time it took to seat them. Graph the median-fit line for the data.

<table>
<thead>
<tr>
<th>People x</th>
<th>3</th>
<th>7</th>
<th>8</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wait Time y (min)</td>
<td>1</td>
<td>5</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

2. Use your median-fit line to predict the time it would take to seat a party of 6.
Objectives
Determine a line of best fit for a set of linear data.
Determine and interpret the correlation coefficient.

Vocabulary
residual
least-squares line
line of best fit
linear regression
correlation coefficient

Who uses this?
Climate scientists can use a least-squares line to study temperature-latitude relationships. (See Example 2.)

Recall that a scatter plot shows two data sets as one set of ordered pairs. A trend line, or line of fit, is a model for the data.

Some trend lines will fit a data set better than others. One way to evaluate how well a line fits a data set is to use residuals. A residual is the signed vertical distance between a data point and a line of fit. The closer the sum of the squared residuals is to 0, the better the line fits the data.

Example 1. Calculating Residuals

The data in the table are graphed along with two lines of fit. For each line, find the sum of the squares of the residuals. Which line is a better fit?

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>6</td>
<td>3</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

Find the residuals.

<table>
<thead>
<tr>
<th>e₁</th>
<th>e₂</th>
<th>e₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

Sum of squared residuals:
(2)² + (-2)² + (1)² = 9

The line \( y = \frac{1}{2}x + 3 \) is a better fit for the data.
1. Two lines of fit for this data are
   \[ y = -\frac{1}{2}x + 6 \] and \[ y = -x + 8. \] For each line, find the sum of the squares of the residuals. Which line is a better fit?

The **least-squares line** for a data set is the line of fit for which the sum of the squares of the residuals is as small as possible. So, the least-squares line is a **line of best fit**. A **line of best fit** is the line that comes closest to all of the points in the data set, using a given process. **Linear regression** is a process of finding the least-squares line.

### Example 2: Finding the Least-Squares Line

The table shows the latitudes and average temperatures of several cities.

<table>
<thead>
<tr>
<th>City</th>
<th>Latitude</th>
<th>Average Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrow, Alaska, USA</td>
<td>71.2° N</td>
<td>-12.7</td>
</tr>
<tr>
<td>Yakutsk, Russia</td>
<td>62.1° N</td>
<td>-10.1</td>
</tr>
<tr>
<td>London, England</td>
<td>51.3° N</td>
<td>10.4</td>
</tr>
<tr>
<td>Chicago, Illinois, USA</td>
<td>41.9° N</td>
<td>10.3</td>
</tr>
<tr>
<td>San Francisco, California, USA</td>
<td>37.5° N</td>
<td>13.8</td>
</tr>
<tr>
<td>Yuma, Arizona, USA</td>
<td>32.7° N</td>
<td>22.8</td>
</tr>
<tr>
<td>Tindouf, Algeria</td>
<td>27.7° N</td>
<td>22.8</td>
</tr>
<tr>
<td>Dakar, Senegal</td>
<td>14.0° N</td>
<td>24.5</td>
</tr>
<tr>
<td>Mangalore, India</td>
<td>12.5° N</td>
<td>27.1</td>
</tr>
</tbody>
</table>

**A** Find an equation for a line of best fit.

Use your calculator. To enter the data, press **STAT** and select **1:Edit**. Enter the latitudes in the L1 column and the average temperatures in the L2 column.

Then press **STAT** and choose **CALC**. Choose **4:LinReg(ax+b)** and press **ENTER**. An equation for a line of best fit is

\[ y \approx -0.69x + 39.11. \]

**B** Interpret the meaning of the slope and y-intercept.

The slope, \(-0.69\), means that for each 1-degree increase in latitude, the average temperature decreases 0.69 °C. The y-intercept, 39.11, means that the average temperature is 39.11 °C at 0° N latitude.

**C** The approximate latitude of Vancouver, Canada, is 49.1° N. Use your equation to predict Vancouver's average temperature.

\[ y \approx -0.69x + 39.11 \]

\[ y \approx -0.69(49.1) + 39.11 \approx 5.23 \]

The average temperature of Vancouver should be close to 5 °C.
2. The table shows the prices and the lengths in yards of several balls of yarn at Knit Mart.

<table>
<thead>
<tr>
<th>Length (yd)</th>
<th>Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1680</td>
<td>65.85</td>
</tr>
<tr>
<td>100</td>
<td>7.85</td>
</tr>
<tr>
<td>153</td>
<td>9.80</td>
</tr>
<tr>
<td>99</td>
<td>10.85</td>
</tr>
<tr>
<td>109</td>
<td>8.35</td>
</tr>
<tr>
<td>109</td>
<td>7.85</td>
</tr>
<tr>
<td>176</td>
<td>19.85</td>
</tr>
<tr>
<td>100</td>
<td>5.35</td>
</tr>
<tr>
<td>1440</td>
<td>65.85</td>
</tr>
<tr>
<td>61</td>
<td>14.85</td>
</tr>
</tbody>
</table>

a. Find an equation for a line of best fit.

b. Interpret the meaning of the slope and y-intercept.

c. Knit Mart also sells yarn in a 1000-yard ball. Use your equation to predict the cost of this yarn.

In Example 2, you may have noticed the last value the calculator gave you, \( r \). This is the correlation coefficient. The correlation coefficient \( r \) is a number, where \(-1 \leq r \leq 1\), that describes how closely the points in a scatter plot cluster around a line of best fit.

Properties of the Correlation Coefficient \( r \)

- \( r \) is a value in the range \(-1 \leq r \leq 1\).
- If \( r = 1 \), the data set forms a straight line with a positive slope.
- If \( r = 0 \), the data set has no correlation.
- If \( r = -1 \), the data set forms a straight line with a negative slope.

r-values close to 1 or \(-1\) indicate a very strong correlation. The closer \( r \) is to 0, the weaker the correlation.

**EXAMPLE 3**

**Correlation Coefficient**

The table shows a relationship between a city's population and the average time the city's citizens spend commuting to work each day.

<table>
<thead>
<tr>
<th>City</th>
<th>Population (thousands)</th>
<th>Average Commute Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albuquerque, NM</td>
<td>505</td>
<td>21.5</td>
</tr>
<tr>
<td>Atlanta, GA</td>
<td>486</td>
<td>31.1</td>
</tr>
<tr>
<td>Austin, TX</td>
<td>710</td>
<td>23.2</td>
</tr>
<tr>
<td>Charlotte, NC</td>
<td>630</td>
<td>25.1</td>
</tr>
<tr>
<td>Chicago, IL</td>
<td>2833</td>
<td>30.6</td>
</tr>
<tr>
<td>Eugene, OR</td>
<td>146</td>
<td>17.9</td>
</tr>
<tr>
<td>Houston, TX</td>
<td>2144</td>
<td>27.7</td>
</tr>
<tr>
<td>Las Vegas, NV</td>
<td>553</td>
<td>25.2</td>
</tr>
<tr>
<td>New York, NY</td>
<td>8496</td>
<td>34.0</td>
</tr>
<tr>
<td>New Orleans, LA</td>
<td>223</td>
<td>24.2</td>
</tr>
</tbody>
</table>
Find an equation for a line of best fit. How well does the line represent the data?

Use your calculator.

Enter the data into the lists \( L1 \) and \( L2 \).

Then press \( \text{STAT} \) and choose \( \text{CALC} \). Choose \( 4: \text{LinReg}(ax+b) \) and press \( \text{ENTER} \). An equation for a line of best fit is \( y \approx 0.001x + 23.8 \). The value of \( r \) is about 0.71, which indicates a moderate positive correlation.

3. Kylie and Marcus designed a quiz to measure how much information adults retain after leaving school. The table below shows the quiz scores of several adults, matched with the number of years each person had been out of school. Find an equation for a line of best fit. How well does the line represent the data?

<table>
<thead>
<tr>
<th>Time Out of School (yr)</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>10</th>
<th>14</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiz Score</td>
<td>85</td>
<td>94</td>
<td>98</td>
<td>75</td>
<td>80</td>
<td>77</td>
<td>63</td>
<td>56</td>
<td>45</td>
<td>50</td>
<td>34</td>
<td>33</td>
</tr>
</tbody>
</table>

Correlation refers to cause-and-effect. If a change in one variable directly causes a change in the other variable, then there is a cause-and-effect relationship between the variables. There is often correlation without causation.

Example 4

Correlation and Causation

The table shows test averages of eight students. The equation of the least-squares line for the data is \( y \approx 0.77x + 18.12 \) and \( r \approx 0.87 \). Discuss correlation and causation for the data set.

<table>
<thead>
<tr>
<th>U.S. History Test Average</th>
<th>90</th>
<th>70</th>
<th>75</th>
<th>100</th>
<th>90</th>
<th>85</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Test Average</td>
<td>80</td>
<td>72</td>
<td>95</td>
<td>92</td>
<td>82</td>
<td>80</td>
<td>92</td>
<td></td>
</tr>
</tbody>
</table>

There is a strong positive correlation between the U.S. history test average and the science test average for these students. There is not a likely cause-and-effect relationship because there is no apparent reason why test scores in one subject would directly affect test scores in the other subject.

4. Eight adults were surveyed about their education and earnings. The table shows the survey results. The equation of the least-squares line for the data is \( y \approx 5.59x - 30.28 \) and \( r \approx 0.86 \). Discuss correlation and causation for the data set.

<table>
<thead>
<tr>
<th>Years of Education</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>14</th>
<th>18</th>
<th>16</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings Last Year (thousand $)</td>
<td>40</td>
<td>65</td>
<td>75</td>
<td>44</td>
<td>70</td>
<td>50</td>
<td>54</td>
<td>86</td>
</tr>
</tbody>
</table>
THINK AND DISCUSS

1. What is the residual for a data point that lies on the line of best fit?

2. GET ORGANIZED Copy and complete the graphic organizer. For each $r$-value, sketch a possible scatter plot and describe the correlation, choosing from the following: strong positive, weak positive, none, strong negative, weak negative.

<table>
<thead>
<tr>
<th>$r$-value</th>
<th>-0.9</th>
<th>-0.4</th>
<th>0</th>
<th>0.4</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scatter Plot</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Description of Correlation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Exercises

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. A signed vertical distance between a data point and its corresponding model point is called a _____?_____ (residual or correlation coefficient)

2. A _____?_____ (least squares line or correlation coefficient) is a measure of how well a line of best fit models a data set.

3. The data in the table are graphed along with two lines of fit. For each line, find the sum of the squares of the residuals. Which line is a better fit for the data?

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>5</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>

4. The table shows numbers of books read by students in an English class over a summer and the students’ grades for the following semester.

<table>
<thead>
<tr>
<th>Books</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>0</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td>65</td>
<td>69</td>
<td>70</td>
<td>73</td>
<td>70</td>
<td>75</td>
<td>78</td>
<td>77</td>
<td>86</td>
<td>85</td>
<td>89</td>
<td>90</td>
<td>95</td>
<td>99</td>
<td>98</td>
</tr>
</tbody>
</table>

   a. Find an equation for a line of best fit.
   b. Interpret the meaning of the slope and $y$-intercept.
   c. Use your equation to predict the grade of a student who reads 15 books.
5. A negative correlation exists between the time Shawnda spends on homework during an evening and the amount of sleep she gets that night. The table shows data for several nights. Find an equation for a line of best fit. How well does the line represent the data?

<table>
<thead>
<tr>
<th>Homework (h)</th>
<th>0.5</th>
<th>0.5</th>
<th>1</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>3</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sleep (h)</td>
<td>8</td>
<td>9</td>
<td>8</td>
<td>8.5</td>
<td>8</td>
<td>7.5</td>
<td>8</td>
<td>7.5</td>
<td>7</td>
<td>7</td>
<td>8.5</td>
<td>6.5</td>
</tr>
</tbody>
</table>

6. Some students were surveyed about how much time they spent playing video games last week and their overall test average. The equation of the least-squares line for the data is \( y \approx -2.93x + 89.70 \) and \( r \approx -0.92 \). Discuss correlation and causation for the data set.

<table>
<thead>
<tr>
<th>Hours Playing Video Games</th>
<th>1</th>
<th>3</th>
<th>3</th>
<th>6</th>
<th>2</th>
<th>1</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Average for all Subjects</td>
<td>80</td>
<td>85</td>
<td>78</td>
<td>70</td>
<td>86</td>
<td>92</td>
<td>60</td>
<td>64</td>
</tr>
</tbody>
</table>

**PRACTICE AND PROBLEM SOLVING**

7. The data in the table are graphed along with two lines of fit. For each line, find the sum of the squares of the residuals. Which line is a better fit for the data?

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>5</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

8. The table shows the mean outside temperature for each of six months and the amount of heating oil used by a family for each of those months.

<table>
<thead>
<tr>
<th>Mean Outside Temperature (°F)</th>
<th>30</th>
<th>28</th>
<th>44</th>
<th>56</th>
<th>62</th>
<th>76</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heating Oil Used (gal)</td>
<td>112</td>
<td>115</td>
<td>94</td>
<td>60</td>
<td>35</td>
<td>12</td>
</tr>
</tbody>
</table>

   a. Find an equation for a line of best fit.
   b. Interpret the meaning of the slope and \( y \)-intercept.
   c. Use your equation to predict the amount of heating oil used in a month in which the mean outside temperature is 20 °F.

9. The table shows the number of customers at a coffee shop and the number of cookies sold for several days. Find an equation for a line of best fit. How well does the line represent the data?

<table>
<thead>
<tr>
<th>Customers</th>
<th>10</th>
<th>12</th>
<th>25</th>
<th>27</th>
<th>40</th>
<th>55</th>
<th>67</th>
<th>109</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cookies Sold</td>
<td>2</td>
<td>6</td>
<td>5</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

10. Some students were surveyed about how much time they spent watching television one week and how much time they spent playing video games the next week. The equation of the least-squares line for the data is \( y \approx 0.76x + 1.63 \) and \( r \approx 0.77 \). Discuss correlation and causation for the data set.

<table>
<thead>
<tr>
<th>Week 1: Hours Watching Television</th>
<th>4</th>
<th>2</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>1</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 2: Hours Playing Video Games</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>
11. **Write About It** Tell which correlation coefficient, \( r = 0.65 \) or \( r = -0.78 \), indicates a stronger linear relationship between two variables. Explain your answer.

12. **Critical Thinking** What can you conclude if the sum of the squared residuals is 0? Explain why the same conclusion might not apply when the sum of the residuals is 0.

13. **Sports** The table shows hits and runs scored by eight New York Yankees in the 2009 baseball season.
   - a. Find the equation of the least-squares line.
   - b. Interpret the meaning of the slope.
   - c. Interpret the meaning of the \( y \)-intercept mathematically.
   - d. Describe any possible correlation for the data set. Use the correlation coefficient to support your answer.
   - e. Use the equation of the least-squares line from part a to predict how many runs a player will score if he gets 100 hits.

14. **Community** The table shows data about temperature and how much bottled water was sold at an annual summer festival in past years. The high temperature for the day of this year’s festival is predicted to be 89 °F. The festival organizer must order bottled water in cases of 100. Find the equation of the least-squares line. Use the equation to decide how many cases the organizer should order.

<table>
<thead>
<tr>
<th>Player</th>
<th>Hits</th>
<th>Runs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jorge Posada</td>
<td>109</td>
<td>55</td>
</tr>
<tr>
<td>Mark Teixeira</td>
<td>178</td>
<td>103</td>
</tr>
<tr>
<td>Robinson Cano</td>
<td>204</td>
<td>103</td>
</tr>
<tr>
<td>Derek Jeter</td>
<td>212</td>
<td>107</td>
</tr>
<tr>
<td>Johnny Damon</td>
<td>155</td>
<td>107</td>
</tr>
<tr>
<td>Melky Cabrera</td>
<td>133</td>
<td>66</td>
</tr>
<tr>
<td>Nick Swisher</td>
<td>124</td>
<td>84</td>
</tr>
<tr>
<td>Hideki Matsui</td>
<td>125</td>
<td>62</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Midtown Summer Fest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>Daily High Temperature (°F)</td>
</tr>
<tr>
<td>Bottled Waters Sold</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regional Historical Museum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>Visitors</td>
</tr>
<tr>
<td>Gift Shop Sales ($)</td>
</tr>
</tbody>
</table>

15. Complete parts a–d for the relationship between the year and the number of visitors.
   - a. Find the equation of the least-squares line and the correlation coefficient.
   - b. Interpret the meaning of the slope and the \( y \)-intercept.
   - c. Is it reasonable to use your equation to make predictions? Explain.
   - d. Is it reasonable to say there is a cause-and-effect relationship? Explain.

16. Complete parts a–d above for the relationship between the number of visitors and the gift shop sales.
17. Which could be the correlation coefficient of this graph?
   A. -1.00  
   B. -0.93  
   C. 0.93  
   D. 1.00

18. The table shows how much time five students studied for a test and their test scores. The equation of a line of fit for the data is \( y = 5x + 60 \). What is the sum of the squares of the residuals for the line of fit?

<table>
<thead>
<tr>
<th>Hours Studying</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Score</td>
<td>60</td>
<td>70</td>
<td>90</td>
<td>80</td>
<td>100</td>
</tr>
</tbody>
</table>

CHALLENGE AND EXTEND

19. The heights and weights of eight basketball players are graphed along with a line of fit.
   a. Find the sum of the squares of the residuals.
   b. Find the mean absolute deviation. (The mean absolute deviation is the mean of the absolute values of the residuals.) Explain why the mean absolute deviation might be more useful than the sum of the squares of the residuals in some cases.

20. Use these facts to complete the data table:
    The equation of a line of fit is \( y = 2x - 3 \).
    The sum of the residuals is 0.
    The sum of the squares of the residuals is 14.
Chess Translations

You can use the game of chess to explore transformations.

A chessboard consists of 64 squares arranged into 8 rows (numbered 1 through 8) and 8 columns (lettered a through h). Each square is named by its column letter and row number. For instance, the square in the lower left corner is a1.

### Selected Rules of Movement

<table>
<thead>
<tr>
<th>Piece</th>
<th>Rules of Movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bishop</td>
<td>Diagonally any number of squares</td>
</tr>
<tr>
<td>King</td>
<td>One square in any direction</td>
</tr>
<tr>
<td>Knight</td>
<td>L-shape: two squares horizontally or vertically and then one square perpendicularly</td>
</tr>
<tr>
<td>Rook</td>
<td>Horizontally or vertically any number of squares</td>
</tr>
</tbody>
</table>

You move each chess piece by applying the rules of movement. Pieces of the same color cannot move onto a space occupied by another piece, and the knight is the only piece that can jump over other pieces.

### Activity

Use the chessboard at right to name all possible locations of the bishop on f7 after one move.

The bishop can move diagonally any number of spaces, but it cannot move into or through any other pieces.

The bishop at f7 can move to any of the marked spaces: e6, e8, g6, g8, or h5.

### Try This

Use the chessboard from the activity to name all possible locations of each piece after one move.

1. the king on b1  
2. the rook on d7  
3. the knight on a6

Use the chessboard from the activity to name all possible locations of each piece after two moves.

4. the king on b1  
5. the rook on d7  
6. the knight on a6

7. **Critical Thinking** In the last move of a chess game a knight is moved to d5. What are the possible squares that it came from?

8. **Make a Conjecture** Explain the connection between the position labeling in chess and points in the coordinate plane.
Exploring Transformations

Objectives
Apply transformations to points and sets of points.
Interpret transformations of real-world data.

Vocabulary
transformation
translation
reflection
stretch
compression

Why learn this?
Changes in recording studio fees can be modeled by transformations. (See Example 4.)

A transformation is a change in the position, size, or shape of a figure. A translation, or slide, is a transformation that moves each point in a figure the same distance in the same direction.

Example 1: Translating Points
Perform the given translation on the point (2, –1). Give the coordinates of the translated point.

A 4 units left
B 2 units right and 3 units up

Perform the given translation on the point (–1, 3). Give the coordinates of the translated point.

1a. 4 units right
1b. 1 unit left and 2 units down

Notice that when you translate left or right, the x-coordinate changes, and when you translate up or down, the y-coordinate changes.

Note: Each point shifts right or left by a number of units. The x-coordinate changes.

<table>
<thead>
<tr>
<th>Translations</th>
<th>Horizontal Translation</th>
<th>Vertical Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Each point shifts right or left by a number of units.</td>
<td>Each point shifts up or down by a number of units.</td>
</tr>
<tr>
<td>x y</td>
<td>(1, 2) → (1 + 3, 2)</td>
<td>(1, 2) → (1, 2 + 2)</td>
</tr>
<tr>
<td></td>
<td>(x, y) → (x + h, y)</td>
<td>(x, y) → (x, y + k)</td>
</tr>
</tbody>
</table>

left if h < 0 right if h > 0 down if k < 0 up if k > 0
A **reflection** is a transformation that flips a figure across a line called the line of reflection. Each reflected point is the same distance from the line of reflection, but on the opposite side of the line.

<table>
<thead>
<tr>
<th>Reflection Across y-axis</th>
<th>Reflection Across x-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each point flips across the y-axis.</td>
<td>Each point flips across the x-axis.</td>
</tr>
<tr>
<td><img src="image" alt="Reflection Across y-axis" /></td>
<td><img src="image" alt="Reflection Across x-axis" /></td>
</tr>
<tr>
<td>(−1, 2) → (1, 2)</td>
<td>(1, 2) → (1, −2)</td>
</tr>
<tr>
<td>(x, y) → (−x, y)</td>
<td>(x, y) → (x, −y)</td>
</tr>
</tbody>
</table>

You can transform a function by transforming its ordered pairs. When a function is translated or reflected, the original graph and the graph of the transformation are **congruent** because the size and shape of the graphs are the same.

**Example 2**

Translating and Reflecting Functions

Use a table to perform each transformation of \( y = f(x) \). Use the same coordinate plane as the original function.

**A** translation 2 units down

Identify important points from the graph and make a table.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y - 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>4</td>
<td>4 − 2 = 2</td>
</tr>
<tr>
<td>−1</td>
<td>0</td>
<td>0 − 2 = −2</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
<td>2 − 2 = 0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2 − 2 = 0</td>
</tr>
</tbody>
</table>

The entire graph shifts 2 units down. Subtract 2 from each y-coordinate.

**B** reflection across y-axis

Identify important points from the graph and make a table.

<table>
<thead>
<tr>
<th>( −x )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1(−2) = 2</td>
<td>−2</td>
<td>4</td>
</tr>
<tr>
<td>−1( −1) = 1</td>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>−1(0) = 0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>−1(2) = −2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Multiply each x-coordinate by −1. The entire graph flips across the y-axis.

For the function from Example 2, use a table to perform each transformation of \( y = f(x) \). Use the same coordinate plane as the original function.

2a. translation 3 units right  
2b. reflection across x-axis
Imagine grasping two points on the graph of a function that lie on opposite sides of the \( y \)-axis. If you pull the points away from the \( y \)-axis, you would create a horizontal stretch of the graph. If you push the points towards the \( y \)-axis, you would create a horizontal compression.

Stretches and compressions are not congruent to the original graph.

<table>
<thead>
<tr>
<th>Stretches and Compressions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Horizontal</strong></td>
</tr>
<tr>
<td>Stretch</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( (4, 0) \rightarrow (2(4), 0) )</td>
</tr>
<tr>
<td>( (x, y) \rightarrow (bx, y) )</td>
</tr>
<tr>
<td>(</td>
</tr>
<tr>
<td>Compression</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>( (4, 0) \rightarrow \left( \frac{1}{2}(4), 0 \right) )</td>
</tr>
<tr>
<td>( (x, y) \rightarrow (bx, y) )</td>
</tr>
<tr>
<td>( 0 &lt;</td>
</tr>
</tbody>
</table>

**Example 3**

**Stretching and Compressing Functions**

Use a table to perform a horizontal compression of \( y = f(x) \) by a factor of \( \frac{1}{2} \). Use the same coordinate plane as the original function.

Identify important points from the graph and make a table.

<table>
<thead>
<tr>
<th>( \frac{1}{2}x )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2}(-1) = -\frac{1}{2} )</td>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>( \frac{1}{2}(0) = 0 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{1}{2}(2) = 1 )</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \frac{1}{2}(4) = 2 )</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

*Multiply each \( x \)-coordinate by \( \frac{1}{2} \).*

**Check It Out!**

3. For the function from Example 3, use a table to perform a vertical stretch of \( y = f(x) \) by a factor of 2. Graph the transformed function on the same coordinate plane as the original function.
**Business Application**

Recording studio fees are usually based on an hourly rate, but the rate can be modified due to various options. The graph shows a basic hourly studio rate. Sketch a graph to represent each situation below and identify the transformation of the original graph that it represents.

**A** The engineer's time is needed, so the hourly rate is 1.5 times the original rate.

If the fees are 1.5 times the basic hourly rate, the value of each y-coordinate would be multiplied by 1.5. This represents a vertical stretch by a factor of 1.5.

**B** A $20 setup fee is added to the basic hourly rate.

If the prices are $20 more than the original estimate, the value of each y-coordinate would increase by 20. This represents a vertical translation up 20 units.

**4. What if...?** Suppose that a discounted rate is \( \frac{3}{4} \) of the original rate. Sketch a graph to represent the situation and identify the transformation of the original graph that it represents.

**THINK AND DISCUSS**

1. Describe two ways to transform \((4, 2)\) to \((2, 2)\).

2. Compare a vertical stretch with a horizontal compression.

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, describe the transformations indicated by the given rule.
GUIDED PRACTICE

1. Vocabulary A transformation that pushes a graph toward the \( x \)-axis is a \( \boxed{\text{reflection or compression}} \).

**SEE EXAMPLE 1**

Perform the given translation on the point \((4, 2)\) and give the coordinates of the translated point.

2. 5 units left

3. 3 units down

4. 1 unit right, 6 units up

**SEE EXAMPLE 2**

Use a table to perform each transformation of \( y = f(x) \). Use the same coordinate plane as the original function.

5. translation 2 units up

6. reflection across the \( y \)-axis

7. reflection across the \( x \)-axis

**SEE EXAMPLE 3**

Use a table to perform each transformation of \( y = f(x) \). Use the same coordinate plane as the original function.

8. horizontal stretch by a factor of 3

9. vertical stretch by a factor of 3

10. vertical compression by a factor of \( \frac{1}{3} \)

**SEE EXAMPLE 4**

**Recreation** The graph shows the price for admission by age at a local zoo. Sketch a graph to represent each situation and identify the transformation of the original graph that it represents.

11. Admission is half price on Wednesdays.

12. To raise funds for endangered species, the zoo charges $1.50 extra per ticket.

13. The maximum age for each ticket price is increased by 5 years.

PRACTICE AND PROBLEM SOLVING

Perform the given translation on \((3, 1)\). Give the coordinates of the translated point.

14. 2 units right

15. 4 units up

16. 5 units left, 4 units down

Use a table to perform each transformation of \( y = f(x) \). Use the same coordinate plane as the original function.

17. translation 2 units down

18. reflection across the \( x \)-axis

19. translation 3 units right

20. reflection across the \( y \)-axis

21. vertical compression by a factor of \( \frac{2}{3} \)

22. horizontal compression by a factor of \( \frac{1}{2} \)

23. horizontal stretch by a factor of \( \frac{3}{2} \)

24. vertical stretch by a factor of 2
**Technology** The graph shows the cost of Web page hosting depending on the Web space used. Sketch a graph to represent each situation and identify the transformation of the original graph that it represents.

25. The prices are reduced by $5.
26. The prices are discounted by 25%.
27. A special is offered for double the amount of Web space for the same price.

**Estimation** The table gives the coordinates for the vertices of a triangle. Estimate the area of each transformed triangle by graphing it and counting the number of squares it covers on the coordinate plane. How does the area of each transformed triangle compare with the area of the original triangle?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

28. reflection across the y-axis
29. 5 units left, 3 units up
30. horizontal stretch by a factor of 2
31. horizontal compression by a factor of \( \frac{2}{3} \)
32. vertical compression by a factor of \( \frac{2}{3} \)
33. reflection across the x-axis
34. 1 unit left, 6 units down
35. vertical stretch by a factor of 3

**Business** An automotive mechanic charges $50 to diagnose the problem in a vehicle and $65 per hour for labor to fix it.

a. If the mechanic increases his diagnostic fee to $60, what kind of transformation is this to the graph of the total repair bill?
b. If the mechanic increases his labor rate to $75 per hour, what kind of transformation is this to the graph of the total repair bill?
c. If it took 3 hours to repair your car, which of the two rate increases would have a greater effect on your total bill?

37. The student council wants to buy vases for the flowers for the school prom. A florist charges a $20 delivery fee plus $1.25 per vase. A home-decorating store charges a $10 delivery fee plus $1.25 per vase.

a. The function \( f(x) = 20 + 1.25x \) models the cost of ordering \( x \) vases from the florist, and the function \( g(x) = 10 + 1.25x \) models the cost of ordering \( x \) vases from the home-decorating store. What do the graphs of these functions look like?
b. How are the graphs related to each other?
c. How could you modify these functions so that their graphs are identical?
d. If the florist decided to waive the $20 delivery fee as long as the number of vases ordered was more than 150, how would the graph of \( f \) change? How would it compare with the graph of the other function?
Transportation Use the graph and the following information for Exercises 39–43.

Roberta left her house at 10:00 A.M. and drove to the library. She was at the library studying until 11:30 A.M. Then she drove to the grocery store. At 12:15 P.M. Roberta left the grocery store and drove home. The graph shows Roberta's position with respect to time.

Sketch a graph to reflect each change to the original story. Assume the time Roberta spends inside each building remains the same.

39. Roberta drove at half the speed from her house to the library.
40. The grocery store she went to is twice as far from the library.
41. The grocery store is 2.5 miles closer to the house than the library is.

Change the original story about Roberta to match each graph.

42. [Graph shows a different pattern from the original graph.]
43. [Graph shows a different pattern from the original graph.]

44. Critical Thinking Suppose two transformations are performed on a single point: a translation and a reflection. Does the order in which the transformations are performed make a difference? Does the type of translation or reflection matter? Explain your reasoning.

45. Write About It Describe how transformations might make graphing easier.

46. The function $c(p) = 0.99p$ represents the cost in dollars of $p$ pounds of peaches. If the cost per pound increases by 10%, how will the graph of the function change?
   - A) Translation 0.1 unit up
   - B) Translation 0.1 unit right
   - C) Horizontal stretch by a factor of 1.1
   - D) Vertical stretch by a factor of 1.1

47. Which transformation would change the point $(5, 3)$ into $(-5, 3)$?
   - F) Reflection across the $x$-axis
   - G) Translation 5 units down
   - H) Reflection across the $y$-axis
   - I) Translation 5 units left

48. The graph of the function $f$ is a line that intersects the $y$-axis at the point $(0, 3)$ and the $x$-axis at the point $(3, 0)$. Which transformation of $f$ does NOT intersect the $y$-axis at the point $(0, 6)$?
   - A) Translation 3 units up
   - B) Translation 3 units right
   - C) Vertical stretch by a factor of 2
   - D) Horizontal compression by a factor of $\frac{1}{2}$
49. Which transformation is displayed in the graph?
   - Reflection across the x-axis
   - Translation 5 units down
   - Reflection across the y-axis
   - Translation 5 units left

50. Which represents a translation 4 units right and 2 units down?
   - From (4, 2) to (0, 0)
   - From (4, -2) to (0, 0)
   - From (-4, -2) to (0, 0)
   - From (-4, 2) to (0, 0)

51. Short Response Graph the points (-1, 3) and (-1, -3). Describe two different transformations that would transform (-1, 3) to (-1, -3).

### CHALLENGE AND EXTEND

52. Suppose the rule \((x, y) \rightarrow (2x, y - 3)\) is used to translate a point. If the coordinates of the translated point are \((22, 7)\), what was the original point?

53. History From 1999 to 2001 the cost for mailing \(n\) first class letters through the United States Postal Service was \(c(n) = 0.33n\). In 2001 the rate was increased by $0.01 per letter. In 2002 the rate was increased an additional $0.03 per letter.
   a. Write an equation that represents the cost of mailing \(n\) first class letters in 2002.
   b. What transformation describes the total change in price?
   c. Graph both functions and estimate the maximum number of first class letters you could mail for $5.00 in both 1999 and 2002.
   d. Explain the effect of the reasonable domain and range for these functions on your answer for part c.

54. Name a point that when reflected across the x-axis has the same coordinates as if it were reflected across the y-axis. How many points are there that satisfy this condition?
The Family of Linear Functions

A family of functions is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family. You can use a graphing calculator to explore families of functions.

Activity

Graph the lines described by \( y = x - 2,\ y = x - 1,\ y = x,\ y = x + 1,\ y = x + 2,\ y = x + 3,\) and \( y = x + 4.\) How does the value of \( b\) affect the graph described by \( y = x + b?\)

1. All of the functions are in the form \( y = x + b.\) Enter them into the \( Y=\) editor.

   \[
   \begin{align*}
   Y1 &= X, 0, x - 2 \quad \text{ENTER} \\
   Y2 &= X, 0, x - 1 \quad \text{ENTER} \\
   Y3 &= X, 0, x \quad \text{ENTER} \\
   Y4 &= X, 0, x + 1 \quad \text{ENTER} \\
   Y5 &= X, 0, x + 2 \quad \text{ENTER} \\
   Y6 &= X, 0, x + 3 \quad \text{ENTER} \\
   \end{align*}
   \]

   and so on.

2. Press \( \text{ZOOM} \) and select 6:Zstandard. Think about the different values of \( b\) as you watch the graphs being drawn. Notice that the lines are all parallel.

3. It appears that the value of \( b\) in \( y = x + b\) shifts the graph up or down—up if \( b\) is positive and down if \( b\) is negative.

Try This

1. Make a prediction about the lines described by \( y = 2x - 3,\ y = 2x - 2,\ y = 2x - 1,\ y = 2x,\ y = 2x + 1,\ y = 2x + 2,\) and \( y = 2x + 3.\) Then graph. Was your prediction correct?

2. Now use your calculator to explore what happens to the graph of \( y = mx\) when you change the value of \( m.\)
   a. Make a Prediction How do you think the lines described by \( y = -2x,\ y = -x,\ y = x,\) and \( y = 2x\) will be related? How will they be alike? How will they be different?
   b. Graph the functions given in part a. Was your prediction correct?
   c. How is the effect of \( m\) different when \( m\) is positive from when \( m\) is negative?
Who uses this?

Business owners can use transformations to show the effects of price changes, such as the price of trophy engraving. (See Example 5.)

**Objective**

Describe how changing slope and y-intercept affect the graph of a linear function.

**Vocabulary**

- family of functions
- parent function
- transformation
- translation
- rotation
- reflection

**Transforming Linear Functions**

A **family of functions** is a set of functions whose graphs have basic characteristics in common. For example, all linear functions form a family because all of their graphs are the same basic shape.

A **parent function** is the most basic function in a family. For linear functions, the parent function is \( f(x) = x \).

The graphs of all other linear functions are **transformations** of the graph of the parent function, \( f(x) = x \). A **transformation** is a change in position or size of a figure.

There are three types of transformations—translations, rotations, and reflections.

Look at the four functions and their graphs below.

Notice that all of the lines above are parallel. The slopes are the same but the y-intercepts are different.

The graphs of \( g(x) = x + 3 \), \( h(x) = x - 2 \), and \( k(x) = x - 4 \) are vertical **translations** of the graph of the parent function, \( f(x) = x \). A **translation** is a type of transformation that moves every point the same distance in the same direction. You can think of a translation as a “slide.”

**Vertical Translation of a Linear Function**

When the y-intercept \( b \) is changed in the function \( f(x) = mx + b \), the graph is translated vertically.

- If \( b \) increases, the graph is translated up.
- If \( b \) decreases, the graph is translated down.
**Example 1**

Translating Linear Functions

Graph \( f(x) = x \) and \( g(x) = x - 5 \). Then describe the transformation from the graph of \( f(x) \) to the graph of \( g(x) \).

![Graphs showing the transformation](image)

The graph of \( g(x) = x - 5 \) is the result of translating the graph of \( f(x) = x \) 5 units down.

**Check It Out!**

1. Graph \( f(x) = x + 4 \) and \( g(x) = x - 2 \). Then describe the transformation from the graph of \( f(x) \) to the graph of \( g(x) \).

**Rotation of a Linear Function**

When the slope \( m \) is changed in the function \( f(x) = mx + b \) it causes a rotation of the graph about the point \((0, b)\), which changes the line’s steepness.

**Example 2**

Rotating Linear Functions

Graph \( f(x) = x + 2 \) and \( g(x) = 2x + 2 \). Then describe the transformation from the graph of \( f(x) \) to the graph of \( g(x) \).

![Graphs showing the transformation](image)

The graph of \( g(x) = 2x + 2 \) is the result of rotating the graph of \( f(x) = x + 2 \) about \((0, 2)\). The graph of \( g(x) \) is steeper than the graph of \( f(x) \).

**Check It Out!**

2. Graph \( f(x) = 3x - 1 \) and \( g(x) = \frac{1}{2}x - 1 \). Then describe the transformation from the graph of \( f(x) \) to the graph of \( g(x) \).
The diagram shows the reflection of the graph of $f(x) = 2x$ across the $y$-axis, producing the graph of $g(x) = -2x$. A reflection is a transformation across a line that produces a mirror image. You can think of a reflection as a “flip” over a line.

**Reflection of a Linear Function**

When the slope $m$ is multiplied by $-1$ in $f(x) = mx + b$, the graph is reflected across the $y$-axis.

**EXAMPLE 3 Reflecting Linear Functions**

Graph $f(x)$. Then reflect the graph of $f(x)$ across the $y$-axis. Write a function $g(x)$ to describe the new graph.

### A

$f(x) = x$

To find $g(x)$, multiply the value of $m$ by $-1$.

In $f(x) = x$, $m = 1$.

$$1(-1) = -1$$

This is the value of $m$ for $g(x)$.

$g(x) = -x$

### B

$f(x) = -4x - 1$

To find $g(x)$, multiply the value of $m$ by $-1$.

In $f(x) = -4x - 1$, $m = -4$.

$$-4(-1) = 4$$

This is the value of $m$ for $g(x)$.

$g(x) = 4x - 1$

### Check It Out!

3. Graph $f(x) = \frac{2}{3}x + 2$. Then reflect the graph of $f(x)$ across the $y$-axis. Write a function $g(x)$ to describe the new graph.
**Example 4**  

**Multiple Transformations of Linear Functions**  

Graph \( f(x) = x \) and \( g(x) = 3x + 1 \). Then describe the transformations from the graph of \( f(x) \) to the graph of \( g(x) \).

Find transformations of \( f(x) = x \) that will result in \( g(x) = 3x + 1 \):

- Multiply \( f(x) \) by 3 to get \( h(x) = 3x \). This rotates the graph about \((0, 0)\) and makes it steeper.
- Then add 1 to \( h(x) \) to get \( g(x) = 3x + 1 \). This translates the graph 1 unit up.

The transformations are a rotation and a translation.

4. Graph \( f(x) = x \) and \( g(x) = -x + 2 \). Then describe the transformations from the graph of \( f(x) \) to the graph of \( g(x) \).

---

**Example 5**  

**Business Application**  

A trophy company charges $175 for a trophy plus $0.20 per letter for the engraving. The total charge for a trophy with \( x \) letters is given by the function \( f(x) = 0.20x + 175 \). How will the graph change if the trophy's cost is lowered to $172? if the charge per letter is raised to $0.50?

\( f(x) = 0.20x + 175 \) is graphed in blue.

If the trophy's cost is lowered to $172, the new function is \( g(x) = 0.20x + 172 \). The original graph will be translated 3 units down.

If the charge per letter is raised to $0.50, the new function is \( h(x) = 0.50x + 175 \). The original graph will be rotated about \((0, 175)\) and become steeper.

5. **What if...?** How will the graph change if the charge per letter is lowered to $0.15? if the trophy's cost is raised to $180?

---

**Think and Discuss**

1. Describe the graph of \( f(x) = x + 3.45 \)
2. Look at the graphs in Example 5. For each line, is every point on the line a solution in this situation? Explain.
3. **Get Organized** Copy and complete the graphic organizer. In each box, sketch a graph of the given transformation of \( f(x) = x \), and label it with a possible equation.
Guided Practice

**Vocabulary** Apply the vocabulary from this lesson to answer each question.

1. Changing the value of \( b \) in \( f(x) = mx + b \) results in a ____ of the graph.
   (translation or reflection)

2. Changing the value of \( m \) in \( f(x) = mx + b \) results in a ____ of the graph.
   (translation or rotation)

Graph \( f(x) \) and \( g(x) \). Then describe the transformation from the graph of \( f(x) \) to the graph of \( g(x) \).

3. \( f(x) = x \), \( g(x) = x - 4 \)  
4. \( f(x) = x \), \( g(x) = x + 1 \)
5. \( f(x) = x \), \( g(x) = x + 2 \)  
6. \( f(x) = x \), \( g(x) = x - 6.5 \)
7. \( f(x) = x \), \( g(x) = \frac{1}{4}x \)  
8. \( f(x) = \frac{1}{5}x + 3 \), \( g(x) = x + 3 \)
9. \( f(x) = 2x - 2 \), \( g(x) = 4x - 2 \)  
10. \( f(x) = x + 1 \), \( g(x) = \frac{1}{2}x + 1 \)

Graph \( f(x) \). Then reflect the graph of \( f(x) \) across the \( y \)-axis. Write a function \( g(x) \) to describe the new graph.

11. \( f(x) = -\frac{1}{5}x \)  
12. \( f(x) = 2x + 4 \)
13. \( f(x) = \frac{1}{3}x - 6 \)  
14. \( f(x) = 5x - 1 \)

Graph \( f(x) \) and \( g(x) \). Then describe the transformations from the graph of \( f(x) \) to the graph of \( g(x) \).

15. \( f(x) = x \), \( g(x) = 2x - 2 \)  
16. \( f(x) = x \), \( g(x) = \frac{1}{3}x + 1 \)
17. \( f(x) = -x - 1 \), \( g(x) = -4x \)  
18. \( f(x) = -x \), \( g(x) = -\frac{1}{2}x - 3 \)

19. **Entertainment** For large parties, a restaurant charges a reservation fee of $25, plus $15 per person. The total charge for a party of \( x \) people is \( f(x) = 15x + 25 \). How will the graph of this function change if the reservation fee is raised to $50? If the per-person charge is lowered to $12?

Practice and Problem Solving

Graph \( f(x) \) and \( g(x) \). Then describe the transformation(s) from the graph of \( f(x) \) to the graph of \( g(x) \).

20. \( f(x) = x \), \( g(x) = x + \frac{1}{2} \)  
21. \( f(x) = x \), \( g(x) = x - 4 \)
22. \( f(x) = \frac{1}{3}x - 1 \), \( g(x) = \frac{1}{10}x - 1 \)  
23. \( f(x) = x + 2 \), \( g(x) = \frac{2}{3}x + 2 \)

Graph \( f(x) \). Then reflect the graph of \( f(x) \) across the \( y \)-axis. Write a function \( g(x) \) to describe the new graph.

24. \( f(x) = 6x \)  
25. \( f(x) = -3x - 2 \)

Graph \( f(x) \) and \( g(x) \). Then describe the transformations from the graph of \( f(x) \) to the graph of \( g(x) \).

26. \( f(x) = 2x \), \( g(x) = 4x - 1 \)  
27. \( f(x) = -7x + 5 \), \( g(x) = -14x \)
28. **School** The number of chaperones on a field trip must include 1 teacher for every 4 students, plus 2 parents total. The function describing the number of chaperones for a trip of \( x \) students is \( f(x) = \frac{1}{4}x + 2 \). How will the graph change if the number of parents is reduced to 0? if the number of teachers is raised to 1 for every 3 students?

Describe the transformation(s) on the graph of \( f(x) = x \) that result in the graph of \( g(x) \). Graph \( f(x) \) and \( g(x) \), and compare the slopes and intercepts.

29. \( g(x) = -x \)  
30. \( g(x) = x + 8 \)  
31. \( g(x) = 3x \)

32. \( g(x) = -\frac{2}{7}x \)  
33. \( g(x) = 6x - 3 \)  
34. \( g(x) = -2x + 1 \)

Sketch the transformed graph. Then write a function to describe your graph.

35. Rotate the graph of \( f(x) = -x + 2 \) until it has the same steepness in the opposite direction.

36. Reflect the graph of \( f(x) = x - 1 \) across the \( y \)-axis, and then translate it 4 units down.

37. Translate the graph of \( f(x) = \frac{1}{6}x - 10 \) six units up.

38. **Hobbies** A book club charges a membership fee of $20 and then $12 for each book purchased.

a. Write and graph a function to represent the cost \( y \) of membership in the club based on the number of books purchased \( x \).

b. **What if...?** Write and graph a second function to represent the cost of membership if the club raises its membership fee to $30.

c. Describe the relationship between your graphs from parts a and b.

Describe the transformation(s) on the graph of \( f(x) = x \) that result in the graph of \( g(x) \).

39. \( g(x) = x - 9 \)  
40. \( g(x) = -x \)  
41. \( g(x) = 5x \)

42. \( g(x) = -\frac{2}{3}x + 1 \)  
43. \( g(x) = -2x \)  
44. \( g(x) = \frac{1}{5}x \)

45. **Careers** Kelly works as a salesperson. She earns a weekly base salary plus a commission that is a percent of her total sales. Her total weekly pay is described by \( f(x) = 0.20x + 300 \), where \( x \) is total sales in dollars.

a. What is Kelly’s weekly base salary?

b. What percent of total sales does Kelly receive as commission?

c. **What if...?** What is the change in Kelly’s salary plan if the weekly pay function changes to \( g(x) = 0.25x + 300 \) to \( h(x) = 0.2x + 400 \)?

46. **Critical Thinking** To transform the graph of \( f(x) = x \) into the graph of \( g(x) = -x \), you can reflect the graph of \( f(x) \) across the \( y \)-axis. Find another transformation that will have the same result.

47. **Write About It** Describe how a reflection across the \( y \)-axis affects each point on a graph. Give an example to illustrate your answer.

48. a. Maria is walking from school to the softball field at a rate of 3 feet per second. Write a rule that gives her distance from school (in feet) as a function of time (in seconds). Then graph.

b. Give a real-world situation that could be described by a line parallel to the one in part a.

c. What does the \( y \)-intercept represent in each of these situations?
49. Which best describes the effect on $f(x) = 2x - 5$ if the slope changes to 10?
   A. Its graph becomes less steep.
   B. Its graph moves 15 units up.
   C. Its graph makes 10 complete rotations.
   D. The x-intercept becomes $\frac{1}{2}$.

50. Given $f(x) = 22x - 182$, which does NOT describe the effect of increasing the $y$-intercept by 182?
   F. The new line passes through the origin.
   G. The new x-intercept is 0.
   H. The new line is parallel to the original.
   J. The new line is steeper than the original.

**CHALLENGE AND EXTEND**

51. You have seen that the graph of $g(x) = x + 3$ is the result of translating the graph of $f(x) = x$ three units up. However, you can also think of this as a horizontal translation—that is, a translation left or right. Graph $g(x) = x + 3$. Describe the horizontal translation of the graph of $f(x) = x$ to get the graph of $g(x) = x + 3$.

52. If $c > 0$, how can you describe the translation that transforms the graph of $f(x) = x$ into the graph of $g(x) = x + c$ as a horizontal translation? $g(x) = x - c$ as a horizontal translation?
Transformations of Functions

Transformations can be used to graph complicated functions by using the graphs of simpler functions called parent functions. The following are examples of parent functions and their graphs.

\[ y = |x| \quad y = \sqrt{x} \quad y = x^2 \]

**Transformation of Parent Function** \( y = f(x) \)

<table>
<thead>
<tr>
<th>Reflection</th>
<th>Vertical Translation</th>
<th>Horizontal Translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Across x-axis: ( y = -f(x) )</td>
<td>( y = f(x) + k )</td>
<td>( y = f(x - h) )</td>
</tr>
<tr>
<td>Across y-axis: ( y = f(-x) )</td>
<td>Up ( k ) units if ( k &gt; 0 )</td>
<td>Right ( h ) units if ( h &gt; 0 )</td>
</tr>
<tr>
<td></td>
<td>Down ( k ) units if ( k &lt; 0 )</td>
<td>Left ( h ) units if ( h &lt; 0 )</td>
</tr>
</tbody>
</table>

**Example**

For the parent function \( y = x^2 \), write a function rule for the given transformation and graph the preimage and image.

**A** a reflection across the x-axis

- Function rule: \( y = -x^2 \)
- Graph:

**B** a translation up 2 units and right 3 units

- Function rule: \( y = (x - 3)^2 + 2 \)
- Graph:

**Try This**

For each parent function, write a function rule for the given transformation and graph the preimage and image.

1. parent function: \( y = x^2 \)
   transformation: a translation down 1 unit and right 4 units

2. parent function: \( y = \sqrt{x} \)
   transformation: a reflection across the x-axis

3. parent function: \( y = |x| \)
   transformation: a translation up 2 units and left 1 unit
Vocabulary

compression  parent function  stretch

correlation  reflection  translation

correlation coefficient  regression  transformation

line of best fit

Complete the sentences below with vocabulary words from the list above.

1. The function \( f(x) = x \) is the ___?___ of the function \( g(x) = 3x - 5 \).

2. A ___?___ is a transformation that moves each point in a figure the same distance in the same direction.

3. The statistical study of the relationship between variables is called ___?___.
The graph shows household alarm monitoring fees. Sketch a graph to represent a \( \frac{1}{2} \) fee reduction on long-term contracts. Then identify the transformation of the original graph that the new graph represents.

Each price is \( \frac{4}{5} \) of the original price. This represents a vertical compression of the graph by a factor of \( \frac{4}{5} \).

EXERCISES

Perform the given transformation to the point \((5, -1)\). Give the coordinates of the new point.

4. 5 units left, 4 units down
5. reflection across the x-axis

The graph shows parking garage fees. Sketch a graph to represent each situation and identify the transformation of the original graph that it represents.

6. The fees are half price on weekends.
7. The fees are increased by 10%.
8. All fees are increased by $1.00.
9-2 Introduction to Parent Functions

**EXAMPLE**

- Identify the parent function for \( g(x) = \sqrt{x - 4} \) from its equation. Then graph \( g \) on your calculator and describe what transformation of the parent function it represents.

\[ g(x) = \sqrt{x - 4} \] is a square-root function.

The graph of the square-root parent function intersects the \( x \)-axis at the point \((0, 0)\).

The graph of the function \( g(x) = \sqrt{x - 4} \) intersects the \( x \)-axis at the point \((4, 0)\).

So \( g(x) = \sqrt{x - 4} \) represents a translation of the square-root parent function 4 units right.

9. \( g(x) = x^2 - 1 \)

10. \( g(x) = -\sqrt{x} \)

11. Graph the data from the table. Describe the parent function that would best approximate the data set. Then use the graph to estimate the tire pressure for a 95-pound rider.

<table>
<thead>
<tr>
<th>Bicycle Road-Tire Pressures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight of Rider (lb)</td>
</tr>
<tr>
<td>Pressure (psi)</td>
</tr>
</tbody>
</table>

**EXERCISES**

Let \( g(x) \) be the indicated transformation of \( f(x) = x \). Write the rule for \( g(x) \).

- horizontal shift 8 units right

- vertical shift 5 units up followed by a vertical stretch by a factor of 3

- horizontal shift 3 units left followed by a vertical shift down 7 units

- vertical shift 5 units up followed by a reflection across the \( x \)-axis

- horizontal shift 12 units right followed by a reflection across the \( y \)-axis

**EXAMPLE**

Let \( g(x) \) be the indicated transformation of \( f(x) = x \). Write the rule for \( g(x) \).

- horizontal shift 5 units left followed by a horizontal stretch by a factor of 3

- Translating \( f(x) = x \) 5 units left replaces each \( x \) with \((x + 5)\).

- Let \( h(x) = f(x + 5) \)

- Replace each \( x \) with \( \left(\frac{x}{3}\right) \).

- \( g(x) = h\left(\frac{x}{3}\right) = \frac{x}{3} + 5 \)

9-3 Transforming Linear Functions

**EXERCISES**

Let \( g(x) \) be the indicated transformation of \( f(x) = x \). Write the rule for \( g(x) \).

- horizontal shift 8 units right

- vertical shift 5 units up followed by a vertical stretch by a factor of 3

- horizontal shift 3 units left followed by a vertical shift down 7 units

- vertical shift 5 units up followed by a reflection across the \( x \)-axis

- horizontal shift 12 units right followed by a reflection across the \( y \)-axis
17. Find the following for this set of data on median income and median home price.

a. Make a scatter plot of the data using median income as the independent variable.

b. Find the correlation coefficient \( r \) and the line of best fit for these data.

<table>
<thead>
<tr>
<th>Median Income (thousands)</th>
<th>Median Home Price (thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>69.5</td>
<td>130.2</td>
</tr>
<tr>
<td>46.3</td>
<td>94.5</td>
</tr>
<tr>
<td>56.7</td>
<td>115.5</td>
</tr>
<tr>
<td>65.2</td>
<td>106.4</td>
</tr>
<tr>
<td>54.7</td>
<td>98.6</td>
</tr>
<tr>
<td>59.6</td>
<td>115.5</td>
</tr>
</tbody>
</table>
1. The table shows how the distance from the top of a building to the horizon depends on the building’s height. Graph the relationship from building height to horizon distance, and identify which parent function best describes the data. Then use your graph to estimate the distance to the horizon from the top of a building with a height of 80 m.

<table>
<thead>
<tr>
<th>Height of Building (m)</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to Horizon (km)</td>
<td>8.0</td>
<td>11.3</td>
<td>15.9</td>
<td>22.5</td>
<td>35.6</td>
</tr>
</tbody>
</table>

Let \( g(x) \) be the indicated transformation(s) of \( f(x) = x \). Write the rule for \( g(x) \).

2. vertical stretch by a factor of 4
3. horizontal translation 6 units right
4. horizontal compression by a factor of \( \frac{1}{6} \) followed by a vertical shift 4 units down

5. A consumer group is studying how hospitals are staffed. Here are the results from eight randomly selected hospitals in a state.

<table>
<thead>
<tr>
<th>Hospital Beds</th>
<th>23</th>
<th>29</th>
<th>35</th>
<th>42</th>
<th>46</th>
<th>54</th>
<th>64</th>
<th>76</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-Time Employees</td>
<td>69</td>
<td>95</td>
<td>118</td>
<td>126</td>
<td>123</td>
<td>178</td>
<td>156</td>
<td>176</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the data with hospital beds as the independent variable.

b. Find the correlation coefficient and the equation of the line of best fit. Draw the line of best fit on your scatter plot.

c. Predict the number of beds in a hospital with 80 full-time employees.

6. Translate \( f(x) = |x| \) so that its vertex is at \((4, -2)\). Then graph.

7. Find \( g(x) \) if \( f(x) = |2x| - 3 \) is stretched horizontally by a factor of 3 and reflected across the \( x \)-axis.
Extending Transformational Geometry

• Apply reflections, translations, and rotations to simple geometric figures in the coordinate plane.
• Understand how symmetry and transformations are related.

Let it Snow!

A blanket of snow is formed by trillions of symmetric crystals. You can use transformations and symmetry to explore snow crystals.
Reading Strategy: Read to Solve Problems

A word problem may be overwhelming at first. Once you identify the important parts of the problem and translate the words into math language, you will find that the problem is similar to others you have solved.

**Reading Tips:**
- ✔ Read each phrase slowly. Write down what the words mean as you read them.
- ✔ Translate the words or phrases into math language.
- ✔ Draw a diagram. Label the diagram so it makes sense to you.
- ✔ Highlight what is being asked.
- ✔ Read the problem again before finding your solution.

Use the Reading Tips to help you understand this problem.

14. After a day hike, a group of hikers set up a camp 3 km east and 7 km north of the starting point. What is the distance from the camp to the starting point?

Use the Distance Formula to find the distance between the camp and the starting point.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

\[ = \sqrt{(3 - 0)^2 + (7 - 0)^2} \approx 7.6 \text{ km} \]

**Try This**

For the following problem, apply the following reading tips. Do not solve.
- • Identify key words.
- • Translate each phrase into math language.
- • Draw a diagram to represent the problem.

1. The lengths of two sides of a triangle are 4 cm and 10 cm. Find the range of possible lengths of the third side.
Who uses this?

Artists use transformations to create decorative patterns. (See Example 4.)

The Alhambra, a 13th-century palace in Granada, Spain, is famous for the geometric patterns that cover its walls and floors. To create a variety of designs, the builders based the patterns on several different transformations.

A transformation is a change in the position, size, or shape of a figure. The original figure is called the preimage. The resulting figure is called the image. A transformation maps the preimage to the image. Arrow notation (→) is used to describe a transformation, and primes (′) are used to label the image.

Transformations

REFLECTION

A reflection (or flip) is a transformation across a line, called the line of reflection. Each point and its image are the same distance from the line of reflection.

ROTATION

A rotation (or turn) is a transformation about a point, called the center of rotation. Each point and its image are the same distance from the center.

TRANSLATION

A translation (or slide) is a transformation in which all the points of a figure move the same distance in the same direction.

Example 1

Identify the transformation. Then use arrow notation to describe the transformation.

The transformation cannot be a translation because each point and its image are not in the same position.

The transformation is a reflection. \( \triangle EFG \rightarrow \triangle EF'G' \)
Identify the transformation. Then use arrow notation to describe the transformation.

The transformation is a 90° rotation. \( \text{RSTU} \rightarrow \text{R'S'T'U'} \)

Identify each transformation. Then use arrow notation to describe the transformation.

1a.

1b.

2.

EXAMPLE 2

Drawing and Identifying Transformations

A figure has vertices at \( A(-1, 4), B(-1, 1), \) and \( C(3, 1) \). After a transformation, the image of the figure has vertices at \( A'(-1, -4), B'(-1, -1), \) and \( C'(3, -1) \). Draw the preimage and image. Then identify the transformation.

- Plot the points. Then use a straightedge to connect the vertices.
- The transformation is a reflection across the \( x \)-axis because each point and its image are the same distance from the \( x \)-axis.

2. A figure has vertices at \( E(2, 0), F(2, -1), G(5, -1), \) and \( H(5, 0) \). After a transformation, the image of the figure has vertices at \( E'(0, 2), F'(1, 2), G'(1, 5), \) and \( H'(0, 5) \). Draw the preimage and image. Then identify the transformation.

To find coordinates for the image of a figure in a translation, add \( a \) to the \( x \)-coordinates of the preimage and add \( b \) to the \( y \)-coordinates of the preimage. Translations can also be described by a rule such as \( (x, y) \rightarrow (x + a, y + b) \).

EXAMPLE 3

Translations in the Coordinate Plane

Find the coordinates for the image of \( \triangle ABC \) after the translation \( (x, y) \rightarrow (x + 3, y - 4) \). Draw the image.

**Step 1** Find the coordinates of \( \triangle ABC \).
- The vertices of \( \triangle ABC \) are \( A(-1, 1), B(-3, 3), \) and \( C(-4, 0) \).
Step 2 Apply the rule to find the vertices of the image.

\[ A'(−1 + 3, 1 − 4) = A'(2, −3) \]
\[ B'(−3 + 3, 3 − 4) = B'(0, −1) \]
\[ C'(−4 + 3, 0 − 4) = C'(−1, −4) \]

Step 3 Plot the points. Then finish drawing the image by using a straightedge to connect the vertices.

3. Find the coordinates for the image of JKLM after the translation \((x, y) \rightarrow (x − 2, y + 4)\). Draw the image.

**Example 4**

Art History Application

The pattern shown is similar to a pattern on a wall of the Alhambra. Write a rule for the translation of square 1 to square 2.

Step 1 Choose 2 points
Choose a point \(A\) on the preimage and a corresponding point \(A'\) on the image. \(A\) has coordinates \((3, 1)\), and \(A'\) has coordinates \((1, 3)\).

Step 2 Translate
To translate \(A\) to \(A'\), 2 units are subtracted from the \(x\)-coordinate and 2 units are added to the \(y\)-coordinate. Therefore, the translation rule is \((x, y) \rightarrow (x − 2, y + 2)\).

4. Use the diagram to write a rule for the translation of square 1 to square 3.

**Think and Discuss**

1. Explain how to recognize a reflection when given a figure and its image.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, sketch an example of each transformation.
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.

1. Given the transformation \( \triangle XYZ \rightarrow \triangle X'Y'Z' \), name the preimage and image of the transformation.

2. The types of transformations of geometric figures in the coordinate plane can be described as a slide, a flip, or a turn. What are the other names used to identify these transformations?

Identify each transformation. Then use arrow notation to describe the transformation.

3. \[ \begin{array}{ccc} & C' & C \\ A & B & A' \end{array} \]

4. \[ \begin{array}{ccc} P' & Q' & P \\ S' & R' & S \end{array} \]

5. A figure has vertices at \( A(-3, 2) \), \( B(-1, -1) \), and \( C(-4, -2) \). After a transformation, the image of the figure has vertices at \( A'(3, 2) \), \( B'(1, -1) \), and \( C'(4, -2) \). Draw the preimage and image. Then identify the transformation.

6. Multi-Step  The coordinates of the vertices of \( \triangle DEF \) are \( D(2, 3) \), \( E(1, 1) \), and \( F(4, 0) \). Find the coordinates for the image of \( \triangle DEF \) after the translation \( (x, y) \rightarrow (x - 3, y - 2) \). Draw the preimage and image.

7. Animation  In an animated film, a simple scene can be created by translating a figure against a still background. Write a rule for the translation that maps the rocket from position 1 to position 2.

PRACTICE AND PROBLEM SOLVING

Identify each transformation. Then use arrow notation to describe the transformation.

8. \[ \begin{array}{ccc} F' & E' & G' \\ & D' & \end{array} \]

9. \[ \begin{array}{ccc} W & X \\ Z & Y \end{array} \]

10. A figure has vertices at \( J(-2, 3) \), \( K(0, 3) \), \( L(0, 1) \), and \( M(-2, 1) \). After a transformation, the image of the figure has vertices at \( J'(2, 1) \), \( K'(4, 1) \), \( L'(4, -1) \), and \( M'(2, -1) \). Draw the preimage and image. Then identify the transformation.
11. **Multi-Step** The coordinates of the vertices of rectangle \(ABCD\) are \(A(-4, 1)\), \(B(1, 1)\), \(C(1, -2)\), and \(D(-4, -2)\). Find the coordinates for the image of rectangle \(ABCD\) after the translation \((x, y) \rightarrow (x + 3, y - 2)\). Draw the preimage and the image.

12. **Travel** Write a rule for the translation that maps the descent of the hot air balloon.

Which transformation is suggested by each of the following?

13. mountain range and its image on a lake
14. straight line path of a band marching down a street
15. wings of a butterfly

Given points \(F(3, 5)\), \(G(-1, 4)\), and \(H(5, 0)\), draw \(\triangle FGH\) and its reflection across each of the following lines.

16. the \(x\)-axis
17. the \(y\)-axis
18. Find the vertices of one of the triangles on the graph. Then use arrow notation to write a rule for translating the other three triangles.

A transformation maps \(A\) onto \(B\) and \(C\) onto \(D\).

19. Name the image of \(A\).
20. Name the preimage of \(B\).
21. Name the image of \(C\).
22. Name the preimage of \(D\).
23. Find the coordinates for the image of \(\triangle RST\) with vertices \(R(1, -4)\), \(S(-1, -1)\), and \(T(-5, 1)\) after the translation \((x, y) \rightarrow (x - 2, y - 8)\).

24. **Critical Thinking** Consider the translations \((x, y) \rightarrow (x + 5, y + 3)\) and \((x, y) \rightarrow (x + 10, y + 5)\). Compare the two translations.

Graph each figure and its image after the given translation.

25. \(\overline{MN}\) with endpoints \(M(2, 8)\) and \(N(-3, 4)\) after the translation \((x, y) \rightarrow (x + 2, y - 5)\)
26. \(\overline{KL}\) with endpoints \(K(-1, 1)\) and \(L(3, -4)\) after the translation \((x, y) \rightarrow (x - 4, y + 3)\)

27. **Write About It** Given a triangle in the coordinate plane, explain how to draw its image after the translation \((x, y) \rightarrow (x + 1, y + 1)\).

28. Greg wants to rearrange the triangular pattern of colored stones on his patio. What combination of transformations could he use to transform \(\triangle CAE\) to the image on the coordinate plane?
29. Which type of transformation maps $\triangle XYZ$ to $\triangle X'Y'Z'$?
   - A Reflection
   - B Rotation
   - C Translation
   - D Not here

30. $\triangle DEF$ has vertices at $D(-4, 2)$, $E(-3, -3)$, and $F(1, 4)$. Which of these points is a vertex of the image of $\triangle DEF$ after the translation $(x, y) \rightarrow (x - 2, y + 1)$?
   - F $(-2, 1)$
   - G $(3, 3)$
   - H $(-5, -2)$
   - I $(-6, -1)$

31. Consider the translation $(1, 4) \rightarrow (-2, 3)$. What number was added to the $x$-coordinate?
   - A $-3$
   - B $-1$
   - C $1$
   - D $7$

32. Consider the translation $(-5, -7) \rightarrow (-2, -1)$. What number was added to the $y$-coordinate?
   - F $-3$
   - G $3$
   - H $6$
   - I $8$

**CHALLENGE AND EXTEND**

33. $\triangle RST$ with vertices $R(-2, -2)$, $S(-3, 1)$, and $T(1, 1)$ is translated by $(x, y) \rightarrow (x - 1, y + 3)$. Then the image, $\triangle R'S'T'$, is translated by $(x, y) \rightarrow (x + 4, y - 1)$, resulting in $\triangle R''S''T''$.
   a. Find the coordinates for the vertices of $\triangle R''S''T''$.
   b. Write a rule for a single translation that maps $\triangle RST$ to $\triangle R''S''T''$.

34. Find the angle through which the minute hand of a clock rotates over a period of 12 minutes.

35. A triangle has vertices $A(1, 0)$, $B(5, 0)$, and $C(2, 3)$. The triangle is rotated 90° counterclockwise about the origin. Draw and label the image of the triangle.

Determine the coordinates for the reflection image of any point $A(x, y)$ across the given line.

36. $x$-axis

37. $y$-axis
Explore Transformations

A transformation is a movement of a figure from its original position (preimage) to a new position (image). In this lab, you will use geometry software to perform transformations and explore their properties.

Use with Transformations in the Coordinate Plane

Activity 1

1. Construct a triangle using the segment tool. Use the text tool to label the vertices $A$, $B$, and $C$.

2. Select points $A$ and $B$ in that order. Choose Mark Vector from the Transform menu.

3. Select $\triangle ABC$ by clicking on all three segments of the triangle.

4. Choose Translate from the Transform menu, using Marked as the translation vector. What do you notice about the relationship between your preimage and its image?

5. What happens when you drag a vertex or a side of $\triangle ABC$?

Try This

For Problems 1 and 2 choose New Sketch from the File menu.

1. Construct a triangle and a segment outside the triangle. Mark this segment as a translation vector as you did in Step 2 of Activity 1. Use Step 4 of Activity 1 to translate the triangle. What happens when you drag an endpoint of the new segment?

2. Instead of translating by a marked vector, use Rectangular as the translation vector and translate by a horizontal distance of 1 cm and a vertical distance of 2 cm. Compare this method with the marked vector method. What happens when you drag a side or vertex of the triangle?

3. Select the angles and sides of the preimage and image triangles. Use the tools in the Measure menu to measure length, angle measure, perimeter, and area. What do you think is true about these two figures?
Activity 2

1. Construct a triangle. Label the vertices \(G, H,\) and \(I.\)

2. Select point \(H\) and choose Mark Center from the Transform menu.

3. Select \(\angle GHI\) by selecting points \(G, H,\) and \(I\) in that order. Choose Mark Angle from the Transform menu.

4. Select the entire triangle \(\triangle GHI\) by dragging a selection box around the figure.

5. Choose Rotate from the Transform menu, using Marked Angle as the angle of rotation.

6. What happens when you drag a vertex or a side of \(\triangle GHI\)?

Try This

For Problems 4–6 choose New Sketch from the File menu.

4. Instead of selecting an angle of the triangle as the rotation angle, draw a new angle outside of the triangle. Mark this angle. Mark \(\angle GHI\) as Center and rotate the triangle. What happens when you drag one of the points that form the rotation angle?

5. Construct \(\triangle QRS,\) a new rotation angle, and a point \(P\) not on the triangle. Mark \(P\) as the center and mark the angle. Rotate the triangle. What happens when you drag \(P\) outside, inside, or on the preimage triangle?

6. Instead of rotating by a marked angle, use Fixed Angle as the rotation method and rotate by a fixed angle measure of 30\(^\circ\). Compare this method with the marked angle method.

7. Using the fixed angle method of rotation, can you find an angle measure that will result in an image figure that exactly covers the preimage figure?
Dilations and Similarity in the Coordinate Plane

**Objectives**
- Apply similarity properties in the coordinate plane.
- Use coordinate proof to prove figures similar.

**Vocabulary**
- dilation
- scale factor

**Who uses this?**
Computer programmers use coordinates to enlarge or reduce images.

Many photographs on the Web are in JPEG format, which is short for Joint Photographic Experts Group. When you drag a corner of a JPEG image in order to enlarge it or reduce it, the underlying program uses coordinates and similarity to change the image’s size.

A **dilation** is a transformation that changes the size of a figure but not its shape. The preimage and the image are always similar. A **scale factor** describes how much the figure is enlarged or reduced. For a dilation with scale factor $k$, you can find the image of a point by multiplying each coordinate by $k$: $(a, b) \rightarrow (ka, kb)$.

**Example 1**

**Computer Graphics Application**

The figure shows the position of a JPEG photo. Draw the border of the photo after a dilation with scale factor $\frac{3}{2}$.

**Step 1** Multiply the vertices of the photo $A(0, 0)$, $B(0, 4)$, $C(3, 4)$, and $D(3, 0)$ by $\frac{3}{2}$.

- $A(0, 0) \rightarrow A'(0 \cdot \frac{3}{2}, 0 \cdot \frac{3}{2}) \rightarrow A'(0, 0)$
- $B(0, 4) \rightarrow B'(0 \cdot \frac{3}{2}, 4 \cdot \frac{3}{2}) \rightarrow B'(0, 6)$
- $C(3, 4) \rightarrow C'(3 \cdot \frac{3}{2}, 4 \cdot \frac{3}{2}) \rightarrow C'(4.5, 6)$
- $D(3, 0) \rightarrow D'(3 \cdot \frac{3}{2}, 0 \cdot \frac{3}{2}) \rightarrow D'(4.5, 0)$

**Step 2** Plot points $A'(0, 0)$, $B'(0, 6)$, $C'(4.5, 6)$, and $D'(4.5, 0)$. Draw the rectangle.

**Helpful Hint**
If the scale factor of a dilation is greater than 1 ($k > 1$), it is an **enlargement**. If the scale factor is less than 1 ($k < 1$), it is a **reduction**.

**Check It Out!**

1. **What if...?** Draw the border of the original photo after a dilation with scale factor $\frac{1}{2}$.
### Example 2
Finding Coordinates of Similar Triangles

Given that \( \triangle AOB \sim \triangle COD \), find the coordinates of \( D \) and the scale factor.

Since \( \triangle AOB \sim \triangle COD \),

\[
\frac{AO}{CO} = \frac{OB}{OD}
\]

\[
\frac{2}{4} = \frac{3}{OD}
\]

Substitute 2 for \( AO \), 4 for \( CO \), and 3 for \( OB \).

\[
2OD = 12 \quad \text{Cross Products Prop.}
\]

Divide both sides by 2.

\[
OD = 6
\]

\( D \) lies on the \( x \)-axis, so its \( y \)-coordinate is 0. Since \( OD = 6 \), its \( x \)-coordinate must be 6. The coordinates of \( D \) are \((6, 0)\).

\[
(M, N) = (3, 0) \rightarrow (3 \cdot 2, 0 \cdot 2) = (6, 0)
\]

so the scale factor is 2.

### Example 3
Proving Triangles Are Similar

Given: \( A(1, 5), B(-1, 3), C(3, 4), D(-3, 1) \), and \( E(5, 3) \)

Prove: \( \triangle ABC \sim \triangle ADE \)

Step 1 Plot the points and draw the triangles.

Step 2 Use the Distance Formula to find the side lengths.

\[
AB = \sqrt{(-1 - 1)^2 + (3 - 5)^2} \quad AC = \sqrt{(3 - 1)^2 + (4 - 5)^2}
\]

\[
= \sqrt{8} = 2\sqrt{2} \quad = \sqrt{5}
\]

\[
AD = \sqrt{(-3 - 1)^2 + (1 - 5)^2} \quad AE = \sqrt{(5 - 1)^2 + (3 - 5)^2}
\]

\[
= \sqrt{32} = 4\sqrt{2} \quad = \sqrt{20} = 2\sqrt{5}
\]

Step 3 Find the similarity ratio.

\[
\frac{AB}{AD} = \frac{2\sqrt{2}}{4\sqrt{2}} \quad \frac{AC}{AE} = \frac{\sqrt{5}}{2\sqrt{5}}
\]

\[
= \frac{1}{4} \quad = \frac{1}{2}
\]

Since \( \frac{AB}{AD} = \frac{AC}{AE} \) and \( \angle A \cong \angle A \) by the Reflexive Property, \( \triangle ABC \sim \triangle ADE \) by SAS ~.

### Check It Out!

2. Given that \( \triangle MON \sim \triangle POQ \) and coordinates \( P(-15, 0), M(-10, 0), \) and \( Q(0, -30) \), find the coordinates of \( N \) and the scale factor.

3. Given: \( R(-2, 0), S(-3, 1), T(0, 1), U(-5, 3), \) and \( V(4, 3) \) Prove: \( \triangle RST \sim \triangle RUV \)
EXAMPLE 4

Using the SSS Similarity Theorem

Graph the image of \( \triangle ABC \) after a dilation with scale factor 2. Verify that \( \triangle A'B'C' \sim \triangle ABC \).

Step 1 Multiply each coordinate by 2 to find the coordinates of the vertices of \( \triangle A'B'C' \).

\[
\begin{align*}
A(2, 3) & \rightarrow A'(2 \cdot 2, 3 \cdot 2) = A'(4, 6) \\
B(0, 1) & \rightarrow B'(0 \cdot 2, 1 \cdot 2) = B'(0, 2) \\
C(3, 0) & \rightarrow C'(3 \cdot 2, 0 \cdot 2) = C'(6, 0)
\end{align*}
\]

Step 2 Graph \( \triangle A'B'C' \).

Step 3 Use the Distance Formula to find the side lengths.

\[
\begin{align*}
AB &= \sqrt{(2 - 0)^2 + (3 - 1)^2} = \sqrt{8} = 2\sqrt{2} \\
A'B' &= \sqrt{(4 - 0)^2 + (6 - 2)^2} = \sqrt{32} = 4\sqrt{2} \\
BC &= \sqrt{(3 - 0)^2 + (0 - 1)^2} = \sqrt{10} \\
B'C' &= \sqrt{(6 - 0)^2 + (0 - 2)^2} = \sqrt{40} = 2\sqrt{10} \\
AC &= \sqrt{(3 - 2)^2 + (0 - 3)^2} = \sqrt{10} \\
A'C' &= \sqrt{(6 - 4)^2 + (0 - 6)^2} = \sqrt{40} = 2\sqrt{10}
\end{align*}
\]

Step 4 Find the similarity ratio.

\[
\frac{A'B'}{AB} = \frac{4\sqrt{2}}{2\sqrt{2}} = 2, \quad \frac{B'C'}{BC} = \frac{2\sqrt{10}}{\sqrt{10}} = 2, \quad \frac{A'C'}{AC} = \frac{2\sqrt{10}}{\sqrt{10}} = 2
\]

Since \( \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} \), \( \triangle ABC \sim \triangle A'B'C' \) by SSS ~.

4. Graph the image of \( \triangle MNP \) after a dilation with scale factor 3. Verify that \( \triangle MNP' \sim \triangle MNP \).

THINK AND DISCUSS

1. \( \triangle JKL \) has coordinates \( J(0, 0), K(0, 2), \) and \( L(3, 0) \). Its image after a dilation has coordinates \( J'(0, 0), K'(0, 8), \) and \( L'(12, 0) \). Explain how to find the scale factor of the dilation.

2. GET ORGANIZED Copy and complete the graphic organizer. Write the definition of a dilation, a property of dilations, and an example and nonexample of a dilation.
GUIDED PRACTICE

**Vocabulary** Apply the vocabulary from this lesson to answer each question.

1. A **?** is a transformation that proportionally reduces or enlarges a figure, such as the pupil of an eye. (*dilation* or *scale factor*)

2. A ratio that describes or determines the dimensional relationship of a figure to that which it represents, such as a map scale of 1 in.:45 ft, is called a **?**. (*dilation* or *scale factor*)

3. **Graphic Design** A designer created this logo for a real estate agent but needs to make the logo twice as large for use on a sign. Draw the logo after a dilation with scale factor 2.

4. Given that \( \triangle AOB \sim \triangle COD \), find the coordinates of \( C \) and the scale factor.

5. Given that \( \triangle ROS \sim \triangle POQ \), find the coordinates of \( S \) and the scale factor.

6. Given: \( A(0, 0), B(-1, 1), C(3, 2), D(-2, 2), \) and \( E(6, 4) \)
   
   **Prove:** \( \triangle ABC \sim \triangle ADE \)

7. Given: \( J(-1, 0), K(-3, -4), L(3, -2), M(-4, -6), \) and \( N(5, -3) \)
   
   **Prove:** \( \triangle JKL \sim \triangle JMN \)

**Multi-Step** Graph the image of each triangle after a dilation with the given scale factor. Then verify that the image is similar to the given triangle.

8. **scale factor 2**

9. **scale factor \( \frac{3}{2} \)**
10. **Advertising** A promoter produced this design for a street festival. She now wants to make the design smaller to use on postcards. Sketch the design after a dilation with scale factor $\frac{1}{2}$.

11. Given that $\triangle UOV \sim \triangle XOY$, find the coordinates of $X$ and the scale factor.

12. Given that $\triangle MON \sim \triangle KOL$, find the coordinates of $K$ and the scale factor.

13. Given: $D(-1, 3)$, $E(-3, -1)$, $F(3, -1)$, $G(-4, -3)$, and $H(5, -3)$
   
   Prove: $\triangle DEF \sim \triangle DGH$

14. Given: $M(0, 10)$, $N(5, 0)$, $P(15, 15)$, $Q(10, -10)$, and $R(30, 20)$
   
   Prove: $\triangle MNP \sim \triangle MQR$

15. Multi-Step
   
   Graph the image of each triangle after a dilation with the given scale factor. Then verify that the image is similar to the given triangle.

16. Critical Thinking
   
   Consider the transformation given by the mapping $(x, y) \rightarrow (2x, 4y)$. Is this transformation a dilation? Why or why not?

17. **ERROR ANALYSIS**
   
   Which solution to find the scale factor of the dilation that maps $\triangle RST$ to $\triangle UVW$ is incorrect? Explain the error.

18. **Write About It**
   
   A dilation maps $\triangle ABC$ to $\triangle A'B'C'$. How is the scale factor of the dilation related to the similarity ratio of $\triangle ABC$ to $\triangle A'B'C'$? Explain.

19. **Multi-Step Test Prep**
   
   a. In order to build a skateboard ramp, Miles draws $\triangle JKL$ on a coordinate plane. One unit on the drawing represents 60 cm of actual distance. Explain how he should assign coordinates for the vertices of $\triangle JKL$.

   b. Graph the image of $\triangle JKL$ after a dilation with scale factor 3.
21. Which coordinates for C make △COD similar to △AOB?

- A (0, 2.4)
- B (0, 2.5)
- C (0, 3)
- D (0, 3.6)

22. A dilation with scale factor 2 maps △RST to △R'S'T'. The perimeter of △RST is 60. What is the perimeter of △R'S'T'?

- F 30
- G 60
- H 120
- J 240

23. Which triangle with vertices D, E, and F is similar to △ABC?

- A D(1, 2), E(3, 2), F(2, 0)
- B D(−1, −2), E(2, −2), F(1, −5)
- C D(1, 2), E(5, 2), F(3, 0)
- D D(−2, −2), E(0, 2), F(−1, 0)

24. Gridded Response ̶̶̶̶̶AB with endpoints A(3, 2) and B(7, 5) is dilated by a scale factor of 3. Find the length of ̶̶̶̶̶A'B'.

CHALLENGE AND EXTEND

25. How many different triangles having ̶̶̶XY as a side are similar to △MNP?

26. △XYZ ~ △MPN. Find the coordinates of Z.

27. A rectangle has two of its sides on the x- and y-axes, a vertex at the origin, and a vertex on the line y = 2x. Prove that any two such rectangles are similar.

28. △ABC has vertices A(0, 1), B(3, 1), and C(1, 3). △DEF has vertices D(1, −1) and E(7, −1). Find two different locations for vertex F so that △ABC ~ △DEF.
Segment Partition

Objectives
Divide a directed line segment into partitions.

Example 1

Finding the Coordinates of a Point in a Directed Line Segment

Find the point \( P \) along the directed line segment from point \( A(-8, -7) \) to point \( B(8, 5) \) that divides the segment in the ratio 3 to 1.

First, find the rise and run of the directed line segment.

\[
\begin{align*}
\text{rise} &= | -7 - 5 | = 12 \\
\text{run} &= | 8 - (-8) | = 16 \\
\end{align*}
\]

Point \( P \) is \( \frac{3}{4} \) of the way between points \( A \) and \( B \), so find \( \frac{3}{4} \) of both the rise and the run:

\[
\begin{align*}
\frac{3}{4} \text{ of rise} &= \frac{3}{4} (12) = 9 \\
\frac{3}{4} \text{ of run} &= \frac{3}{4} (16) = 12 \\
\end{align*}
\]

Point \( P \) is 9 units up and 12 units right from point \( A \). Its coordinates are \((-8 + 12, -7 + 9)\), or \((4, 2)\).

Check It Out!

1. Find the point \( Q \) along the directed line segment from point \( R(-2, 4) \) to point \( S(18, -6) \) that divides the segment in the ratio 3 to 7.
**Example 2**

Using Construction to Draw a Point in a Directed Line Segment

Given the directed line segment from $A$ to $B$, construct a point $G$ that divides the segment in the ratio 1 to 1 from $A$ to $B$.

Use a straightedge to draw the ray $AC$. The exact measure of the angle is not important, but the construction is easiest for angles from about 30° to 60°.

Place the compass point on $A$ and draw an arc through $AC$. Label the intersection $D$. Using the same compass setting, draw another arc centered on $D$, and label the intersection $E$.

Connect points $B$ and $E$. Construct an angle congruent to $\angle AEB$ with $D$ as its vertex. Label the intersection of the angle with $AB$ as point $F$.

Point $F$ divides the segment in the ratio 1 to 2.

---

**Exercises**

1. Find the point $P$ along the directed line segment from point $A$ to point $B$ that divides the segment in the ratio 2 to 5.

2. Draw a directed line segment from $A$ to $B$, then construct point $P$ that divides the segment in the ratio 2 to 3 from point $B$ to point $A$.

3. Find the point $P$ along the directed line segment from point $A$ to point $B$ that divides the segment in the ratio 1 to 6.

3. Draw a directed line segment from $A$ to $B$, then construct point $P$ that divides the segment in the ratio 4 to 1 from point $A$ to point $B$. 

---

68 Chapter 10 Similarity
Direct Variation

For two similar figures, the measure of each point was multiplied by the same scale factor. Is the relationship between the scale factor and the perimeter of the figure a direct variation?

Recall from algebra that if \( y \) varies directly as \( x \), then \( y = kx \), or \( \frac{y}{x} = k \), where \( k \) is the constant of variation.

**Example**

A rectangle has a length of 4 ft and a width of 2 ft. Find the relationship between the scale factors of similar rectangles and their corresponding perimeters. If the relationship is a direct variation, find the constant of variation.

**Step 1** Make a table to record data.

<table>
<thead>
<tr>
<th>Scale Factor ( x )</th>
<th>Length ( \ell = x(4) )</th>
<th>Width ( w = x(2) )</th>
<th>Perimeter ( P = 2\ell + 2w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \ell = \frac{1}{2}(4) = 2 )</td>
<td>( w = \frac{1}{2}(2) = 1 )</td>
<td>( 2(2) + 2(1) = 6 )</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>8</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>10</td>
<td>60</td>
</tr>
</tbody>
</table>

**Step 2** Graph the points \((\frac{1}{2}, 6), (2, 24), (3, 36), (4, 48), (5, 60)\).

Since the points are collinear and the line that contains them includes the origin, the relationship is a direct variation.

**Step 3** Find the equation of direct variation.

\[
y = kx
\]

\[
60 = k(5) \quad \text{Substitute 60 for } y \text{ and 5 for } x.
\]

\[
12 = k \quad \text{Divide both sides by 5.}
\]

\[
y = 12x \quad \text{Substitute 12 for } k.
\]

Thus the constant of variation is 12.

**Try This**

Use the scale factors given in the above table. Find the relationship between the scale factors of similar figures and their corresponding perimeters. If the relationship is a direct variation, find the constant of variation.

1. regular hexagon with side length 6
2. triangle with side lengths 3, 6, and 7
3. square with side length 3
An **isometry** is a transformation that does not change the shape or size of a figure. Reflections, translations, and rotations are all isometries. Isometries are also called **congruence transformations** or **rigid motions**.

Recall that a reflection is a transformation that moves a figure (the preimage) by flipping it across a line. The reflected figure is called the image. A reflection is an isometry, so the image is always congruent to the preimage.

**Example 1**

Tell whether each transformation appears to be a reflection. Explain.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes; the image appears to be flipped across a line.</td>
<td>No; the figure does not appear to be flipped.</td>
</tr>
</tbody>
</table>

**Construction**

**Reflect a Figure Using Patty Paper**

1. Draw a triangle and a line of reflection on a piece of patty paper.
2. Fold the patty paper back along the line of reflection.
3. Trace the triangle. Then unfold the paper.

Draw a segment from each vertex of the preimage to the corresponding vertex of the image. Your construction should show that the line of reflection is the perpendicular bisector of every segment connecting a point and its image.
Reflections

A reflection is a transformation across a line, called the line of reflection, so that the line of reflection is the perpendicular bisector of each segment joining each point and its image.

**EXAMPLE 2**

**Drawing Reflections**

Copy the quadrilateral and the line of reflection.

Draw the reflection of the quadrilateral across the line.

**Step 1** Through each vertex draw a line perpendicular to the line of reflection.

**Step 2** Measure the distance from each vertex to the line of reflection. Locate the image of each vertex on the opposite side of the line of reflection and the same distance from it.

**Step 3** Connect the images of the vertices.

2. Copy the quadrilateral and the line of reflection. Draw the reflection of the quadrilateral across the line.

**EXAMPLE 3**

**Problem-Solving Application**

A trail designer is planning two trails that connect campsites $A$ and $B$ to a point on the river. He wants the total length of the trails to be as short as possible. Where should the trail meet the river?

1. **Understand the Problem**

   The problem asks you to locate point $X$ on the river so that $AX + XB$ has the least value possible.

2. **Make a Plan**

   Let $B'$ be the reflection of point $B$ across the river. For any point $X$ on the river, $XB' \cong XB$, so $AX + XB = AX + XB'$. $AX + XB'$ is least when $A$, $X$, and $B'$ are collinear.

3. **Solve**

   Reflect $B$ across the river to locate $B'$. Draw $AB'$ and locate $X$ at the intersection of $AB'$ and the river.

4. **Look Back**

   To verify your answer, choose several possible locations for $X$ and measure the total length of the trails for each location.

**What if...?** If $A$ and $B$ were the same distance from the river, what would be true about $AX$ and $BX$?
**Example 4**

**Drawing Reflections in the Coordinate Plane**

Reflect the figure with the given vertices across the given line.

**A** \(M(1, 2), N(1, 4), P(3, 3); y\)-axis

The reflection of \((x, y)\) is \((-x, y)\).

- \(M(1, 2) \rightarrow M'(\,-1, 2)\)
- \(N(1, 4) \rightarrow N'(\,-1, 4)\)
- \(P(3, 3) \rightarrow P'(\,-3, 3)\)

Graph the preimage and image.

**B** \(D(2, 0), E(2, 2), F(5, 2), G(5, 1); y = x\)

The reflection of \((x, y)\) is \((y, x)\).

- \(D(2, 0) \rightarrow D'(0, 2)\)
- \(E(2, 2) \rightarrow E'(2, 2)\)
- \(F(5, 2) \rightarrow F'(2, 5)\)
- \(G(5, 1) \rightarrow G'(1, 5)\)

Graph the preimage and image.

4. Reflect the rectangle with vertices \(S(3, 4), T(3, 1), U(-2, 1),\) and \(V(-2, 4)\) across the \(x\)-axis.

---

**Think and Discuss**

1. Acute scalene \(\triangle ABC\) is reflected across \(BC\). Classify quadrilateral \(ABA'C\). Explain your reasoning.

2. Point \(A'\) is a reflection of point \(A\) across line \(\ell\). What is the relationship of \(\ell\) to \(AA'\)?

3. **Get Organized** Copy and complete the graphic organizer.

<table>
<thead>
<tr>
<th>Line of Reflection</th>
<th>Image of ((a, b))</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)-axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y)-axis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y = x)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. **Vocabulary** If a transformation is an *isometry*, how would you describe the relationship between the preimage and the image?

2. Tell whether each transformation appears to be a reflection.

3. 

4. 

5. 

6. 

7. 

8. **City Planning** The towns of San Pablo and Tanner are located on the same side of Highway 105. Two access roads are planned that connect the towns to a point \( P \) on the highway. Draw a diagram that shows where point \( P \) should be located in order to make the total length of the access roads as short as possible.

9. Reflect the figure with the given vertices across the given line.

10. 

11. 

12. 

13. 

14. 

15. 

16. 

**PRACTICE AND PROBLEM SOLVING**

Tell whether each transformation appears to be a reflection.

13. 

14. 

15. 

16. 

See Extra Practice for more Skills Practice and Applications Practice exercises.
Multi-Step Copy each figure and the line of reflection. Draw the reflection of the figure across the line.

17. 

18. 

19. Recreation Cara is playing pool. She wants to hit the ball at point A without hitting the ball at point B. She has to bounce the cue ball, located at point C, off the side rail and into her ball. Draw a diagram that shows the exact point along the rail that Cara should aim for.

Reflect the figure with the given vertices across the given line.
20. \(A(-3, 2), B(0, 2), C(-2, 0); y\)-axis
21. \(M(-4, -1), N(-1, -1), P(-2, -2); y = x\)
22. \(J(1, 2), K(-2, -1), L(3, -1); x\)-axis
23. \(S(-1, 1), T(1, 4), U(3, 2), V(1, -3); y = x\)

Copy each figure. Then complete the figure by drawing the reflection image across the line.
24. 
25. 
26. 

Chemistry In chemistry, *chiral* molecules are mirror images of each other. Although they have similar structures, chiral molecules can have very different properties. For example, the compound \(R-(+)-\)limonene smells like oranges, while its mirror image, \(S-(\text{-})\)-limonene, smells like lemons. Use the figure and the given line of reflection to draw \(S-(\text{-})\)-limonene.

Each figure shows a preimage and image under a reflection. Copy the figure and draw the line of reflection.
27. 
28. 
29. 
30. 

Use arrow notation to describe the mapping of each point when it is reflected across the given line.
31. \((5, 2); x\)-axis
32. \((-3, -7); y\)-axis
33. \((0, 12); x\)-axis
34. \((-3, -6); y = x\)
35. \((0, -5); y = x\)
36. \((4, 4); y = x\)
37. The figure shows one hole of a miniature golf course.
   a. Is it possible to hit the ball in a straight line from the tee \( T \) to the hole \( H \)?
   b. Find the coordinates of \( H' \), the reflection of \( H \) across \( BC \).
   c. The point at which a player should aim in order to make a hole in one is the intersection of \( TH' \) and \( BC \). What are the coordinates of this point?

38. **Critical Thinking** Sketch the next figure in the sequence below.

   ![](image.png)

39. **Critical Thinking** Under a reflection in the coordinate plane, the point \((3, 5)\) is mapped to the point \((5, 3)\). What is the line of reflection? Is this the only possible line of reflection? Explain.

Draw the reflection of the graph of each function across the given line.

40. \( x \)-axis

   ![](image.png)

41. \( y \)-axis

   ![](image.png)

42. **Write About It** Imagine reflecting all the points in a plane across line \( \ell \). Which points remain fixed under this transformation? That is, for which points is the image the same as the preimage? Explain.

**Construction** Use the construction of a line perpendicular to a given line through a given point and the construction of a segment congruent to a given segment to construct the reflection of each figure across a line.

43. a point

44. a segment

45. a triangle

46. Daryl is using a coordinate plane to plan a garden. He draws a flower bed with vertices \((3, 1)\), \((3, 4)\), \((-2, 4)\), and \((-2, 1)\). Then he creates a second flower bed by reflecting the first one across the \( x \)-axis. Which of these is a vertex of the second flower bed?

   - A \((-2, -4)\)
   - B \((-3, 1)\)
   - C \((2, 1)\)
   - D \((-3, -4)\)
47. In the reflection shown, the shaded figure is the preimage. Which of these represents the mapping?

- [F] \( MJNP \rightarrow DSWG \)
- [G] \( DGWS \rightarrow MJNP \)
- [H] \( JMPN \rightarrow GWSD \)
- [I] \( PMJN \rightarrow SDGW \)

48. What is the image of the point \((-3, 4)\) when it is reflected across the \(y\)-axis?

- [A] \((4, -3)\)
- [B] \((-3, -4)\)
- [C] \((3, 4)\)
- [D] \((-4, -3)\)

**CHALLENGE AND EXTEND**

Find the coordinates of the image when each point is reflected across the given line.

49. \((4, 2); y = 3\)  
50. \((-3, 2); x = 1\)  
51. \((3, 1); y = x + 2\)

52. Prove that the reflection image of a segment is congruent to the preimage.

Given: \(A'B'\) is the reflection image of \(AB\) across line \(\ell\).
Prove: \(AB \cong A'B'\)
Plan: Draw auxiliary lines \(AA'\) and \(BB'\) as shown. First prove that \(\triangle ACD \cong \triangle A'C'D\). Then use CPCTC to conclude that \(\angle CDA \cong \angle CDA'\). Therefore \(\triangle ADB \cong \triangle A'DB'\), which makes it possible to prove that \(\triangle ADB \cong \triangle A'DB'\). Finally use CPCTC to conclude that \(AB \cong A'B'\).

Once you have proved that the reflection image of a segment is congruent to the preimage, how could you prove the following? Write a plan for each proof.

53. If \(A'B'\) is the reflection of \(AB\), then \(AB = A'B'\).

54. If \(\angle A'B'C'\) is the reflection of \(\angle ABC\), then \(m\angle ABC = m\angle A'B'C'\).

55. The reflection \(\triangle A'B'C'\) is congruent to the preimage \(\triangle ABC\).

56. If point \(C\) is between points \(A\) and \(B\), then the reflection \(C'\) is between \(A'\) and \(B'\).

57. If points \(A, B,\) and \(C\) are collinear, then the reflections \(A', B',\) and \(C'\) are collinear.
Who uses this?
Marching band directors use translations to plan their bands’ field shows. (See Example 4.)

A translation is a transformation where all the points of a figure are moved the same distance in the same direction. A translation is an isometry, so the image of a translated figure is congruent to the preimage.

**Example 1**

**Identifying Translations**

Tell whether each transformation appears to be a translation. Explain.

- **A**
  - No; not all of the points have moved the same distance.

- **B**
  - Yes; all of the points have moved the same distance in the same direction.

**Check It Out!**

Tell whether each transformation appears to be a translation.

1a.

1b.

**Construction**

Translate a Figure Using Patty Paper

1. Draw a triangle and a translation vector on a sheet of paper.
2. Place a sheet of patty paper on top of the diagram. Trace the triangle and vector.
3. Slide the bottom paper in the direction of the vector until the head of the top vector aligns with the tail of the bottom vector. Trace the triangle.

Draw a segment from each vertex of the preimage to the corresponding vertex of the image. Your construction should show that every segment connecting a point and its image is the same length as the translation vector. These segments are also parallel to the translation vector.
A translation is a transformation along a vector such that each segment joining a point and its image has the same length as the vector and is parallel to the vector.

**EXAMPLE 2**

**Drawing Translations**

Copy the triangle and the translation vector. Draw the translation of the triangle along \( \vec{v} \).

**Step 1** Draw a line parallel to the vector through each vertex of the triangle.

**Step 2** Measure the length of the vector. Then, from each vertex mark off this distance in the same direction as the vector, on each of the parallel lines.

**Step 3** Connect the images of the vertices.

2. Copy the quadrilateral and the translation vector. Draw the translation of the quadrilateral along \( \vec{w} \).

Recall that a vector in the coordinate plane can be written as \( \langle a, b \rangle \), where \( a \) is the horizontal change and \( b \) is the vertical change from the initial point to the terminal point.
3. **EXAMPLE 3** Drawing Translations in the Coordinate Plane

Translate the triangle with vertices $A(-2,-4)$, $B(-1,-2)$, and $C(-3,0)$ along the vector $(2,4)$.

The image of $(x, y)$ is $(x + 2, y + 4)$.

- $A(-2,-4) \rightarrow A'(0,0)$
- $B(-1,-2) \rightarrow B'(1,2)$
- $C(-3,0) \rightarrow C'(-1,4)$

Graph the preimage and image.

3. Translate the quadrilateral with vertices $R(2,5)$, $S(0,2)$, $T(1,-1)$, and $U(3,1)$ along the vector $(-3,-3)$.

4. **EXAMPLE 4** Entertainment Application

In a marching drill, it takes 8 steps to march 5 yards. A drummer starts 8 steps to the left and 8 steps up from the center of the field. She marches 16 steps to the right to her second position. Then she marches 24 steps down the field to her final position. What is the drummer’s final position? What single translation vector moves her from the starting position to her final position?

The drummer’s starting coordinates are $(-8,8)$.

Her second position is $(-8 + 16, 8) = (8,8)$.

Her final position is $(8, 8 - 24) = (8, -16)$.

The vector that moves her directly from her starting position to her final position is $(16,0) + (0,-24) = (16,-24)$.

4. **What if…?** Suppose another drummer started at the center of the field and marched along the same vectors as above. What would this drummer’s final position be?

**THINK AND DISCUSS**

1. Point $A'$ is a translation of point $A$ along $\vec{v}$. What is the relationship of $\vec{v}$ to $AA'$?

2. $AB$ is translated to form $A'B'$. Classify quadrilateral $AA'B'B$. Explain your reasoning.

3. **GET ORGANIZED** Copy and complete the graphic organizer.
GUIDED PRACTICE

SEE EXAMPLE 1
Tell whether each transformation appears to be a translation.
1. 
2. 
3. 
4. 

SEE EXAMPLE 2
Multi-Step Copy each figure and the translation vector. Draw the translation of the figure along the given vector.
5. 
6. 

SEE EXAMPLE 3
Translate the figure with the given vertices along the given vector.
7. \(A(-4, -4), B(-2, -3), C(-1, 3); (5, 0)\)
8. \(R(-3, 1), S(-2, 3), T(2, 3), U(3, 1); (0, -4)\)
9. \(J(-2, 2), K(-1, 2), L(-1, -2), M(-3, -1); (3, 2)\)

SEE EXAMPLE 4
10. Art The Zulu people of southern Africa are known for their beadwork. To create a typical Zulu pattern, translate the polygon with vertices \((1, 5), (2, 3), (1, 1),\) and \((0, 3)\) along the vector \((0, -4)\). Translate the image along the same vector. Repeat to generate a pattern. What are the vertices of the fourth polygon in the pattern?

PRACTICE AND PROBLEM SOLVING
Tell whether each transformation appears to be a translation.
11. 
12. 
13. 
14. 

Multi-Step Copy each figure and the translation vector. Draw the translation of the figure along the given vector.

15. \[ \begin{align*} \vec{v} \end{align*} \]

16. \[ \begin{align*} \vec{w} \end{align*} \]

Translate the figure with the given vertices along the given vector.

17. \( P(-1, 2), Q(1, -1), R(3, 1), S(2, 3); \langle -3, 0 \rangle \)

18. \( A(1, 3), B(-1, 2), C(2, 1), D(4, 2); \langle -3, -3 \rangle \)

19. \( D(0, 15), E(-10, 5), F(10, -5); \langle 5, -20 \rangle \)

20. Animation An animator draws the ladybug shown and then translates it along the vector \( \langle 1, 1 \rangle \), followed by a translation of the new image along the vector \( \langle 2, 2 \rangle \), followed by a translation of the second image along the vector \( \langle 3, 3 \rangle \).
   a. Sketch the ladybug’s final position.
   b. What single vector moves the ladybug from its starting position to its final position?

Draw the translation of the graph of each function along the given vector.

21. \( \langle 3, 0 \rangle \)

22. \( \langle -1, -1 \rangle \)

23. Probability The point \( P(3, 2) \) is translated along one of the following four vectors chosen at random: \( \langle -3, 0 \rangle, \langle -1, -4 \rangle, \langle 3, -2 \rangle \), and \( \langle 2, 3 \rangle \). Find the probability of each of the following.
   a. The image of \( P \) is in the fourth quadrant.
   b. The image of \( P \) is on an axis.
   c. The image of \( P \) is at the origin.

24. The figure shows one hole of a miniature golf course and the path of a ball from the tee \( T \) to the hole \( H \).
   a. What translation vector represents the path of the ball from \( T \) to \( \overline{DC} \)?
   b. What translation vector represents the path of the ball from \( \overline{DC} \) to \( \overline{HR} \)?
   c. Show that the sum of these vectors is equal to the vector that represents the straight path from \( T \) to \( H \).
Each figure shows a preimage (blue) and its image (red) under a translation. Copy the figure and draw the vector along which the polygon is translated.

25.

26.

27. **Critical Thinking** The points of a plane are translated along the given vector \( \overrightarrow{AB} \). Do any points remain fixed under this transformation? That is, are there any points for which the image coincides with the preimage? Explain.

28. **Carpentry** Carpenters use a tool called *adjustable parallels* to set up level work areas and to draw parallel lines. Describe how a carpenter could use this tool to translate a given point along a given vector. What additional tools, if any, would be needed?

Find the vector associated with each translation. Then use arrow notation to describe the mapping of the preimage to the image.

29. the translation that maps point \( A \) to point \( B \)

30. the translation that maps point \( B \) to point \( A \)

31. the translation that maps point \( C \) to point \( D \)

32. the translation that maps point \( E \) to point \( B \)

33. the translation that maps point \( C \) to the origin

34. **Multi-Step** The rectangle shown is translated two-thirds of the way along one of its diagonals. Find the area of the region where the rectangle and its image overlap.

35. **Write About It** Point \( P \) is translated along the vector \( \langle a, b \rangle \). Explain how to find the distance between point \( P \) and its image.

**Construction** Use the construction of a line parallel to a given line through a given point and the construction of a segment congruent to a given segment to construct the translation of each figure along a vector.

36. a point  
37. a segment  
38. a triangle

39. What is the image of \( P(1, 3) \) when it is translated along the vector \( \langle -3, 5 \rangle \)?

   - A) \((-2, 8)\)  
   - B) \((0, 6)\)  
   - C) \((1, 3)\)  
   - D) \((0, 4)\)

40. After a translation, the image of \( A(-6, -2) \) is \( B(-4, -4) \). What is the image of the point \( (3, -1) \) after this translation?

   - F) \((-5, 1)\)  
   - G) \((5, -3)\)  
   - H) \((5, 1)\)  
   - I) \((-5, -3)\)
41. Which vector translates point \( Q \) to point \( P \)?

- A: \((-2, -4)\)
- B: \((4, -2)\)
- C: \((-2, 4)\)
- D: \((2, -4)\)

**CHALLENGE AND EXTEND**

42. The point \( M(1, 2) \) is translated along a vector that is parallel to the line \( y = 2x + 4 \). The translation vector has magnitude \( \sqrt{5} \). What are the possible images of point \( M \)?

43. A cube has edges of length 2 cm. Point \( P \) is translated along \( \vec{u} \), \( \vec{v} \), and \( \vec{w} \) as shown.

a. Describe a single translation vector that maps point \( P \) to point \( Q \).

b. Find the magnitude of this vector to the nearest hundredth.

44. Prove that the translation image of a segment is congruent to the preimage.

**Given:** \( \overline{A'B'} \) is the translation image of \( \overline{AB} \).

**Prove:** \( \overline{AB} \cong \overline{A'B'} \)

**Hint:** Draw auxiliary lines \( \overline{AA'} \) and \( \overline{BB'} \).

Once you have proved that the translation image of a segment is congruent to the preimage, how could you prove the following? Write a plan for each proof.

45. If \( \overline{A'B'} \) is a translation of \( \overline{AB} \), then \( AB = A'B' \).

46. If \( \angle A'B'C' \) is a translation of \( \angle ABC \), then \( m\angle ABC = m\angle A'B'C' \).

47. The translation \( \triangle A'B'C' \) is congruent to the preimage \( \triangle ABC \).

48. If point \( C \) is between points \( A \) and \( B \), then the translation \( C' \) is between \( A' \) and \( B' \).

49. If points \( A \), \( B \), and \( C \) are collinear, then the translations \( A' \), \( B' \), and \( C' \) are collinear.
Who uses this?
Astronomers can use properties of rotations to analyze photos of star trails. (See Exercise 35.)

Remember that a rotation is a transformation that turns a figure around a fixed point, called the center of rotation. A rotation is an isometry, so the image of a rotated figure is congruent to the preimage.

Example 1
Identifying Rotations

Tell whether each transformation appears to be a rotation. Explain.

A
Yes; the figure appears to be turned around a point.

B
No; the figure appears to be flipped, not turned.

Check It Out!
Tell whether each transformation appears to be a rotation.

1a.

1b.

Construction
Rotate a Figure Using Patty Paper

1. On a sheet of paper, draw a triangle and a point. The point will be the center of rotation.

2. Place a sheet of patty paper on top of the diagram. Trace the triangle and the point.

3. Hold your pencil down on the point and rotate the bottom paper counterclockwise. Trace the triangle.

Draw a segment from each vertex to the center of rotation. Your construction should show that a point’s distance to the center of rotation is equal to its image’s distance to the center of rotation. The angle formed by a point, the center of rotation, and the point’s image is the angle by which the figure was rotated.
Rotations

A rotation is a transformation about a point $P$, called the center of rotation, such that each point and its image are the same distance from $P$, and such that all angles with vertex $P$ formed by a point and its image are congruent. In the figure, $\angle APA'$ is the angle of rotation.

Example 2

Drawing Rotations

Copy the figure and the angle of rotation. Draw the rotation of the triangle about point $P$ by $m\angle A$.

Step 1 Draw a segment from each vertex to point $P$.

Step 2 Construct an angle congruent to $\angle A$ onto each segment. Measure the distance from each vertex to point $P$ and mark off this distance on the corresponding ray to locate the image of each vertex.

Step 3 Connect the images of the vertices.

CHECK IT OUT! 2. Copy the figure and the angle of rotation. Draw the rotation of the segment about point $Q$ by $m\angle X$.

Rotations in the Coordinate Plane

If the angle of a rotation in the coordinate plane is not a multiple of $90^\circ$, you can use sine and cosine ratios to find the coordinates of the image.
**Example 3**

**Drawing Rotations in the Coordinate Plane**

Rotate \( \triangle ABC \) with vertices \( A(2, -1) \), \( B(4, 1) \), and \( C(3, 3) \) by 90° about the origin.

The rotation of \((x, y)\) is \((-y, x)\).

- \( A(2, -1) \rightarrow A'(1, 2) \)
- \( B(4, 1) \rightarrow B'(-1, 4) \)
- \( C(3, 3) \rightarrow C'(-3, 3) \)

Graph the preimage and image.

**Example 4**

**Engineering Application**

The London Eye observation wheel has a radius of 67.5 m and takes 30 minutes to make a complete rotation. A car starts at position \((67.5, 0)\). What are the coordinates of the car's location after 5 minutes?

**Step 1** Find the angle of rotation. Five minutes is \(\frac{5}{30} = \frac{1}{6}\) of a complete rotation, or \(\frac{1}{6}(360°) = 60°\).

**Step 2** Draw a right triangle to represent the car's location \((x, y)\) after a rotation of 60° about the origin.

**Step 3** Use the cosine ratio to find the \(x\)-coordinate.

\[
\cos 60° = \frac{x}{67.5}
\]

\[
x = 67.5 \cos 60° \approx 33.8 \quad \text{Solve for } x.
\]

**Step 4** Use the sine ratio to find the \(y\)-coordinate.

\[
\sin 60° = \frac{y}{67.5}
\]

\[
y = 67.5 \sin 60° \approx 58.5 \quad \text{Solve for } y.
\]

The car's location after 5 minutes is approximately \((33.8, 58.5)\).

**Think and Discuss**

1. Describe the image of a rotation of a figure by an angle of 360°.

2. Point \( A' \) is a rotation of point \( A \) about point \( P \). What is the relationship of \( AP \) to \( A'P \)?

3. **Get Organized**

   Copy and complete the graphic organizer.

<table>
<thead>
<tr>
<th>Reflection</th>
<th>Translation</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Definition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Example</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
GUIDED PRACTICE

Tell whether each transformation appears to be a rotation.

1. [Diagram]
2. [Diagram]
3. [Diagram]
4. [Diagram]

Copy each figure and the angle of rotation. Draw the rotation of the figure about point \( P \) by \( \angle A \).

5. [Diagram]
6. [Diagram]

Rotate the figure with the given vertices about the origin using the given angle of rotation.

7. \( A(1, 0), B(3, 2), C(5, 0); 90^\circ \)
8. \( J(2, 1), K(4, 3), L(2, 4), M(-1, 2); 90^\circ \)
9. \( D(2, 3), E(-1, 2), F(2, 1); 180^\circ \)
10. \( P(-1, -1), Q(-4, -2), R(0, -2); 180^\circ \)

Animation An artist uses a coordinate plane to plan the motion of an animated car. To simulate the car driving around a curve, the artist places the car at the point \( (10, 0) \) and then rotates it about the origin by \( 30^\circ \). Give the car’s final position, rounding the coordinates to the nearest tenth.

PRACTICE AND PROBLEM SOLVING

Tell whether each transformation appears to be a rotation.

12. [Diagram]
13. [Diagram]
14. [Diagram]
15. [Diagram]
Copy each figure and the angle of rotation. Draw the rotation of the figure about point $P$ by $\angle A$.

16. \[ \triangle \]

17. \[ \triangle \]

Rotate the figure with the given vertices about the origin using the given angle of rotation.

18. $E(-1, 2), F(3, 1), G(2, 3); 90^\circ$
19. $A(-1, 0), B(-1, -3), C(1, -3), D(1, 0); 90^\circ$
20. $P(0, -2), Q(2, 0), R(3, -3); 180^\circ$
21. $L(2, 0), M(-1, -2), N(2, -2); 180^\circ$

22. **Architecture** The CN Tower in Toronto, Canada, features a revolving restaurant that takes 72 minutes to complete a full rotation. A table that is 50 feet from the center of the restaurant starts at position $(50, 0)$. What are the coordinates of the table after 6 minutes? Round coordinates to the nearest tenth.

Copy each figure. Then draw the rotation of the figure about the red point using the given angle measure.

23. $90^\circ$
24. $180^\circ$
25. $180^\circ$

26. Point $Q$ has coordinates $(2, 3)$. After a rotation about the origin, the image of point $Q$ lies on the $y$-axis.
   a. Find the angle of rotation to the nearest degree.
   b. Find the coordinates of the image of point $Q$. Round to the nearest tenth.

Rectangle $RSTU$ is the image of rectangle $LMNP$ under a $180^\circ$ rotation about point $A$. Name each of the following.

27. the image of point $N$
28. the preimage of point $S$
29. the image of $\overline{MN}$
30. the preimage of $\overline{TU}$

31. A miniature golf course includes a hole with a windmill. Players must hit the ball through the opening at the base of the windmill while the blades rotate.
   a. The blades take 20 seconds to make a complete rotation. Through what angle do the blades rotate in 4 seconds?
   b. Find the coordinates of point $A$ after 4 seconds. (Hint: $(4, 3)$ is the center of rotation.)
Each figure shows a preimage and its image under a rotation. Copy the figure and locate the center of rotation.

32.  

33.  

34.  

35. **Astronomy** The photograph was made by placing a camera on a tripod and keeping the camera's shutter open for a long time. Because of Earth's rotation, the stars appear to rotate around Polaris, also known as the North Star.
   a. **Estimation** Estimate the angle of rotation of the stars in the photo.
   b. **Estimation** Use your result from part a to estimate the length of time that the camera's shutter was open. (Hint: If the shutter was open for 24 hours, the stars would appear to make one complete rotation around Polaris.)

36. **Estimation** In the diagram, \( \triangle ABC \rightarrow \triangle A'B'C' \) under a rotation about point \( P \).
   a. Estimate the angle of rotation.
   b. Explain how you can draw two segments and can then use a protractor to measure the angle of rotation.
   c. Copy the figure. Use the method from part b to find the angle of rotation. How does your result compare to your estimate?

37. **Critical Thinking** A student wrote the following in his math journal. "Under a rotation, every point moves around the center of rotation by the same angle measure. This means that every point moves the same distance." Do you agree? Explain.

Use the figure for Exercises 38–40.

38. Sketch the image of pentagon \( ABCDE \) under a rotation of 90° about the origin. Give the vertices of the image.

39. Sketch the image of pentagon \( ABCDE \) under a rotation of 180° about the origin. Give the vertices of the image.

40. **Write About It** Is the image of \( ABCDE \) under a rotation of 180° about the origin the same as its image under a reflection across the \( x \)-axis? Explain your reasoning.

41. **Construction** Copy the figure. Use the construction of an angle congruent to a given angle to construct the image of point \( X \) under a rotation about point \( P \) by \( m\angle A \).
42. What is the image of the point $(-2, 5)$ when it is rotated about the origin by $90^\circ$?

- A $(-5, 2)$
- B $(5, -2)$
- C $(-5, -2)$
- D $(2, -5)$

43. The six cars of a Ferris wheel are located at the vertices of a regular hexagon. Which rotation about point $P$ maps car $A$ to car $C$?

- F $60^\circ$
- G $90^\circ$
- H $120^\circ$
- I $135^\circ$

44. **Gridded Response** Under a rotation about the origin, the point $(-3, 4)$ is mapped to the point $(3, -4)$. What is the measure of the angle of rotation?

### CHALLENGE AND EXTEND

45. **Engineering** Gears are used to change the speed and direction of rotating parts in pieces of machinery. In the diagram, suppose gear $B$ makes one complete rotation in the counterclockwise direction. Give the angle of rotation and direction for the rotation of gear $A$. Explain how you got your answer.

46. **Given:** $A'B'$ is the rotation image of $AB$ about point $P$.
**Prove:** $AB \cong A'B'$

*(Hint: Draw auxiliary lines $AP, BP, A'P$, and $B'P$ and show that $\triangle APB \cong \triangle A'PB'$)*

Once you have proved that the rotation image of a segment is congruent to the preimage, how could you prove the following? Write a plan for each proof.

47. If $A'B'$ is a rotation of $AB$, then $AB = A'B'$.

48. If $\angle A'B'C$ is a rotation of $\angle ABC$, then $m\angle ABC = m\angle A'B'C$.

49. The rotation $\triangle A'B'C$ is congruent to the preimage $\triangle ABC$.

50. If point $C$ is between points $A$ and $B$, then the rotation $C'$ is between $A'$ and $B'$.

51. If points $A$, $B$, and $C$ are collinear, then the rotations $A'$, $B'$, and $C'$ are collinear.
Diatoms are microscopic algae that are found in aquatic environments. Scientists use a system that was developed in the 1970s to classify diatoms based on their symmetry.

A figure has symmetry if there is a transformation of the figure such that the image coincides with the preimage.

**Line Symmetry**

A figure has line symmetry (or reflection symmetry) if it can be reflected across a line so that the image coincides with the preimage. The line of symmetry (also called the axis of symmetry) divides the figure into two congruent halves.

**EXAMPLE 1**

Identifying Line Symmetry

Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.

- **A**
  - yes; one line of symmetry

- **B**
  - no line symmetry

- **C**
  - yes; five lines of symmetry

**CHECK IT OUT!**

Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.

- 1a.
- 1b. **B**
- 1c.
Rotational Symmetry

A figure has rotational symmetry (or radial symmetry) if it can be rotated about a point by an angle greater than 0° and less than 360° so that the image coincides with the preimage.

The angle of rotational symmetry is the smallest angle through which a figure can be rotated to coincide with itself. The number of times the figure coincides with itself as it rotates through 360° is called the order of the rotational symmetry.

**Example 2** Identifying Rotational Symmetry

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Symmetry</th>
<th>Angle of Symmetry</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>yes</td>
<td>180°</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>no</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>yes</td>
<td>60°</td>
<td>6</td>
</tr>
</tbody>
</table>

**Check It Out!**

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

2a.  
2b.  
2c.  

**Example 3** Biology Application

Describe the symmetry of each diatom. Copy the shape and draw any lines of symmetry. If there is rotational symmetry, give the angle and order.

A line symmetry and rotational symmetry; angle of rotational symmetry: 180°; order: 2

B line symmetry and rotational symmetry; angle of rotational symmetry: 120°; order: 3

**Check It Out!**

Describe the symmetry of each diatom. Copy the shape and draw any lines of symmetry. If there is rotational symmetry, give the angle and order.

3a.  
3b.  

10-6 Symmetry 93
A three-dimensional figure has *plane symmetry* if a plane can divide the figure into two congruent reflected halves.

A three-dimensional figure has *symmetry about an axis* if there is a line about which the figure can be rotated (by an angle greater than 0° and less than 360°) so that the image coincides with the preimage.

**Example 4**

**Identifying Symmetry in Three Dimensions**

Tell whether each figure has plane symmetry, symmetry about an axis, or neither.

**A** trapezoidal prism

- plane symmetry

**B** equilateral triangular prism

- plane symmetry and symmetry about an axis

**Check It Out!**

Tell whether each figure has plane symmetry, symmetry about an axis, or no symmetry.

4a. cone

4b. pyramid

**Think and Discuss**

1. Explain how you could use scissors and paper to cut out a shape that has line symmetry.

2. Describe how you can find the angle of rotational symmetry for a regular polygon with *n* sides.

3. **Get Organized** Copy and complete the graphic organizer. In each region, draw a figure with the given type of symmetry.
GUIDED PRACTICE

Vocabulary  Apply the vocabulary from this lesson to answer each question.

1. Describe the **line of symmetry** of an isosceles triangle.
2. The capital letter T has ___ (line symmetry or rotational symmetry).

**SEE EXAMPLE 1**

Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.

3. 
4. 
5. 

**SEE EXAMPLE 2**

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

6. 
7. 
8. 

**SEE EXAMPLE 3**

9. **Architecture** The Pentagon in Alexandria, Virginia, is the world's largest office building. Copy the shape of the building and draw all lines of symmetry. Give the angle and order of rotational symmetry.

**SEE EXAMPLE 4**

Tell whether each figure has plane symmetry, symmetry about an axis, or neither.

10. prism
11. cylinder
12. rectangular prism

PRACTICE AND PROBLEM SOLVING

Tell whether each figure has line symmetry. If so, copy the shape and draw all lines of symmetry.

13. 
14. 
15. 

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

16. 
17. 
18.
19. **Art** Op art is a style of art that uses optical effects to create an impression of movement in a painting or sculpture. The painting at right, Vega-Tek, by Victor Vasarely, is an example of op art. Sketch the shape in the painting and draw any lines of symmetry. If there is rotational symmetry, give the angle and order.

20. Tell whether each figure has plane symmetry, symmetry about an axis, or neither.
   - sphere
   - triangular pyramid
   - torus

21. Draw a triangle with the following number of lines of symmetry. Then classify the triangle.
   - exactly one line of symmetry
   - three lines of symmetry
   - no lines of symmetry

22. **Data Analysis** The graph shown, called the *standard normal curve*, is used in statistical analysis. The area under the curve is 1 square unit. There is a vertical line of symmetry at \(x = 0\). The areas of the shaded regions are indicated on the graph.

   26. Find the area under the curve for \(x > 0\).
   27. Find the area under the curve for \(x > 2\).
   28. If a point under the curve is selected at random, what is the probability that the \(x\)-value of the point will be between \(-1\) and 1?

29. Tell whether the figure with the given vertices has line symmetry and/or rotational symmetry. Give the angle and order if there is rotational symmetry. Draw the figure and any lines of symmetry.
   - \(A(-2, 2), B(2, 2), C(1, -2), D(-1, -2)\)
   - \(R(-3, 3), S(3, 3), T(3, -3), U(-3, -3)\)
   - \(J(4, 4), K(-2, 2), L(2, -2)\)
   - \(A(3, 1), B(0, 2), C(-3, 1), D(-3, -1), E(0, -2), F(3, -1)\)

30. **Art** The Chokwe people of Angola are known for their traditional sand designs. These complex drawings are traced out to illustrate stories that are told at evening gatherings. Classify the symmetry of the Chokwe design shown.

31. **Algebra** Graph each function. Tell whether the graph has line symmetry and/or rotational symmetry. If there is rotational symmetry, give the angle and order. Write the equations of any lines of symmetry.
   - \(y = x^2\)
   - \(y = (x - 2)^2\)
   - \(y = x^3\)
37. This woodcut, entitled *Circle Limit III*, was made by Dutch artist M. C. Escher.
   a. Does the woodcut have line symmetry? If so, describe the lines of symmetry. If not, explain why not.
   b. Does the woodcut have rotational symmetry? If so, give the angle and order of the symmetry. If not, explain why not.
   c. Does your answer to part b change if color is not taken into account? Explain.

Classify the quadrilateral that meets the given conditions. First make a conjecture and then verify your conjecture by drawing a figure.

38. two lines of symmetry perpendicular to the sides and order-2 rotational symmetry
39. no line symmetry and order-2 rotational symmetry
40. two lines of symmetry through opposite vertices and order-2 rotational symmetry
41. four lines of symmetry and order-4 rotational symmetry
42. one line of symmetry through a pair of opposite vertices and no rotational symmetry

43. Physics High-speed photography makes it possible to analyze the physics behind a water splash. When a drop lands in a bowl of liquid, the splash forms a crown of evenly spaced points. What is the angle of rotational symmetry for a crown with 24 points?

44. Critical Thinking What can you conclude about a rectangle that has four lines of symmetry? Explain.

45. Geography The Isle of Man is an island in the Irish Sea. The island’s symbol is a triskelion that consists of three running legs radiating from the center. Describe the symmetry of the triskelion.

46. Critical Thinking Draw several examples of figures that have two perpendicular lines of symmetry. What other type of symmetry do these figures have? Make a conjecture based on your observation.

Each figure shows part of a shape with a center of rotation and a given rotational symmetry. Copy and complete each figure.

47. order 4
48. order 6
49. order 2

50. Write About It Explain the connection between the angle of rotational symmetry and the order of the rotational symmetry. That is, if you know one of these, explain how you can find the other.
51. What is the order of rotational symmetry for the hexagon shown?
   A. 2  B. 3  C. 4  D. 6

52. Which of these figures has exactly four lines of symmetry?
   F. Regular octagon  H. Isosceles triangle
   G. Equilateral triangle  I. Square

53. Consider the graphs of the following equations. Which graph has the y-axis as a line of symmetry?
   A. \( y = (x - 3)^2 \)  B. \( y = x^3 \)  C. \( y = x^2 - 3 \)
   D. \( y = |x + 3| \)

54. Donnell designed a garden plot that has rotational symmetry, but not line symmetry. Which of these could be the shape of the plot?
   F.  G.  H.  I.

CHALLENGE AND EXTEND

55. A regular polygon has an angle of rotational symmetry of 5°. How many sides does the polygon have?

56. How many lines of symmetry does a regular \( n \)-gon have if \( n \) is even? if \( n \) is odd? Explain your reasoning.

Find the equation of the line of symmetry for the graph of each function.

57. \( y = (x + 4)^2 \)  58. \( y = |x - 2| \)  59. \( y = 3x^2 + 5 \)

Give the number of axes of symmetry for each regular polyhedron. Describe all axes of symmetry.

60. cube  61. tetrahedron  62. octahedron
If you rotate a rectangle around one of its sides, the path it makes through space is a cylinder. A **solid of revolution** is a three-dimensional figure that is formed by rotating a two-dimensional shape around an axis.

**Objectives**
Understand how solids can be produced by rotating a two-dimensional figure through space.

**Vocabulary**
solid of revolution

---

**EXAMPLE 1**
**Sketching a Solid of Revolution**

Draw the solid of revolution formed by the shape rotated around the axis given. Describe the resulting shape.

A right triangle rotated around an axis that passes through one of the legs forms a cone.

**CHECK IT OUT!**
1. Draw the solid of revolution formed by the given shape rotated around the axis given. Describe the resulting shape.

---

**EXAMPLE 2**
**Recreation Application**

A chess pawn is a solid of revolution. Draw a two-dimensional shape and an axis of rotation that could form the pawn.

*The two-dimensional shape should match the outline of one side of the pawn.*
2. Draw a two-dimensional shape and axis of rotation that could form the sports drink bottle.

Draw the solid of revolution formed by each shape rotated around the axis given. Describe the resulting shape.

1. 

2. 

Draw a two-dimensional shape and axis of rotation that could form each figure.

3. 

4. 

Draw the solid of revolution formed by each shape rotated around the $z$-axis. Then find the volume of the solid to the nearest tenth of a unit.

5. 

6. 

7. **Critical Thinking** If you find the cross section of a solid of revolution in a plane that’s perpendicular to the axis of rotation, you will always get the same shape. What is it? Use drawings to support your answer.

8. **Write About It** Will rotation of the blue figure about either axis shown in the figure produce a sphere? Explain why or why not.

9. **Challenge** Is an oblique cylinder a solid of revolution? Explain your reasoning.
**Objectives**

Use transformations to draw tessellations.
Identify regular and semiregular tessellations and figures that will tessellate.

**Vocabulary**

- translation symmetry
- frieze pattern
- glide reflection symmetry
- tessellation
- regular tessellation
- semiregular tessellation

---

**Who uses this?**

Repeating patterns play an important role in traditional Native American art.

A pattern has **translation symmetry** if it can be translated along a vector so that the image coincides with the preimage. A **frieze pattern** is a pattern that has translation symmetry along a line.

Both of the frieze patterns shown below have translation symmetry. The pattern on the right also has **glide reflection symmetry**. A pattern with **glide reflection symmetry** coincides with its image after a glide reflection.

---

**Example 1**

**Art Application**

Identify the symmetry in each frieze pattern.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Symmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>translation symmetry and glide reflection symmetry</td>
</tr>
<tr>
<td>B</td>
<td>translation symmetry</td>
</tr>
</tbody>
</table>

---

**Helpful Hint**

When you are given a frieze pattern, you may assume that the pattern continues forever in both directions.

---

**Identify the symmetry in each frieze pattern.**

1a. translation symmetry and glide reflection symmetry

1b. translation symmetry

---

A **tessellation**, or *tiling*, is a repeating pattern that completely covers a plane with no gaps or overlaps. The measures of the angles that meet at each vertex must add up to 360°.

In the tessellation shown, each angle of the quadrilateral occurs once at each vertex. Because the angle measures of any quadrilateral add to 360°, any quadrilateral can be used to tessellate the plane. Four copies of the quadrilateral meet at each vertex.
The angle measures of any triangle add up to 180°. This means that any triangle can be used to tessellate a plane. Six copies of the triangle meet at each vertex, as shown.

\[ m\angle 1 + m\angle 2 + m\angle 3 = 180° \]
\[ m\angle 1 + m\angle 2 + m\angle 3 + m\angle 1 + m\angle 2 + m\angle 3 = 360° \]

**Example 2**

**Using Transformations to Create Tessellations**

Copy the given figure and use it to create a tessellation.

A

**Step 1** Rotate the triangle 180° about the midpoint of one side.

**Step 2** Translate the resulting pair of triangles to make a row of triangles.

**Step 3** Translate the row of triangles to make a tessellation.

B

**Step 1** Rotate the quadrilateral 180° about the midpoint of one side.

**Step 2** Translate the resulting pair of quadrilaterals to make a row of quadrilaterals.

**Step 3** Translate the row of quadrilaterals to make a tessellation.

**Check It Out**

2. Copy the given figure and use it to create a tessellation.

A regular tessellation is formed by congruent regular polygons. A semiregular tessellation is formed by two or more different regular polygons, with the same number of each polygon occurring in the same order at every vertex.

Every vertex has two squares and three triangles in this order: square, triangle, square, triangle, triangle.
EXAMPLE 3
Classifying Tessellations

Classify each tessellation as regular, semiregular, or neither.

A  
Two regular octagons and one square meet at each vertex. The tessellation is semiregular.

B  
Only squares are used. The tessellation is regular.

C  
Irregular hexagons are used in the tessellation. It is neither regular nor semiregular.

EXAMPLE 4
Determining Whether Polygons Will Tessellate

Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

A  
No; each angle of the pentagon measures 108°, and 108 is not a divisor of 360°.

B  
Yes; two octagons and one square meet at each vertex.  
\[135° + 135° + 90° = 360°\]

Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

4a.  
4b.
THINK AND DISCUSS

1. Explain how you can identify a frieze pattern that has glide reflection symmetry.
2. Is it possible to tessellate a plane using circles? Why or why not?
3. GET ORGANIZED Copy and complete the graphic organizer.

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.
1. Sketch a pattern that has glide reflection symmetry.
2. Describe a real-world example of a regular tessellation.

SEE EXAMPLE 1
Transportation The tread of a tire is the part that makes contact with the ground. Various tread patterns help improve traction and increase durability. Identify the symmetry in each tread pattern.

3. 
4. 
5. 

SEE EXAMPLE 2
Copy the given figure and use it to create a tessellation.

6. 
7. 
8. 

SEE EXAMPLE 3
Classify each tessellation as regular, semiregular, or neither.

9. 
10. 
11. 

SEE EXAMPLE 4
Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

12. 
13. 
14.
**PRACTICE AND PROBLEM SOLVING**

**Interior Decorating** Identify the symmetry in each wallpaper border.

15.  
16.  
17.  

Copy the given figure and use it to create a tessellation.

18.  
19.  
20.  

Classify each tessellation as regular, semiregular, or neither.

21.  
22.  
23.  

Determine whether the given regular polygon(s) can be used to form a tessellation. If so, draw the tessellation.

24.  
25.  
26.  

27. **Physics** A truck moving down a road creates whirling pockets of air called a *vortex train*. Use the figure to classify the symmetry of a vortex train.

Identify all of the types of symmetry (translation, glide reflection, and/or rotation) in each tessellation.

28.  
29.  
30.  

Tell whether each statement is sometimes, always, or never true.

31. A triangle can be used to tessellate a plane.
32. A frieze pattern has glide reflection symmetry.
33. The angles at a vertex of a tessellation add up to 360°.
34. It is possible to use a regular pentagon to make a regular tessellation.
35. A semiregular tessellation includes scalene triangles.
36. Many of the patterns in M. C. Escher's works are based on simple tessellations. For example, the pattern at right is based on a tessellation of equilateral triangles. Identify the figure upon which each pattern is based.

![Pattern A](image1)

![Pattern B](image2)

37. Use the given figure to draw a frieze pattern with the given symmetry.
38. translation symmetry

39. translation symmetry

40. glide reflection symmetry

41. **Optics** A kaleidoscope is formed by three mirrors joined to form the lateral surface of a triangular prism. Copy the triangular faces and reflect it over each side. Repeat to form a tessellation. Describe the symmetry of the tessellation.

42. **Critical Thinking** The pattern on a soccer ball is a tessellation of a sphere using regular hexagons and regular pentagons. Can these two shapes be used to tessellate a plane? Explain your reasoning.

43. **Chemistry** A *polymer* is a substance made of repeating chemical units or molecules. The *repeat unit* is the smallest structure that can be repeated to create the chain. Draw the repeat unit for polypropylene, the polymer shown below.

\[
-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{CH}-\text{CH}_2-\text{CH}-\text{CH}_2-\text{CH}_3
\]

44. The *dual* of a tessellation is formed by connecting the centers of adjacent polygons with segments. Copy or trace the semiregular tessellation shown and draw its dual. What type of polygon makes up the dual tessellation?

45. **Write About It** You can make a regular tessellation from an equilateral triangle, a square, or a regular hexagon. Explain why these are the only three regular tessellations that are possible.
46. Which frieze pattern has glide reflection symmetry?

- A
- B
- C
- D

47. Which shape CANNOT be used to make a regular tessellation?

- Equilateral triangle
- Regular pentagon
- Square
- Regular hexagon

48. Which pair of regular polygons can be used to make a semiregular tessellation?

- A
- B
- C
- D

---

**CHALLENGE AND EXTEND**

49. Some shapes can be used to tessellate a plane in more than one way. Three tessellations that use the same rectangle are shown. Draw a parallelogram and draw at least three tessellations using that parallelogram.

Determine whether each figure can be used to tessellate three-dimensional space.

- 50.
- 51.
- 52.
**Vocabulary**

- center of dilation
- composition of transformations
- enlargement
- frieze pattern
- glide reflection
- glide reflection symmetry
- isometry
- line symmetry
- line of symmetry
- reduction
- regular tessellation
- rotational symmetry
- semiregular tessellation
- symmetry
- tessellation
- translation symmetry

Complete the sentences below with vocabulary words from the list above.

1. A(n) **regular tessellation** is a pattern formed by congruent regular polygons.
2. A pattern that has translation symmetry along a line is called a(n) **translation symmetry**.
3. A transformation that does not change the size or shape of a figure is a(n) **isometry**.
4. One transformation followed by another is called a(n) **composition of transformations**.

**10-1 Reflections**

**Example**

Reflect the figure with the given vertices across the given line.

\[ A(1, -2), B(4, -3), C(3, 0); y = x \]

To reflect across the line \( y = x \), interchange the \( x \)- and \( y \)-coordinates of each point. The images of the vertices are \( A'(-2, 1) \), \( B'(-3, 4) \), and \( C'(0, 3) \).

**Exercises**

Tell whether each transformation appears to be a reflection.

5. [Diagram of a figure]

6. [Diagram of a figure]

7. [Diagram of a figure]

8. [Diagram of a figure]

Reflect the figure with the given vertices across the given line.

9. \( E(-3, 2), F(0, 2), G(-2, 5); x \)-axis

10. \( J(2, -1), K(4, -2), L(4, -3), M(2, -3); y \)-axis

11. \( P(2, -2), Q(4, -2), R(3, -4); y = x \)

12. \( A(2, 2), B(-2, 2), C(-1, 4); y = x \)
10-2 Translations

**Example**

Translate the figure with the given vertices along the given vector.

\[ D(-4, 4), E(-4, 2), F(-1, 1), G(-2, 3); \langle 5, -5 \rangle \]

To translate along \( \langle 5, -5 \rangle \), add 5 to the \( x \)-coordinate of each point and add -5 to the \( y \)-coordinate of each point. The vertices of the image are \( D'(1, -1) \), \( E'(1, -3) \), \( F'(4, -4) \), and \( G'(3, -2) \).

**Exercises**

Tell whether each transformation appears to be a translation.

13. [Diagram]
14. [Diagram]
15. [Diagram]
16. [Diagram]

17. \( R(1, -1), S(1, -3), T(4, -3), U(4, -1); \langle -5, 2 \rangle \)
18. \( A(-4, -1), B(-3, 2), C(-1, -2); \langle 6, 0 \rangle \)
19. \( M(1, 4), N(4, 4), P(3, 1); \langle -3, -3 \rangle \)
20. \( D(3, 1), E(2, -2), F(3, -4), G(4, -2); \langle -6, 2 \rangle \)

10-3 Rotations

**Example**

Rotate the figure with the given vertices about the origin using the given angle of rotation.

\( A(-2, 0), B(-1, 3), C(-4, 3); 180^\circ \)

To rotate by \( 180^\circ \), find the opposite of the \( x \)- and \( y \)-coordinate of each point. The vertices of the image are \( A'(2, 0), B'(1, -3) \), and \( C'(4, -3) \).

**Exercises**

Tell whether each transformation appears to be a rotation.

21. [Diagram]
22. [Diagram]
23. [Diagram]
24. [Diagram]

25. \( A(1, 3), B(4, 1), C(4, 4); 90^\circ \)
26. \( A(1, 3), B(4, 1), C(4, 4); 180^\circ \)
27. \( M(2, 2), N(5, 2), P(3, -2), Q(0, -2); 90^\circ \)
28. \( G(-2, 1), H(-3, -2), J(-1, -4); 180^\circ \)
**10-6 Compositions of Transformations**

**Example**

- Draw the result of the composition of isometries.

Translate $\triangle MNP$ along $\vec{v}$ and then reflect it across line $\ell$.

First draw $\triangle M'N'P'$, the translation image of $\triangle MNP$. Then reflect $\triangle M'N'P'$ across line $\ell$ to find the final image, $\triangle M''N''P''$.

**Exercises**

Draw the result of the composition of isometries.

29. Translate $ABCD$ along $\vec{v}$ and then reflect it across line $m$.

30. Reflect $\triangle JKL$ across line $m$ and then rotate it $90^\circ$ about point $P$.

---

**10-5 Symmetry**

**Examples**

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

- no rotational symmetry

The figure coincides with itself when it is rotated by $90^\circ$. Therefore the angle of rotational symmetry is $90^\circ$. The order of symmetry is 4.

**Exercises**

Tell whether each figure has line symmetry. If so, copy the figure and draw all lines of symmetry.

31.

32.

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry and the order of symmetry.

33.

34.

35.

36.
**Tessellations**

**Examples**

- Copy the given figure and use it to create a tessellation. Rotate the quadrilateral 180° about the midpoint of one side.

  Translate the resulting pair of quadrilaterals to make a row.

  Translate the row to make a tessellation.

- Classify the tessellation as regular, semiregular, or neither.

  The tessellation is made of two different regular polygons, and each vertex has the same polygons in the same order. Therefore the tessellation is semiregular.

**Exercises**

Copy the given figure and use it to create a tessellation.

37. 
38. 
39. 
40. 
41. 
42.

Classify each tessellation as regular, semiregular, or neither.

**Dilations**

**Example**

- Draw the image of the figure with the given vertices under a dilation centered at the origin using the given scale factor.
  \(A(0, -2), B(2, -2), C(2, 0)\); scale factor: 2

  Multiply the \(x\)- and \(y\)-coordinates of each point by 2. The vertices of the image are \(A'(0, -4)\), \(B'(4, -4)\), and \(C'(4, 0)\).

**Exercises**

Tell whether each transformation appears to be a dilation.

43. 
44. 
45. \(R(0, 0), S(4, 4), T(4, -4)\); scale factor: \(-\frac{1}{2}\)
46. \(D(0, 2), E(-2, 2), F(-2, 0)\); scale factor: \(-2\)
Tell whether each transformation appears to be a reflection.

1.

2.

Tell whether each transformation appears to be a translation.

3.

4.

5. An interior designer is using a coordinate grid to place furniture in a room. The position of a sofa is represented by a rectangle with vertices (1, 3), (2, 2), (5, 5), and (4, 6). He decides to move the sofa by translating it along the vector \(-1, -1\). Draw the sofa in its final position.

Tell whether each transformation appears to be a rotation.

6.

7.

8. Rotate rectangle \(DEFG\) with vertices \(D(1, -1), E(4, -1), F(4, -3),\) and \(G(1, -3)\) about the origin by 180°.

9. Rectangle \(ABCD\) with vertices \(A(3, -1), B(3, -2), C(1, -2),\) and \(D(1, -1)\) is reflected across the \(y\)-axis, and then its image is reflected across the \(x\)-axis. Describe a single transformation that moves the rectangle from its starting position to its final position.

10. Tell whether the “no entry” sign has line symmetry. If so, copy the sign and draw all lines of symmetry.

11. Tell whether the “no entry” sign has rotational symmetry. If so, give the angle of rotational symmetry and the order of the symmetry.

12. Copy the given figure and use it to create a tessellation.

13.

14.

15. Classify the tessellation shown as regular, semiregular, or neither.

Tell whether each transformation appears to be a dilation.

16.

17.

18. Draw the image of \(\triangle ABC\) with vertices \(A(2, -1), B(1, -4),\) and \(C(4, -4)\) under a dilation centered at the origin with scale factor \(-\frac{1}{2}\).
11-1 Congruent Triangles
LAB Explore SSS and SAS Triangle Congruence
11-2 Triangle Congruence: SSS and SAS
LAB Predict Other Triangle Congruence Relationships
11-3 Triangle Congruence: ASA, AAS, and HL

**Flexible Creations**
When you turn a kaleidoscope, the shapes flip to form a variety of designs. You can create flexagons that also flip to form patterns.
Reading Strategy: Read Geometry Symbols

In Geometry we often use symbols to communicate information. When studying each lesson, read both the symbols and the words slowly and carefully. Reading aloud can sometimes help you translate symbols into words.

Throughout this course, you will use these symbols and combinations of these symbols to represent various geometric statements.

<table>
<thead>
<tr>
<th>Symbol Combinations</th>
<th>Translated into Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ST \parallel UV )</td>
<td>Line ( ST ) is parallel to line ( UV ).</td>
</tr>
<tr>
<td>( BC \perp GH )</td>
<td>Segment ( BC ) is perpendicular to segment ( GH ).</td>
</tr>
<tr>
<td>( p \rightarrow q )</td>
<td>If ( p ), then ( q ).</td>
</tr>
<tr>
<td>( m\angle QRS = 45^\circ )</td>
<td>The measure of angle ( QRS ) is 45 degrees.</td>
</tr>
<tr>
<td>( \angle CDE \cong \angle LMN )</td>
<td>Angle ( CDE ) is congruent to angle ( LMN ).</td>
</tr>
</tbody>
</table>

Try This

Rewrite each statement using symbols.

1. the absolute value of 2 times pi
2. The measure of angle 2 is 125 degrees.
3. Segment \( XY \) is perpendicular to line \( BC \).
4. If not \( p \), then not \( q \).

Translate the symbols into words.

5. \( m\angle FGH = m\angle VWX \)
6. \( \overline{ZA} \parallel \overline{TU} \)
7. \( \sim p \rightarrow q \)
8. \( \overline{ST} \) bisects \( \angle TSU \).
Who uses this?
Machinists used triangles to construct a model of the International Space Station’s support structure.

Geometric figures are congruent if they are the same size and shape. Corresponding angles and corresponding sides are in the same position in polygons with an equal number of sides. Two polygons are congruent if and only if their corresponding angles and sides are congruent. Thus triangles that are the same size and shape are congruent.

Helpful Hint
Two vertices that are the endpoints of a side are called consecutive vertices. For example, P and Q are consecutive vertices.

Properties of Congruent Polygons

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Corresponding Angles</th>
<th>Corresponding Sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \triangle ABC \cong \triangle DEF )</td>
<td>( \angle A \cong \angle D ), ( \angle B \cong \angle E ), ( \angle C \cong \angle F )</td>
<td>( \overline{AB} \cong \overline{DE} ), ( \overline{BC} \cong \overline{EF} ), ( \overline{AC} \cong \overline{DF} )</td>
</tr>
<tr>
<td>( \triangle PQRST \cong \triangle WXZY )</td>
<td>( \angle P \cong \angle W ), ( \angle Q \cong \angle X ), ( \angle R \cong \angle Y ), ( \angle S \cong \angle Z )</td>
<td>( \overline{PQ} \cong \overline{WX} ), ( \overline{QR} \cong \overline{XY} ), ( \overline{RS} \cong \overline{YZ} ), ( \overline{PS} \cong \overline{WZ} )</td>
</tr>
</tbody>
</table>

To name a polygon, write the vertices in consecutive order. For example, you can name polygon \( PQRST \) as \( QRSP \) or \( SRQP \), but not as \( PRQS \). In a congruence statement, the order of the vertices indicates the corresponding parts.

Example 1
\( \triangle RST \) and \( \triangle XYZ \) represent the triangles of the space station’s support structure. If \( \triangle RST \cong \triangle XYZ \), identify all pairs of congruent corresponding parts.

Angles: \( \angle R \cong \angle X \), \( \angle S \cong \angle Y \), \( \angle T \cong \angle Z \)

Sides: \( \overline{RS} \cong \overline{XY} \), \( \overline{ST} \cong \overline{YZ} \), \( \overline{RT} \cong \overline{XZ} \)

Check It Out!
1. If polygon \( LMNP \cong polygon EFGH \), identify all pairs of corresponding congruent parts.
Using Corresponding Parts of Congruent Triangles

Given: \( \triangle EFH \cong \triangle GFH \)

**A** Find the value of \( x \).

\[ \angle FHE \text{ and } \angle FHG \text{ are rt. } \]

\[ m\angle FHE = m\angle FHG \]

\[ (6x - 12) = 90 \]

\[ 6x = 102 \]

\[ x = 17 \]

**B** Find \( m\angle GFH \).

\[ m\angle EFH + m\angle FHE + m\angle E = 180° \]

\[ m\angle EFH + 90 + 21.6 = 180 \]

\[ m\angle EFH + 111.6 = 180 \]

\[ m\angle EFH = 68.4° \]

\[ \angle GFH \cong \angle EFH \]

\[ m\angle GFH = m\angle EFH \]

\[ m\angle GFH = 68.4° \]

**CHECK IT OUT!**

Given: \( \triangle ABC \cong \triangle DEF \)

2a. Find the value of \( x \).

2b. Find \( m\angle F \).

**Example 3**

Proving Triangles Congruent

Given: \( \angle P \) and \( \angle M \) are right angles.

\( R \) is the midpoint of \( PM \).

\( PQ \cong MN, QR \cong NR \)

Prove: \( \triangle PQR \cong \triangle MNR \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle P ) and ( \angle M ) are rt. ( \triangle )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle P \cong \angle M )</td>
<td>2. Rt. ( \angle \cong ) Thm.</td>
</tr>
<tr>
<td>3. ( \angle PRQ \cong \angle MRN )</td>
<td>3. Vert. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>4. ( \angle Q \cong \angle N )</td>
<td>4. Third ( \triangle ) Thm.</td>
</tr>
<tr>
<td>5. ( R ) is the mdpt. of ( PM ).</td>
<td>5. Given</td>
</tr>
<tr>
<td>6. ( PR \cong MR )</td>
<td>6. Def. of mdpt.</td>
</tr>
<tr>
<td>7. ( PQ \cong MN, QR \cong NR )</td>
<td>7. Given</td>
</tr>
<tr>
<td>8. ( \triangle PQR \cong \triangle MNR )</td>
<td>8. Def. of ( \cong \triangle )</td>
</tr>
</tbody>
</table>

**CHECK IT OUT!**

3. Given: \( \overline{AD} \) bisects \( \overline{BE} \).

\( \overline{BE} \) bisects \( \overline{AD} \).

\( AB \cong DE, \angle A \cong \angle D \)

Prove: \( \triangle ABC \cong \triangle DEC \)
**Example 4**

**Engineering Application**

The bars that give structural support to a roller coaster form triangles. Since the angle measures and the lengths of the corresponding sides are the same, the triangles are congruent.

Given: \( \overline{JK} \perp \overline{KL}, \overline{ML} \perp \overline{KL}, \angle KLI \equiv \angle KLM, \overline{JK} \equiv \overline{ML}, \overline{JL} \equiv \overline{MK} \)

Prove: \( \triangle JKL \equiv \triangle MLK \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{JK} \perp \overline{KL}, \overline{ML} \perp \overline{KL} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle KLI \equiv \angle KLM )</td>
<td>2. Def. of ( \perp ) lines</td>
</tr>
<tr>
<td>3. ( \angle JKL \equiv \angle MLK )</td>
<td>3. Rt. ( \equiv ) Thm.</td>
</tr>
<tr>
<td>4. ( \angle KJI \equiv \angle LKM )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( \angle KJI \equiv \angle LMK )</td>
<td>5. Third ( \equiv ) Thm.</td>
</tr>
<tr>
<td>6. ( \overline{JK} \equiv \overline{ML}, \overline{JL} \equiv \overline{MK} )</td>
<td>6. Given</td>
</tr>
<tr>
<td>7. ( \overline{KL} \equiv \overline{LK} )</td>
<td>7. Reflex. Prop. of ( \equiv )</td>
</tr>
<tr>
<td>8. ( \triangle JKL \equiv \triangle MLK )</td>
<td>8. Def. of ( \equiv \triangle )</td>
</tr>
</tbody>
</table>

4. Use the diagram to prove the following.

Given: \( \overline{MK} \) bisects \( \overline{JL} \), \( \overline{JL} \) bisects \( \overline{MK} \), \( \overline{JK} \equiv \overline{ML}, \overline{JK} \parallel \overline{ML} \)

Prove: \( \triangle JKN \equiv \triangle LMN \)

**Think and Discuss**

1. A roof truss is a triangular structure that supports a roof. How can you be sure that two roof trusses are the same size and shape?

2. **GET Organized** Copy and complete the graphic organizer. In each box, name the congruent corresponding parts.
GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

1. An everyday meaning of corresponding is “matching.” How can this help you find the corresponding parts of two triangles?

2. If $\triangle ABC \cong \triangle RST$, what angle corresponds to $\angle S$?

Given: $\triangle RST \cong \triangle LMN$. Identify the congruent corresponding parts.

3. $\overline{RS} \cong \underline{?}$
4. $\overline{LN} \cong \underline{?}$
5. $\angle S \cong \underline{?}$
6. $\overline{TS} \cong \underline{?}$
7. $\angle L \cong \underline{?}$
8. $\angle N \cong \underline{?}$

Given: $\triangle FGH \cong \triangle JKL$. Find each value.

9. $KL$
10. $x$

11. Given: $E$ is the midpoint of $\overline{AC}$ and $\overline{BD}$.

$AB \equiv CD, AB \parallel CD$

Prove: $\triangle ABE \cong \triangle CDE$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $AB \parallel CD$</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. $\angle ABE \equiv \angle CDE$, $\angle BAE \equiv \angle DCE$</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>3. $AB \equiv CD$</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. $E$ is the mdpt. of $\overline{AC}$ and $\overline{BD}$.</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>5. $\angle AEB \equiv \angle CED$</td>
<td>5. Def. of mdpt.</td>
</tr>
<tr>
<td>7. $\triangle ABE \equiv \triangle CDE$</td>
<td>7. g. ?</td>
</tr>
</tbody>
</table>

12. Engineering The geodesic dome shown is a 14-story building that models Earth. Use the given information to prove that the triangles that make up the sphere are congruent.

Given: $\overline{SU} \equiv \overline{ST} \equiv \overline{SR}$, $\overline{TU} \equiv \overline{TR}$, $\angle UST \equiv \angle RST$, and $\angle U \equiv \angle R$

Prove: $\triangle RTS \cong \triangle UTS$
PRACTICE AND PROBLEM SOLVING

Given: Polygon $CDEF \cong$ polygon $KLMN$. Identify the congruent corresponding parts.

13. $\overline{DE} \cong \ ?$

14. $\overline{KN} \cong \ ?$

15. $\angle F \cong \ ?$

16. $\angle L \cong \ ?$

Given: $\triangle ABD \cong \triangle CBD$. Find each value.

17. $m\angle C$

18. $y$

19. Given: $\overline{MP}$ bisects $\angle NMR$. $P$ is the midpoint of $\overline{NR}$. $\overline{MN} \cong \overline{MR}$, $\angle N \cong \angle R$

Prove: $\triangle MNP \cong \triangle MRP$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle N \cong \angle R$</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. $\overline{MP}$ bisects $\angle NMR$.</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>3. c. ?</td>
<td>3. Def. of $\angle$ bisector</td>
</tr>
<tr>
<td>5. P is the mdpt of $\overline{NR}$.</td>
<td>5. e. ?</td>
</tr>
<tr>
<td>7. $\overline{MN} \cong \overline{MR}$</td>
<td>7. g. ?</td>
</tr>
<tr>
<td>8. $\overline{MP} \cong \overline{MP}$</td>
<td>8. h. ?</td>
</tr>
<tr>
<td>9. $\triangle MNP \cong \triangle MRP$</td>
<td>9. Def. of $\cong \triangle$</td>
</tr>
</tbody>
</table>

20. Hobbies In a garden, triangular flower beds are separated by straight rows of grass as shown.

Given: $\angle ADC$ and $\angle BCD$ are right angles. $\overline{AC} \cong \overline{BD}$, $\overline{AD} \cong \overline{BC}$, $\angle DAC \cong \angle CBD$

Prove: $\triangle ADC \cong \triangle BCD$

21. For two triangles, the following corresponding parts are given: $\overline{GS} \cong \overline{KP}$, $\overline{GR} \cong \overline{KH}$, $\overline{SR} \cong \overline{PH}$, $\angle S \cong \angle P$, $\angle G \cong \angle K$, and $\angle R \cong \angle H$.

Write three different congruence statements.

22. The two polygons in the diagram are congruent. Complete the following congruence statement for the polygons. $\text{polygon } R \ ? \cong \text{polygon } V \ ?$

Write and solve an equation for each of the following.

23. $\triangle ABC \cong \triangle DEF$. $AB = 2x - 10$, and $DE = x + 20$.

Find the value of $x$ and $AB$.

24. $\triangle IKL \cong \triangle MNP$. $m\angle L = (x^2 + 10)^\circ$, and $m\angle P = (2x^2 + 1)^\circ$. What is $m\angle L$?

25. Polygon $ABCD \cong$ polygon $PQRS$. $BC = 6x + 5$, and $QR = 5x + 7$.

Find the value of $x$ and $BC$. 

Extra Practice

See Extra Practice for more Skills Practice and Applications Practice exercises.
26. Many origami models begin with a square piece of paper, \(JKLM\), that is folded along both diagonals to make the creases shown. \(\overline{JL}\) and \(\overline{MK}\) are perpendicular bisectors of each other, and \(\angle NML \cong \angle NKL\).
   a. Explain how you know that \(\overline{KL}\) and \(\overline{ML}\) are congruent.
   b. Prove \(\triangle NML \cong \triangle NKL\).

27. Draw a diagram and then write a proof.
   Given: \(BD \perp AC, D\) is the midpoint of \(AC, AB \cong CB, \) and \(BD\) bisects \(\angle ABC\).
   Prove: \(\triangle ABD \cong \triangle CBD\)

28. Critical Thinking Draw two triangles that are not congruent but have an area of 4 cm\(^2\) each.

29. //ERROR ANALYSIS// Given \(\triangle MPQ \cong \triangle EDF\).
   Two solutions for finding \(m \angle E\) are shown.
   Which is incorrect? Explain the error.

30. Write About It Given the diagram of the triangles, is there enough information to prove that \(\triangle HKL\) is congruent to \(\triangle YWX\)? Explain.

31. Which congruence statement correctly indicates that the two given triangles are congruent?
   \(\triangle ABC \cong \triangle FDE\)  \(\triangle ABC \cong \triangle DEF\)

32. \(\triangle MNP \cong \triangle RST\). What are the values of \(x\) and \(y\)?
   \(x = 26, y = 21\frac{1}{3}\)  \(x = 25, y = 20\frac{2}{3}\)
   \(x = 27, y = 20\)  \(x = 30\frac{1}{3}, y = 16\frac{2}{3}\)

33. \(\triangle ABC \cong \triangle XYZ\). \(m \angle A = 47.1^\circ\), and \(m \angle C = 13.8^\circ\). Find \(m \angle Y\).
   \(A\) 13.8  \(C\) 76.2
   \(B\) 42.9  \(D\) 119.1

34. \(\triangle MNR \cong \triangle SPQ\), \(NL = 18, SP = 33, SR = 10, RQ = 24,\) and \(QP = 30\). What is the perimeter of \(\triangle MNR\)?
   \(F\) 79  \(H\) 87
   \(G\) 85  \(J\) 97
35. **Multi-Step** Given that the perimeter of $TUVW$ is 149 units, find the value of $x$. Is $\triangle TUV \cong \triangle TWV$? Explain.

36. **Multi-Step** Polygon $ABCD \cong$ polygon $EFGH$. $\angle A$ is a right angle. $m\angle E = (y^2 - 10)^\circ$, and $m\angle H = (2y^2 - 132)^\circ$. Find $m\angle D$.

37. **Given:** $\overline{RS} \cong \overline{RT}$, $\angle S \cong \angle T$
   **Prove:** $\triangle RST \cong \triangle RTS$

---

**Career Path**

**Q:** What math classes did you take in high school?
**A:** Algebra 1 and 2, Geometry, Precalculus

**Q:** What kind of degree or certification will you receive?
**A:** I will receive an associate’s degree in applied science. Then I will take an exam to be certified as an EMT or paramedic.

**Q:** How do you use math in your hands-on training?
**A:** I calculate dosages based on body weight and age. I also calculate drug doses in milligrams per kilogram per hour or set up an IV drip to deliver medications at the correct rate.

**Q:** What are your future career plans?
**A:** When I am certified, I can work for a private ambulance service or with a fire department. I could also work in a hospital, transporting critically ill patients by ambulance or helicopter.
Explore SSS and SAS
Triangle Congruence

You have used the definition of congruent triangles to prove triangles congruent. To use the definition, you need to prove that all three pairs of corresponding sides and all three pairs of corresponding angles are congruent.

In this lab, you will discover some shortcuts for proving triangles congruent.

Activity 1

1. Measure and cut six pieces from the straws: two that are 2 inches long, two that are 4 inches long, and two that are 5 inches long.

2. Cut two pieces of string that are each about 20 inches long.

3. Thread one piece of each size of straw onto a piece of string. Tie the ends of the string together so that the pieces of straw form a triangle.

4. Using the remaining pieces, try to make another triangle with the same side lengths that is not congruent to the first triangle.

Try This

1. Repeat Activity 1 using side lengths of your choice. Are your results the same?

2. Do you think it is possible to make two triangles that have the same side lengths but that are not congruent? Why or why not?

3. How does your answer to Problem 2 provide a shortcut for proving triangles congruent?

4. Complete the following conjecture based on your results. Two triangles are congruent if _______? _______.
**Activity 2**

1. Measure and cut two pieces from the straws: one that is 4 inches long and one that is 5 inches long.

2. Use a protractor to help you bend a paper clip to form a 30° angle.

3. Place the pieces of straw on the sides of the 30° angle. The straws will form two sides of your triangle.

4. Without changing the angle formed by the paper clip, use a piece of straw to make a third side for your triangle, cutting it to fit as necessary. Use additional paper clips or string to hold the straws together in a triangle.

**Try This**

5. Repeat Activity 2 using side lengths and an angle measure of your choice. Are your results the same?

6. Suppose you know two side lengths of a triangle and the measure of the angle between these sides. Can the length of the third side be any measure? Explain.

7. How does your answer to Problem 6 provide a shortcut for proving triangles congruent?

8. Use the two given sides and the given angle from Activity 2 to form a triangle that is not congruent to the triangle you formed. *(Hint: One of the given sides does not have to be adjacent to the given angle.)*

9. Complete the following conjecture based on your results.
   Two triangles are congruent if ______?_______.
Objectives
Apply SSS and SAS to construct triangles and to solve problems.
Prove triangles congruent by using SSS and SAS.

Vocabulary
triangle rigidity
included angle

Who uses this?
Engineers used the property of triangle rigidity to design the internal support for the Statue of Liberty and to build bridges, towers, and other structures. (See Example 2.)

Recall that you proved triangles congruent by showing that all six pairs of corresponding parts were congruent.

The property of triangle rigidity gives you a shortcut for proving two triangles congruent. It states that if the side lengths of a triangle are given, the triangle can have only one shape.

For example, you only need to know that two triangles have three pairs of congruent corresponding sides. This can be expressed as the following postulate.

**Postulate 11-2-1**  
**Side-Side-Side (SSS) Congruence**

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

**Postulate**

<table>
<thead>
<tr>
<th>POSTULATE</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.</td>
<td>[Diagram: △ABC with sides 4 cm, 7 cm, 6 cm]</td>
<td>△ABC ≅ △FDE</td>
</tr>
</tbody>
</table>

**Example 1**

Use SSS to explain why △PQR ≅ △PSR.

It is given that \( \overline{PQ} \cong \overline{PS} \) and that \( \overline{QR} \cong \overline{SR} \). By the Reflexive Property of Congruence, \( \overline{PR} \cong \overline{PR} \). Therefore △PQR ≅ △PSR by SSS.

1. Use SSS to explain why △ABC ≅ △CDA.

An included angle is an angle formed by two adjacent sides of a polygon. \( \angle B \) is the included angle between sides \( \overline{AB} \) and \( \overline{BC} \).
It can also be shown that only two pairs of congruent corresponding sides are needed to prove the congruence of two triangles if the included angles are also congruent.

**Postulate 11-2-2**

**Side-Angle-Side (SAS) Congruence**

<table>
<thead>
<tr>
<th>POSTULATE</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.</td>
<td></td>
<td>$\triangle ABC \cong \triangle EFD$</td>
</tr>
</tbody>
</table>

**Example 2**

**Engineering Application**

The figure shows part of the support structure of the Statue of Liberty. Use SAS to explain why $\triangle KPN \cong \triangle LPM$.

It is given that $\overline{KP} \cong \overline{LP}$ and that $\overline{NP} \cong \overline{MP}$.

By the Vertical Angles Theorem, $\angle KPN \cong \angle LPM$.

Therefore $\triangle KPN \cong \triangle LPM$ by SAS.

2. Use SAS to explain why $\triangle ABC \cong \triangle DBC$.

The SAS Postulate guarantees that if you are given the lengths of two sides and the measure of the included angle, you can construct one and only one triangle.

**Construction**

**Congruent Triangles Using SAS**

Use a straightedge to draw two segments and one angle, or copy the given segments and angle.

1. Construct $\overline{AB}$ congruent to one of the segments.

2. Construct $\angle A$ congruent to the given angle.

3. Construct $\overline{AC}$ congruent to the other segment. Draw $\overline{CB}$ to complete $\triangle ABC$. 
EXAMPLE 3

Verifying Triangle Congruence

Show that the triangles are congruent
for the given value of the variable.

A \( \triangle UVW \cong \triangle YXZ, \quad x = 3 \)
\[ \begin{align*}
ZY &= x - 1 \\
&= 3 - 1 = 2 \\
XZ &= x = 3 \\
XY &= 3x - 5 \\
&= 3(3) - 5 = 4 \\
\overline{UV} &\cong \overline{YX}, \overline{VW} \cong \overline{XZ}, \text{ and } \overline{UW} \cong \overline{YZ}.
\end{align*} \]
So \( \triangle UVW \cong \triangle YXZ \) by SSS.

B \( \triangle DEF \cong \triangle JGH, \quad y = 7 \)
\[ \begin{align*}
JG &= 2y + 1 \\
&= 2(7) + 1 \\
&= 15 \\
GH &= y^2 - 4y + 3 \\
&= (7)^2 - 4(7) + 3 \\
&= 24 \\
m\angle G &= 12y + 42 \\
&= 12(7) + 42 \\
&= 126^\circ \\
\overline{DE} &\cong \overline{JG}, \overline{EF} \cong \overline{GH}, \text{ and } \angle E \cong \angle G.
\end{align*} \]
So \( \triangle DEF \cong \triangle JGH \) by SAS.

3. Show that \( \triangle ADB \cong \triangle CDB \) when \( t = 4 \).

EXAMPLE 4

Proving Triangles Congruent

Given: \( \ell \parallel m, \overline{EG} \cong \overline{HF} \)
Prove: \( \triangle EGF \cong \triangle HFG \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{EG} \cong \overline{HF} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \ell \parallel m )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \angle EGF \cong \angle HFG )</td>
<td>3. Alt. Int. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>4. ( \overline{FG} \cong \overline{GF} )</td>
<td>4. Reflex Prop. of ( \cong )</td>
</tr>
<tr>
<td>5. ( \triangle EGF \cong \triangle HFG )</td>
<td>5. SAS Steps 1, 3, 4</td>
</tr>
</tbody>
</table>

4. Given: \( \overline{QP} \) bisects \( \angle RQS \), \( \overline{QR} \cong \overline{QS} \)
Prove: \( \triangle RQP \cong \triangle SQP \)
**THINK AND DISCUSS**

1. Describe three ways you could prove that $\triangle ABC \cong \triangle DEF$.

2. Explain why the SSS and SAS Postulates are shortcuts for proving triangles congruent.

3. **GET ORGANIZED** Copy and complete the graphic organizer. Use it to compare the SSS and SAS postulates.

---

**GUIDED PRACTICE**

1. **Vocabulary** In $\triangle RST$ which angle is the included angle of sides $\overline{ST}$ and $\overline{TR}$?

   Use SSS to explain why the triangles in each pair are congruent.

2. $\triangle ABD \cong \triangle CDB$

3. $\triangle MNP \cong \triangle MQP$

---

**SEE EXAMPLE 1**

4. **Sailing** Signal flags are used to communicate messages when radio silence is required. The Zulu signal flag means, "I require a tug." $GJ = GH = GL = GK = 20$ in. Use SAS to explain why $\triangle JGK \cong \triangle LGH$.

---

**SEE EXAMPLE 2**

5. $\triangle GHJ \cong \triangle IHJ$, $x = 4$

6. $\triangle RST \cong \triangle TUR$, $x = 18$

---

**SEE EXAMPLE 3**
7. Given: \( \overline{JK} \cong \overline{ML}, \angle JKL \cong \angle MLK \)
Prove: \( \triangle JKL \cong \triangle MLK \)

Proof:

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>1. ( \overline{JK} \cong \overline{ML} )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. b. ?</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( \overline{KL} \cong \overline{LK} )</td>
<td>3. c. ?</td>
</tr>
<tr>
<td>4. ( \triangle JKL \cong \triangle MLK )</td>
<td>4. d. ?</td>
</tr>
</tbody>
</table>

**PRACTICE AND PROBLEM SOLVING**

Use SSS to explain why the triangles in each pair are congruent.

8. \( \triangle BCD \cong \triangle EDC \)

9. \( \triangle GJK \cong \triangle GJL \)

10. **Theater** The lights shining on a stage appear to form two congruent right triangles. Given \( \overline{EC} \cong \overline{DB} \), use SAS to explain why \( \triangle ECB \cong \triangle DBC \).

Show that the triangles are congruent for the given value of the variable.

11. \( \triangle MNP \cong \triangle QNP, \ y = 3 \)

12. \( \triangle XYZ \cong \triangle STU, \ t = 5 \)

13. Given: \( B \) is the midpoint of \( \overline{DC} \). \( AB \perp \overline{DC} \)
Prove: \( \triangle ABD \cong \triangle ABC \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( B ) is the mdpt. of ( \overline{DC} ).</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. b. ?</td>
<td>2. Def. of mdpt.</td>
</tr>
<tr>
<td>3. c. ?</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \triangle ABD ) and ( \triangle ABC ) are rt. ( \triangle )</td>
<td>4. d. ?</td>
</tr>
<tr>
<td>5. ( \triangle ABD \cong \triangle ABC )</td>
<td>5. e. ?</td>
</tr>
<tr>
<td>6. f. ?</td>
<td>6. Reflex. Prop. of ( \cong )</td>
</tr>
<tr>
<td>7. ( \triangle ABD \cong \triangle ABC )</td>
<td>7. g. ?</td>
</tr>
</tbody>
</table>
Which postulate, if any, can be used to prove the triangles congruent?

14. 

15. 

16. 

17. 

18. Explain what additional information, if any, you would need to prove \( \triangle ABC \cong \triangle DEC \) by each postulate.
   a. SSS
   b. SAS

Multi-Step Graph each triangle. Then use the Distance Formula and the SSS Postulate to determine whether the triangles are congruent.

19. \( \triangle QRS \) and \( \triangle TUV \)
   Q(−2, 0), R(1, −2), S(−3, −2)
   T(5, 1), U(3, −2), V(3, 2)

20. \( \triangle ABC \) and \( \triangle DEF \)
   A(2, 3), B(3, −1), C(7, 2)
   D(−3, 1), E(1, 2), F(−3, 5)

21. Given: \( \angle ZVY \cong \angle WYV \)
    \( \angle ZVW \cong \angle WYZ \)
    \( VV \cong \overline{YZ} \)

Prove: \( \triangle ZVY \cong \triangle WYV \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle ZVY \cong \angle WYV ), ( \angle ZVW \cong \angle WYZ )</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. ( m\angle ZVY = m\angle WYV ), ( m\angle ZVW = m\angle WYZ )</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>3. ( m\angle ZVY + m\angle ZVW = m\angle WYV + m\angle WYZ )</td>
<td>3. Add. Prop. of ( = )</td>
</tr>
<tr>
<td>4. ( \angle WVV \cong \angle ZYV )</td>
<td>4. ( \angle ) Add. Post.</td>
</tr>
<tr>
<td>5. ( \angle WVV \cong \angle ZYV )</td>
<td>5. d. ?</td>
</tr>
<tr>
<td>6. ( VV \cong \overline{YZ} )</td>
<td>6. e. ?</td>
</tr>
<tr>
<td>7. f. ?</td>
<td>7. Reflex. Prop. of ( \cong )</td>
</tr>
<tr>
<td>8. ( \triangle ZVY \cong \triangle WYV )</td>
<td>8. g. ?</td>
</tr>
</tbody>
</table>

22. The diagram shows two triangular trusses that were built for the roof of a doghouse.
   a. You can use a protractor to check that \( \angle A \) and \( \angle D \) are right angles. Explain how you could make just two additional measurements on each truss to ensure that the trusses are congruent.
   b. You verify that the trusses are congruent and find that \( AB = AC = 2.5 \text{ ft} \). Find the length of \( EF \) to the nearest tenth. Explain.
23. **Critical Thinking** Draw two isosceles triangles that are not congruent but that have a perimeter of 15 cm each.

24. \( \triangle ABC \cong \triangle ADC \) for what value of \( x \)? Explain why the SSS Postulate can be used to prove the two triangles congruent.

25. **Ecology** A wing deflector is a triangular structure made of logs that is filled with large rocks and placed in a stream to guide the current or prevent erosion. Wing deflectors are often used in pairs. Suppose an engineer wants to build two wing deflectors. The logs that form the sides of each wing deflector are perpendicular. How can the engineer make sure that the two wing deflectors are congruent?

26. **Write About It** If you use the same two sides and included angle to repeat the construction of a triangle, are your two constructed triangles congruent? Explain.

27. **Construction** Use three segments (SSS) to construct a scalene triangle. Suppose you then use the same segments in a different order to construct a second triangle. Will the result be the same? Explain.

28. Which of the three triangles below can be proven congruent by SSS or SAS?

   ![Triangle Diagrams]
   
   - **A** I and II
   - **B** II and III
   - **C** I and III
   - **D** I, II, and III

29. What is the perimeter of polygon \( ABCD \)?

   - **F** 29.9 cm
   - **G** 39.8 cm
   - **H** 49.8 cm
   - **I** 59.8 cm

30. Jacob wants to prove that \( \triangle FGH \cong \triangle JKL \) using SAS. He knows that \( FG = JK \) and \( FH = JL \). What additional piece of information does he need?

   - **A** \( \angle F \cong \angle J \)
   - **B** \( \angle G \cong \angle K \)
   - **C** \( \angle H \cong \angle L \)
   - **D** \( \angle F \cong \angle G \)

31. What must the value of \( x \) be in order to prove that \( \triangle EFG \cong \triangle EHG \) by SSS?

   - **F** 1.5
   - **G** 4.25
   - **H** 4.67
   - **I** 5.5
32. Given: $\angle ADC$ and $\angle BCD$ are supplementary. $\overline{AD} \cong \overline{CB}$
Prove: $\triangle ADB \cong \triangle CBD$
(Hint: Draw an auxiliary line.)

33. Given: $\angle QPS \cong \angle TPR$, $\overline{PQ} \cong \overline{PT}$, $\overline{PR} \cong \overline{PS}$
Prove: $\triangle PQR \cong \triangle PTS$

**Algebra** Use the following information for Exercises 34 and 35.
Find the value of $x$. Then use SSS or SAS to write a paragraph proof showing that two of the triangles are congruent.

34. $m\angle FKJ = 2x^\circ$
   $m\angle KFJ = (3x + 10)^\circ$
   $KJ = 4x + 8$
   $HJ = 6(x - 4)$

35. $\overline{FJ}$ bisects $\angle KFH$.
   $m\angle KFJ = (2x + 6)^\circ$
   $m\angle KFJ = (3x - 21)^\circ$
   $FK = 8x - 45$
   $FH = 6x + 9$

**Using Technology**

Use geometry software to complete the following.

1. Draw a triangle and label the vertices $A$, $B$, and $C$.
   Draw a point and label it $D$. Mark a vector from $A$ to $B$ and translate $D$ by the marked vector. Label the image $E$.
   Draw $\overline{DE}$. Mark $\angle BAC$ and rotate $\overline{DE}$ about $D$ by the marked angle. Mark $\angle ABC$ and rotate $\overline{DE}$ about $E$ by the marked angle. Label the intersection $F$.

2. Drag $A$, $B$, and $C$ to different locations.
   What do you notice about the two triangles?

3. Write a conjecture about $\triangle ABC$ and $\triangle DEF$.

4. Test your conjecture by measuring the sides and angles of $\triangle ABC$ and $\triangle DEF$. 
Activity 1

1. Construct $\angle CAB$ measuring $45^\circ$ and $\angle EDF$ measuring $110^\circ$.

2. Move $\angle EDF$ so that $\overrightarrow{DE}$ overlays $\overrightarrow{BA}$. Where $\overrightarrow{DF}$ and $\overrightarrow{AC}$ intersect, label the point $G$. Measure $\angle DGA$.

3. Move $\angle CAB$ to the left and right without changing the measures of the angles. Observe what happens to the size of $\angle DGA$.

4. Measure the distance from $A$ to $D$. Try to change the shape of the triangle without changing $AD$ and the measures of $\angle A$ and $\angle D$.

Try This

1. Repeat Activity 1 using angle measures of your choice. Are your results the same? Explain.

2. Do the results change if one of the given angles measures $90^\circ$?

3. What theorem proves that the measure of $\angle DGA$ in Step 2 will always be the same?

4. In Step 3 of the activity, the angle measures in $\triangle ADG$ stayed the same as the size of the triangle changed. Does Angle-Angle-Angle, like Side-Side-Side, make only one triangle? Explain.

5. Repeat Step 4 of the activity but measure the length of $\overline{AG}$ instead of $\overline{AD}$. Are your results the same? Does this lead to a new congruence postulate or theorem?

6. If you are given two angles of a triangle, what additional piece of information is needed so that only one triangle is made? Make a conjecture based on your findings in Step 5.
**Activity 2**

1. Construct \( \overline{YZ} \) with a length of 6.5 cm.

2. Using \( \overline{YZ} \) as a side, construct \( \angle XYZ \) measuring 43°.

3. Draw a circle at \( Z \) with a radius of 5 cm. Construct \( \overline{ZW} \), a radius of circle \( Z \).

4. Move \( W \) around circle \( Z \). Observe the possible shapes of \( \triangle YZW \).

**Try This**

7. In Step 4 of the activity, how many different triangles were possible? Does Side-Side-Angle make only one triangle?

8. Repeat Activity 2 using an angle measure of 90° in Step 2 and a circle with a radius of 7 cm in Step 3. How many different triangles are possible in Step 4?

9. Repeat the activity again using a measure of 90° in Step 2 and a circle with a radius of 8.25 cm in Step 3. Classify the resulting triangle by its angle measures.

10. Based on your results, complete the following conjecture. In a Side-Side-Angle combination, if the corresponding nonincluded angles are ____?, then only one triangle is possible.
**Objectives**

Apply ASA, AAS, and HL to construct triangles and to solve problems.
Prove triangles congruent by using ASA, AAS, and HL.

**Vocabulary**

included side

---

**Why use this?**

Bearings are used to convey direction, helping people find their way to specific locations.

Participants in an *orienteering* race use a map and a compass to find their way to checkpoints along an unfamiliar course. Directions are given by *bearings*, which are based on compass headings. For example, to travel along the bearing S 43° E, you face south and then turn 43° to the east.

An *included side* is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an *included side*.

**Postulate 11-3-1**

**Angle-Side-Angle (ASA) Congruence**

<table>
<thead>
<tr>
<th>POSTULATE</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.</td>
<td>△ABC ≅ △DEF</td>
<td></td>
</tr>
</tbody>
</table>

---

**EXAMPLE 1**

Organizers of an orienteering race are planning a course with checkpoints A, B, and C. Does the table give enough information to determine the location of the checkpoints?

**Problem-Solving Application**

<table>
<thead>
<tr>
<th></th>
<th>Bearing</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A to B</td>
<td>N 55° E</td>
<td>7.6 km</td>
</tr>
<tr>
<td>B to C</td>
<td>N 26° W</td>
<td></td>
</tr>
<tr>
<td>C to A</td>
<td>S 20° W</td>
<td></td>
</tr>
</tbody>
</table>

**Understand the Problem**

The answer is whether the information in the table can be used to find the position of checkpoints A, B, and C.

List the *important information*: The bearing from A to B is N 55° E. From B to C is N 26° W, and from C to A is S 20° W. The distance from A to B is 7.6 km.
Make a Plan

Draw the course using vertical lines to show north-south directions. Then use these parallel lines and the alternate interior angles to help find angle measures of \( \triangle ABC \).

Solve

\[
\begin{align*}
    \angle CAB &= 55^\circ - 20^\circ = 35^\circ \\
    \angle CBA &= 180^\circ - (26^\circ + 55^\circ) = 99^\circ
\end{align*}
\]

You know the measures of \( \angle CAB \) and \( \angle CBA \) and the length of the included side \( AB \). Therefore by ASA, a unique triangle \( ABC \) is determined.

Look Back

One and only one triangle can be made using the information in the table, so the table does give enough information to determine the location of all the checkpoints.

1. **What if...?** If 7.6 km is the distance from \( B \) to \( C \), is there enough information to determine the location of all the checkpoints? Explain.

**Example 2** Applying ASA Congruence

Determine if you can use ASA to prove \( \triangle UVX \cong \triangle WVX \). Explain.

\[
\angle UXV \cong \angle WXV \text{ as given. Since } \angle WVX \text{ is a right angle that forms a linear pair with } \angle UVX, \angle WVX \cong \angle UVX. \text{ Also } VX \cong VX \text{ by the Reflexive Property. Therefore } \triangle UVX \cong \triangle WVX \text{ by ASA.}
\]

2. Determine if you can use ASA to prove \( \triangle NKL \cong \triangle LMN \). Explain.

**Construction** Congruent Triangles Using ASA

Use a straightedge to draw a segment and two angles, or copy the given segment and angles.

1. Construct \( \overline{CD} \) congruent to the given segment.
2. Construct \( \angle C \) congruent to one of the angles.
3. Construct \( \angle D \) congruent to the other angle.
4. Label the intersection of the rays as \( E \).

\( \triangle CDE \)
You can use the Third Angles Theorem to prove another congruence relationship based on ASA. This theorem is Angle-Angle-Side (AAS).

**Theorem 11-3-2**  
**Angle-Angle-Side (AAS) Congruence**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.</td>
<td><img src="image" alt="Diagram of triangles GHJ and KLM" /></td>
<td>$\triangle GHJ \cong \triangle KLM$</td>
</tr>
</tbody>
</table>

**Proof**

**Angle-Angle-Side Congruence**

Given: $\angle G \cong \angle K$, $\angle J \cong \angle M$, $\overline{HJ} \cong \overline{LM}$  
Prove: $\triangle GHJ \cong \triangle KLM$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle G \cong \angle K$, $\angle J \cong \angle M$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle H \cong \angle L$</td>
<td>2. Third $\triangle$ Thm.</td>
</tr>
<tr>
<td>3. $\overline{HJ} \cong \overline{LM}$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\triangle GHJ \cong \triangle KLM$</td>
<td>4. ASA Steps 1, 3, and 2</td>
</tr>
</tbody>
</table>

**Example 3**

**Using AAS to Prove Triangles Congruent**

Use AAS to prove the triangles congruent.

Given: $AB \parallel ED$, $BC \cong DC$  
Prove: $\triangle ABC \cong \triangle EDC$

Proof:

1. $BC \cong DC$  
   Given
2. $\angle B \cong \angle D$  
   Alt. Int. $\triangle$ Thm.
3. $\angle A \cong \angle E$  
   Alt. Int. $\triangle$ Thm.
4. $\triangle ABC \cong \triangle EDC$  
   AAS

**Check It Out!**

3. Use AAS to prove the triangles congruent.
   Given: JL bisects $\angle KLM$, $\angle K \cong \angle M$
   Prove: $\triangle JKL \cong \triangle JML$

There are four theorems for right triangles that are not used for acute or obtuse triangles. They are Leg-Leg (LL), Hypotenuse-Angle (HA), Leg-Angle (LA), and Hypotenuse-Leg (HL). You will prove LL, HA, and LA in Exercises 21, 23, and 33.
THEOREM HYPOTHESIS CONCLUSION

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

\[ \triangle ABC \cong \triangle DEF \]

You will prove the Hypotenuse-Leg Theorem in lesson Isosceles and Equilateral Triangles

**Example 4**

Applying HL Congruence

Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.

**A** \( \triangle VWX \) and \( \triangle YXW \)

According to the diagram, \( \triangle VWX \) and \( \triangle YXW \) are right triangles that share hypotenuse \( WX \). \( WX \cong XW \) by the Reflexive Property. It is given that \( WV \cong XY \), therefore \( \triangle VWX \cong \triangle YXW \) by HL.

**B** \( \triangle VWZ \) and \( \triangle YXZ \)

This conclusion cannot be proved by HL. According to the diagram, \( \triangle VWZ \) and \( \triangle YXZ \) are right triangles, and \( WV \cong XY \). You do not know that hypotenuse \( WZ \) is congruent to hypotenuse \( XZ \).

**Check It Out!**

4. Determine if you can use the HL Congruence Theorem to prove \( \triangle ABC \cong \triangle DCB \). If not, tell what else you need to know.

**Think and Discuss**

1. Could you use AAS to prove that these two triangles are congruent? Explain.

2. The arrangement of the letters in ASA matches the arrangement of what parts of congruent triangles? Include a sketch to support your answer.

3. GET ORGANIZED Copy and complete the graphic organizer. In each column, write a description of the method and then sketch two triangles, marking the appropriate congruent parts.
GUIDED PRACTICE

1. **Vocabulary** A triangle contains \( \triangle ABC \) and \( \triangle ACB \) with \( \overline{BC} \) “closed in” between them. How would this help you remember the definition of *included side*?

2. **Surveying** Use the table for Exercises 2 and 3.
   A landscape designer surveyed the boundaries of a triangular park. She made the following table for the dimensions of the land.

<table>
<thead>
<tr>
<th>A to B</th>
<th>B to C</th>
<th>C to A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing</td>
<td>E S 25° E</td>
<td>N 62° W</td>
</tr>
<tr>
<td>Distance</td>
<td>115 ft</td>
<td>?</td>
</tr>
</tbody>
</table>

   2. Draw the plot of land described by the table.
   Label the measures of the angles in the triangle.

   3. Does the table have enough information to determine the locations of points \( A, B, \) and \( C \)? Explain.

   4. **Determine if you can use ASA to prove the triangles congruent. Explain.**
   \( \triangle VRS \) and \( \triangle VTS \), given that \( \overline{VS} \) bisects \( \angle RST \) and \( \angle RVT \)

   5. **\( \triangle DEH \) and \( \triangle FGH \)**

   6. **Use AAS to prove the triangles congruent.**
   Given: \( \angle R \) and \( \angle P \) are right angles.
   \( \overline{QR} \parallel \overline{SP} \)
   Prove: \( \triangle QPS \cong \triangle SRQ \)
   Proof:
   \[
   \text{a. } \angle R \text{ and } \angle P \text{ are rt. } \Delta. \\
   \text{b. } \angle R \cong \angle P \\
   \text{c. } \overline{QR} \parallel \overline{SP} \\
   \text{d. } \triangle QPS \cong \triangle SRQ
   \]

   7. **Determine if you can use the HL Congruence Theorem to prove the triangles congruent. If not, tell what else you need to know.**
   \( \triangle ABC \) and \( \triangle CDA \)

   8. \( \triangle XYV \) and \( \triangle ZYV \)
PRACTICE AND PROBLEM SOLVING

Surveying Use the table for Exercises 9 and 10.
From two different observation towers a fire is sighted. The locations of the towers are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>X to Y</th>
<th>X to F</th>
<th>Y to F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing</td>
<td>E</td>
<td>N 53° E</td>
<td>N 16° W</td>
</tr>
<tr>
<td>Distance</td>
<td>6 km</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

9. Draw the diagram formed by observation tower X, observation tower Y, and the fire F. Label the measures of the angles.

10. Is there enough information given in the table to pinpoint the location of the fire? Explain.

Determine if you can use ASA to prove the triangles congruent. Explain.

11. ΔMKJ and ΔMKL

12. ΔRST and ΔTUR

13. Given: AB ≅ DE, ∠C ≅ ∠F
   Prove: ΔABC ≅ ΔDEF

Proof:

14. ΔGHJ and ΔJKG

15. ΔABE and ΔDCE, given that E is the midpoint of AD and BC

Multi-Step For each pair of triangles write a triangle congruence statement. Identify the transformation that moves one triangle to the position of the other triangle.

16.

17.

18. Critical Thinking Side-Side-Angle (SSA) cannot be used to prove two triangles congruent. Draw a diagram that shows why this is true.

Math History

Euclid wrote the mathematical text The Elements around 2300 years ago. It may be the second most reprinted book in history.
19. A carpenter built a truss to support the roof of a doghouse.

a. The carpenter knows that $\overline{KJ} \cong \overline{MJ}$. Can the carpenter conclude that $\triangle KJL \cong \triangle MJL$? Why or why not?

b. Suppose the carpenter also knows that $\angle JKL$ is a right angle. Which theorem can be used to show that $\triangle KJL \cong \triangle MJL$?

20. ERROR ANALYSIS Two proofs that $\triangle EFH \cong \triangle GHF$ are given. Which is incorrect? Explain the error.

A

It is given that $\overline{EF} \parallel \overline{GH}$. By the Alam. Int. $\angle$ Thm., $\angle EFH \cong \angle GHF$.

$\angle E \cong \angle G$ by the Rt. $\angle$ Thm. By the Reflex. Prop. of $\cong$, $\overline{HF} \cong \overline{HF}$.

So by AAS, $\triangle EFH \cong \triangle GHF$.

B

$\overline{HF}$ is the hyp. of both rt. $\triangle$. $\overline{HF} \cong \overline{HF}$ by the Reflex. Prop. of $\cong$. Since the opp. sides of a rect. are $\cong$, $\overline{EF} \cong \overline{GH}$. So by HL, $\triangle EFH \cong \triangle FHG$.

21. Write a paragraph proof of the Leg-Leg (LL) Congruence Theorem. If the legs of one right triangle are congruent to the corresponding legs of another right triangle, the triangles are congruent.

22. Use AAS to prove the triangles congruent.

Given: $\overline{AD} \parallel \overline{BC}$, $\overline{AD} \cong \overline{CB}$

Prove: $\triangle AED \cong \triangle CEB$

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{AD} \parallel \overline{BC}$</td>
<td>1. a. ?</td>
</tr>
<tr>
<td>2. $\angle DAE \cong \angle BCE$</td>
<td>2. b. ?</td>
</tr>
<tr>
<td>5. e. ?</td>
<td>5. f. ?</td>
</tr>
</tbody>
</table>

23. Prove the Hypotenuse-Angle (HA) Theorem.

Given: $\overline{KM} \perp \overline{JL}$, $\overline{JM} \cong \overline{LM}$, $\angle JMK \cong \angle LMK$

Prove: $\triangle KJM \cong \triangle LKM$

24. Write About It The legs of both right $\triangle DEF$ and right $\triangle RST$ are 3 cm and 4 cm. They each have a hypotenuse 5 cm in length. Describe two different ways you could prove that $\triangle DEF \cong \triangle RST$.

25. Construction Use the method for constructing perpendicular lines to construct a right triangle.

26. What additional congruence statement is necessary to prove $\triangle XWY \cong \triangle XWZ$ by ASA?

A $\angle XWZ \cong \angle XWY$

B $\angle VUY \cong \angle WUZ$

C $\overline{VZ} \cong \overline{WY}$

D $\overline{XZ} \cong \overline{XY}$
27. Which postulate or theorem justifies the congruence statement \( \triangle STU \cong \triangle VUT \)?
   - (A) ASA
   - (B) SSS
   - (C) HL
   - (D) SAS

28. Which of the following congruence statements is true?
   - (A) \( \angle A \cong \angle B \)
   - (B) \( \triangle AED \cong \triangle CEB \)
   - (C) \( \overline{CE} \cong \overline{DE} \)
   - (D) \( \triangle AED \cong \triangle BEC \)

29. In \( \triangle RST \), \( RT = 6y - 2 \). In \( \triangle UVW \), \( UW = 2y + 7 \). \( \angle R \cong \angle U \), and \( \angle S \cong \angle V \).
   What must be the value of \( y \) in order to prove that \( \triangle RST \cong \triangle UVW \)?
   - (F) 1.25
   - (G) 2.25
   - (H) 9.0
   - (I) 11.5

30. Extended Response Draw a triangle. Construct a second triangle that has the same angle measures but is not congruent. Compare the lengths of each pair of corresponding sides. Consider the relationship between the lengths of the sides and the measures of the angles. Explain why Angle-Angle-Angle (AAA) is not a congruence principle.

31. Sports This bicycle frame includes \( \triangle VSU \) and \( \triangle VTU \), which lie in intersecting planes. From the given angle measures, can you conclude that \( \triangle VSU \cong \triangle VTU \)? Explain.
   \[
   \begin{align*}
   \text{m} \angle VUS &= (7y - 2) {}^\circ \\
   \text{m} \angle VUT &= (5 \frac{1}{2} x - \frac{1}{2}) {}^\circ \\
   \text{m} \angle USV &= 5 \frac{2}{3} y {}^\circ \\
   \text{m} \angle UTV &= (4x + 8) {}^\circ \\
   \text{m} \angle SVU &= (3y - 6) {}^\circ \\
   \text{m} \angle TVU &= 2x {}^\circ 
   \end{align*}
   \]

32. Given: \( \triangle ABC \) is equilateral. \( C \) is the midpoint of \( \overline{DE} \). \( \angle DAC \) and \( \angle EBC \) are congruent and supplementary.
   Prove: \( \triangle DAC \cong \triangle EBC \)

33. Write a two-column proof of the Leg-Angle (LA) Congruence Theorem. If a leg and an acute angle of one right triangle are congruent to the corresponding parts of another right triangle, the triangles are congruent. (Hint: There are two cases to consider.)

34. If two triangles are congruent by ASA, what theorem could you use to prove that the triangles are also congruent by AAS? Explain.
Vocabulary

acute triangle  dilation  isometry
auxiliary line  equiangular triangle  legs of an isosceles triangle
base  equilateral triangle
base angle  exterior  obtuse triangle
congruent polygons  exterior angle  right triangle
corollary  included angle  rigid transformation
corresponding angles  included side  scalene triangle
corresponding sides  interior  triangle rigidity
CPCTC  interior angle  vertex angle

Complete the sentences below with vocabulary words from the list above.

1. A(n) ____?___ is a triangle with at least two congruent sides.
2. A name given to matching angles of congruent triangles is ____?___.
3. A(n) ____?___ is the common side of two consecutive angles in a polygon.

11-1 Congruence and Transformations

**EXAMPLE**

- Determine whether the polygons are congruent.
  
  A(1, 1), B(4, 1), C(5, 3)
  
  P(–2, 3), Q(1, 3), R(2, 5)
  
  The triangles are congruent because ABC can be mapped to PQR by a translation:
  
  (x, y) → (x – 3, y + 2).

**EXERCISES**

4. Determine whether ABC and PQR are congruent. Support your answer by describing a transformation.
   
   A(3, –1), B(4, 0), C(5, 3)
   
   P(–3, 1), Q(–4, 0), R(–5, –3)

5. Apply the transformation M : (x, y) → (4x, 4y) to DEF with vertices D(1, 2), E(2, –3), and F(4, 0). Name the coordinates of the image points. Identify and describe the transformation.

11-2 Classifying Triangles

**EXAMPLE**

- Classify the triangle by its angle measures and side lengths.
  
  isosceles right triangle

**EXERCISES**

Classify each triangle by its angle measures and side lengths.

6.

7. 135°
11-3 Angle Relationships in Triangles

**Example**
- Find \( m\angle S \).

\[
12x = 3x + 42 + 6x \\
12x = 9x + 42 \\
3x = 42 \\
x = 14 \\
m\angle S = 6(14) = 84^\circ
\]

11-4 Congruent Triangles

**Example**
- Given: \( \triangle DEF \cong \triangle JKL \). Identify all pairs of congruent corresponding parts. Then find the value of \( x \).

The congruent pairs follow: \( \angle D \cong \angle J \), \( \angle E \cong \angle K \), \( \angle F \cong \angle L \), \( DE \cong JK \), \( EF \cong KL \), and \( DF \cong JL \).

Since \( m\angle E = m\angle K \), \( 90 = 8x - 22 \). After 22 is added to both sides, \( 112 = 8x \). So \( x = 14 \).

11-5 Triangle Congruence: SSS and SAS

**Example**
- Given: \( \overline{RS} \cong \overline{UT} \), and \( \overline{VS} \cong \overline{VT} \). \( V \) is the midpoint of \( \overline{RU} \).

Prove: \( \triangle RSV \cong \triangle UTV \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{RS} \cong \overline{UT} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{VS} \cong \overline{VT} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( V ) is the mdpt. of ( \overline{RU} .</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overline{RV} \cong \overline{UV} )</td>
<td>4. Def. of mdpt.</td>
</tr>
<tr>
<td>5. ( \triangle RSV \cong \triangle UTV )</td>
<td>5. SSS Steps 1, 2, 4</td>
</tr>
</tbody>
</table>

**Exercises**

- Find \( m\angle N \).

8. \( N \)

9. In \( \triangle LMN \), \( m\angle L = 8x \), \( m\angle M = (2x + 1)^\circ \), and \( m\angle N = (6x - 1)^\circ \).

- Find \( m\angle S \).

- \( 12x = 3x + 42 + 6x \)

- \( 12x = 9x + 42 \)

- \( 3x = 42 \)

- \( x = 14 \)

- \( m\angle S = 6(14) = 84^\circ \)

- Given: \( \triangle PQR \cong \triangle XYZ \). Identify the congruent corresponding parts.

- \( \overline{PR} \cong \overline{?} \)

- \( \angle Y \cong \overline{?} \)

- Given: \( \triangle ABC \cong \triangle CDA \) Find each value.

- \( AB \)

- \( CD \)

- \( 15 - 4y \)

- \( 3y + 1 \)

- Given: \( \triangle DEF \cong \triangle JKL \). Identify all pairs of congruent corresponding parts. Then find the value of \( x \).

- \( D \)

- \( E \)

- \( F \)

- \( J \)

- \( K \)

- \( L \)

- \( \overline{DE} \cong \overline{JK} \)

- \( \overline{EF} \cong \overline{KL} \)

- \( \overline{DF} \cong \overline{JL} \)

- Since \( m\angle E = m\angle K \), \( 90 = 8x - 22 \). After 22 is added to both sides, \( 112 = 8x \). So \( x = 14 \).

- **Exercises**

- Given: \( \overline{PQ} \parallel \overline{RQ} \), and \( \overline{PR} \parallel \overline{QR} \). \( P \) is the midpoint of \( \overline{RU} \).

Prove: \( \triangle PRS \cong \triangle QRS \)

Proof:

<table>
<thead>
<tr>
<th>Statements</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1. ( \overline{PR} \parallel \overline{QR} )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \overline{PR} \parallel \overline{QR} )</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. ( P ) is the mdpt. of ( \overline{RU} .</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. ( \overline{PV} \cong \overline{QV} )</td>
<td>4. Def. of mdpt.</td>
</tr>
<tr>
<td>5. ( \triangle PRS \cong \triangle QRS )</td>
<td>5. SSS Steps 1, 2, 4</td>
</tr>
</tbody>
</table>

- Given: \( \overline{RS} \cong \overline{UT} \), and \( \overline{VS} \cong \overline{VT} \). \( V \) is the midpoint of \( \overline{RU} \).

Prove: \( \triangle RSV \cong \triangle UTV \)

Proof:

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</tr>
<tr>
<td>5. ( \angle RSV \cong \angle UTV )</td>
<td>5. SSS Steps 1, 2, 4</td>
</tr>
</tbody>
</table>

- Given: \( \overline{AB} \parallel \overline{DE} \), \( \overline{DB} \parallel \overline{AE} \)

Prove: \( \triangle ADB \cong \triangle DAE \)

- Given: \( \overline{GJ} \) bisects \( \overline{FH} \), and \( \overline{FH} \) bisects \( \overline{GJ} \). \( \angle FGK \cong \angle HJK \)

Prove: \( \triangle FGK \cong \triangle HJK \)
11-6 Triangle Congruence: ASA, AAS, and HL

**EXAMPLES**

- **Given:** $B$ is the midpoint of $\overline{AE}$.
  \[ \angle A \cong \angle E, \quad \angle ABC \cong \angle EBD \]
  Prove: $\triangle ABC \cong \triangle EBD$

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle A \cong \angle E$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle ABC \cong \angle EBD$</td>
<td>2. Given</td>
</tr>
<tr>
<td>3. $B$ is the mdpt. of $\overline{AE}$</td>
<td>3. Given</td>
</tr>
<tr>
<td>4. $\overline{AB} \cong \overline{EB}$</td>
<td>4. Def. of mdpt.</td>
</tr>
<tr>
<td>5. $\triangle ABC \cong \triangle EBD$</td>
<td>5. ASA Steps 1, 4, 2</td>
</tr>
</tbody>
</table>

16. **Given:** $C$ is the midpoint of $\overline{AG}$.
  \[ \overline{HA} \parallel \overline{GB} \]
  Prove: $\triangle HAC \cong \triangle BGC$

17. **Given:** $\overline{WX} \perp \overline{XZ}$, $\overline{YZ} \perp \overline{ZX}$, $\overline{WZ} \cong \overline{YX}$
  Prove: $\triangle WZX \cong \triangle YXZ$

18. **Given:** $\angle S$ and $\angle V$ are right angles.
  \[ RT \cong UW, \quad m\angle T = m\angle W \]
  Prove: $\triangle RST \cong \triangle UVW$

11-7 Triangle Congruence: CPCTC

**EXAMPLES**

- **Given:** $\overline{JL}$ and $\overline{HK}$ bisect each other.
  Prove: $\angle JHG \cong \angle LKG$

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\overline{JL}$ and $\overline{HK}$ bisect each other.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\overline{JG} \cong \overline{LG}$, and $\overline{HG} \cong \overline{KG}$.</td>
<td>2. Def. of bisect</td>
</tr>
<tr>
<td>3. $\angle JGH \cong \angle LKG$.</td>
<td>3. Vert. ( \Delta ) Thm.</td>
</tr>
<tr>
<td>4. $\triangle JHG \cong \triangle LKG$.</td>
<td>4. SAS Steps 2, 3</td>
</tr>
<tr>
<td>5. $\angle JHG \cong \angle LKG$.</td>
<td>5. CPCTC</td>
</tr>
</tbody>
</table>

19. **Given:** $M$ is the midpoint of $\overline{BD}$.
  \[ \overline{BC} \cong \overline{DC} \]
  Prove: $\angle 1 \cong \angle 2$

20. **Given:** $\overline{PQ} \cong \overline{RQ}$, $\overline{PS} \cong \overline{RS}$
  Prove: $\overline{QS}$ bisects $\angle PQR$.

21. **Given:** $H$ is the midpoint of $\overline{GJ}$.
  \[ L \text{ is the midpoint of } \overline{MK}, \quad \overline{GM} \cong \overline{JL}, \overline{GJ} \cong \overline{KM}, \quad \angle G \cong \angle K \]
  Prove: $\angle GMH \cong \angle KJL$
11-8 Introduction to Coordinate Proof

**Examples**

- Given: ∠B is a right angle in isosceles right △ABC. E is the midpoint of AB, D is the midpoint of CB. AB ≅ CB
  
  Prove: CE ≅ AD

  Proof: Use the coordinates A(0, 2a), B(0, 0), and C(2a, 0). Draw AD and CE.

  ![Diagram](image)

  By the Midpoint Formula,
  
  \[ E = \left( \frac{0 + 0}{2}, \frac{2a + 0}{2} \right) = (0, a) \]

  \[ D = \left( \frac{0 + 2a}{2}, \frac{0 + 0}{2} \right) = (a, 0) \]

  By the Distance Formula,
  
  \[ CE = \sqrt{(2a - 0)^2 + (0 - a)^2} \]

  \[ = \sqrt{4a^2 + a^2} = a\sqrt{5} \]

  \[ AD = \sqrt{(a - 0)^2 + (0 - 2a)^2} \]

  \[ = \sqrt{a^2 + 4a^2} = a\sqrt{5} \]

  Thus CE ≅ AD by the definition of congruence.

**Exercises**

Position each figure in the coordinate plane and give the coordinates of each vertex.

22. a right triangle with leg lengths r and s
23. a rectangle with length 2p and width p
24. a square with side length 8m

For exercises 25 and 26 assign coordinates to each vertex and write a coordinate proof.

25. Given: In rectangle ABCD, E is the midpoint of AB, F is the midpoint of BC, G is the midpoint of CD, and H is the midpoint of AD.

   Prove: EF ≅ GH

26. Given: △PQR has a right ∠Q. M is the midpoint of PR.

   Prove: MP = MQ = MR

27. Show that a triangle with vertices at (3, 5), (3, 2), and (2, 5) is a right triangle.

11-9 Isosceles and Equilateral Triangles

**Example**

- Find the value of x.

  \[ m\angle D + m\angle E + m\angle F = 180^\circ \]

  by the Triangle Sum Theorem.

  \[ m\angle E = m\angle F \]

  by the Isosceles Triangle Theorem.

  \[ m\angle D + 2m\angle E = 180^\circ \]

  Substitution

  \[ 42 + 2(3x) = 180 \]

  Substitute the given values.

  \[ 6x = 138 \]

  Simplify.

  \[ x = 23 \]

  Divide both sides by 6.

**Exercises**

Find each value.

28. x

29. RS

30. Given: △ACD is isosceles with ∠D as the vertex angle. B is the midpoint of AC.

   AB = x + 5, BC = 2x - 3, and CD = 2x + 6.

   Find the perimeter of △ACD.
1. Classify \(\triangle ACD\) by its angle measures.

Classify each triangle by its side lengths.
2. \(\triangle ACD\) 
3. \(\triangle ABC\) 
4. \(\triangle ABD\)

5. While surveying the triangular plot of land shown, a surveyor finds that \(m\angle S = 43^\circ\). The measure of \(\angle RTP\) is twice that of \(\angle RTS\). What is \(m\angle R\)?

Given: \(\triangle XYZ \cong \triangle JKL\)
Identify the congruent corresponding parts.
6. \(\overline{JL} \cong \overline{?}\) 
7. \(\angle Y \cong \overline{?}\) 
8. \(\angle L \cong \overline{?}\) 
9. \(\overline{YZ} \cong \overline{?}\)

10. Given: \(T\) is the midpoint of \(\overline{PR}\) and \(\overline{SQ}\).
Prove: \(\triangle PTS \cong \triangle RTQ\)

11. The figure represents a walkway with triangular supports. Given that \(\overline{GJ}\) bisects \(\angle HGK\) and \(\angle H \cong \angle K\), use AAS to prove \(\triangle HGJ \cong \triangle KGJ\)

12. Given: \(\overline{AB} \cong \overline{DC}\), \(\overline{AB} \perp \overline{AC}\), \(\overline{DC} \perp \overline{DB}\)
Prove: \(\triangle ABC \cong \triangle DCB\)

13. Given: \(\overline{PQ} \parallel \overline{SR}\), \(\angle S \cong \angle Q\)
Prove: \(\overline{PS} \parallel \overline{QR}\)

14. Position a right triangle with legs 3 m and 4 m long in the coordinate plane.
Give the coordinates of each vertex.

15. Assign coordinates to each vertex and write a coordinate proof.
   Given: Square \(\overline{ABCD}\)
   Prove: \(\overline{AC} \cong \overline{BD}\)

Find each value.
16. \(y\) 
17. \(m\angle S\)

18. Given: Isosceles \(\triangle ABC\) has coordinates \(A(2a, 0), B(0, 2b),\) and \(C(-2a, 0)\).
    \(D\) is the midpoint of \(\overline{AC}\), and \(E\) is the midpoint of \(\overline{AB}\).
Prove: \(\triangle AED\) is isosceles.
Extra Practice ........................................ EPS2
Skills Practice ........................................ EPS2
Applications Practice .......................... EPA2

Problem-Solving Handbook .............. PS2
  Draw a Diagram ................................. PS2
  Make a Model ................................ PS3
  Guess and Test ................................. PS4
  Work Backward ............................... PS5
  Find a Pattern ................................. PS6
  Make a Table ................................. PS7
  Solve a Simpler Problem .............. PS8
  Use Logical Reasoning .................. PS9
  Use a Venn Diagram ..................... PS10
  Make an Organized List ............... PS11

Selected Answers .............................. SA2
Glossary ......................................... G1
Index ............................................. IN1
Draw a Diagram
You can draw a diagram to help you visualize what the words of a problem are describing.

**EXAMPLE**

A gardener wants to plant a 2.5-foot-wide border of flowers around a rectangular herb garden. The herb garden is 12 feet long and 7.5 feet wide. What is the area of the border?

1. **Understand the Problem**
   You need to find the area of the garden’s border. You are given the garden’s dimensions and the width of the border.

2. **Make a Plan**
   Draw and label a diagram of the herb garden with the surrounding border. Find the dimensions of the outer rectangle. Then find the area of the inner rectangle and subtract to find the area of the border.

3. **Solve**
   length of outer rectangle: $2.5\, \text{ft} + 12\, \text{ft} + 2.5\, \text{ft} = 17\, \text{ft}$
   width of outer rectangle: $2.5\, \text{ft} + 7.5\, \text{ft} + 2.5\, \text{ft} = 12.5\, \text{ft}$
   Find the area of each rectangle:
   area of outer rectangle: $17\, \text{ft} \times 12.5\, \text{ft} = 212.5\, \text{ft}^2$
   area of inner rectangle: $12\, \text{ft} \times 7.5\, \text{ft} = 90\, \text{ft}^2$
   Subtract:
   area of border: $212.5\, \text{ft}^2 - 90\, \text{ft}^2 = 122.5\, \text{ft}^2$

4. **Look Back**
   To check your answer, solve the problem in a different way. Divide the border into four parts and find the area of each part. Then add the areas.
   $17\, \text{ft} \times 2.5\, \text{ft} = 42.5\, \text{ft}^2$
   $7.5\, \text{ft} \times 2.5\, \text{ft} = 18.75\, \text{ft}^2$
   $17\, \text{ft} \times 2.5\, \text{ft} = 42.5\, \text{ft}^2$
   $7.5\, \text{ft} \times 2.5\, \text{ft} = 18.75\, \text{ft}^2$
   $42.5\, \text{ft}^2 + 42.5\, \text{ft}^2 + 18.75\, \text{ft}^2 + 18.75\, \text{ft}^2 = 122.5\, \text{ft}^2$

**PRACTICE**

1. A circular fish pond is surrounded by a circular border of stones that is 18 inches wide. The fish pond is 4 feet in diameter. What is the area of the border? (Use 3.14 for π.)

2. Thirty-two teams are in the first round of a softball tournament. A team is eliminated as soon as it loses a game. How many games need to be played to determine the winner? (*Hint:* Use a tree diagram.)
Mr. Duncan is using blue and white square tiles to create a pattern on his kitchen wall. The entire design will have 8 rows with 15 tiles in each row. The bottom row alternates colors starting with blue, and the row above that alternates colors starting with white. He will continue this alternating pattern so that the same two colors are never next to each other. How many of each color tile does Mr. Duncan need to complete the entire design?

1. Understand the Problem
   You need to find how many of each color tile are needed. You know the number of rows and the number of tiles in each row. The colors alternate so that the same two colors are never next to each other.

2. Make a Plan
   Use blocks (preferably blue and white, but any two colors would work) to make a model of the first two rows. Count how many of each color you use. Then multiply to find how many of each color would be used in the entire design.

3. Solve
   Create the bottom row. Start with a blue block and alternate colors across the row until you have used 15 blocks.
   
   ![Bottom Row Model]

   Create the row above the bottom row. Start with a white block.
   
   ![Row Above Bottom Model]

   There will be a total of 8 rows: 4 that start with blue and 4 that start with white. Count how many of each color are used above and multiply each number by 4.
   - blue: \(15 \times 4 = 60\)
   - white: \(15 \times 4 = 60\)

   Mr. Duncan needs 60 blue tiles and 60 white tiles.

4. Look Back
   The grid is 15 units by 8 units, so there are \(15 \times 8 = 120\) squares in the grid. Add the number of blue and white tiles to see if the sum is 120: \(60 + 60 = 120\).

**Practice**

1. Mr. Duncan decides to tile another area of his kitchen wall. This design will have 12 rows with 10 tiles in each row. The bottom row will repeat this pattern: blue, white, blue, blue, blue, white. The row above the bottom row will repeat this pattern: white, green, white, white, green, white. He will use these two patterns for each of the remaining rows so that the first colors of each row always alternate. How many of each color tile will Mr. Duncan need?
Guess and Test

The guess and test strategy can be used when you cannot think of another way to solve the problem. Begin by making a reasonable guess, and then test it to see whether your guess is correct. If not, adjust the guess accordingly and test again. Keep guessing and testing until you correctly solve the problem.

EXAMPLE

The manager of a college computer lab purchased 24 printers at a total cost of $3120. Some of the printers were laser, and some were ink jet. The laser printers cost $250 each, and the ink jet printers cost $70 each. How many of each type of printer did the manager purchase?

1. Understand the Problem
   You know the cost of each type of printer and the total number of printers. You need to find the number of each type of printer purchased.

2. Make a Plan
   Make reasonable first guesses for each type of printer. The sum must be 24. Then multiply each guess by the cost of each printer. Find the total and compare it to $3120. Adjust the guess as needed and continue until you find the solution.

3. Solve
   Use a table to organize your guesses.

<table>
<thead>
<tr>
<th>Laser Printers</th>
<th>Ink Jet Printers</th>
<th>Total Printers</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st guess</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>24</td>
<td>12($250) + 12($70)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$3000 + $840 = $3840</td>
</tr>
<tr>
<td>2nd guess</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>18</td>
<td>24</td>
<td>6($250) + 18($70)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$1500 + $1260 = $2760</td>
</tr>
<tr>
<td>3rd guess</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>16</td>
<td>24</td>
<td>8($250) + 16($70)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$2000 + $1120 = $3120</td>
</tr>
</tbody>
</table>

   The manager purchased 8 laser printers and 16 ink jet printers.

4. Look Back
   The total spent is $3120, and the total number of printers is 24. The solution is correct.

PRACTICE

1. All 350 seats were sold for a concert in the park. Adult tickets cost $15, and child tickets cost $5. Ticket sales totaled $4350. How many of each type of ticket were sold?

2. Jane is 3 times as old as Theo. Luke is 5 years older than Theo. Zoe is 8 years younger than twice Theo’s age, and Cassie is 6 years younger than Theo. The sum of their ages is 71. How old is each person?
Work Backward

You can work backward to solve a problem when you know the ending value and are asked to find the initial value.

**EXAMPLE**

Lee Ann is taking a vacation in Paris, France. Her flight arrived in Paris at 9:35 A.M. on Tuesday. The plane left New York City and flew for 7 hours and 55 minutes to Nice, France, where there was a layover of 1 hour 12 minutes. From Nice the plane flew 1 hour and 25 minutes to Paris. Paris time is 6 hours ahead of New York City time. What time did the plane leave New York City?

1. **Understand the Problem**
   You are asked to find the time that the plane left New York City. You know when the flight arrived in Paris, the length of the stops that were made along the way, and the time difference between New York City and Paris.

2. **Make a Plan**
   Work backward from the time the plane arrived in Paris, using inverse operations. Then apply the time difference between the two cities.

3. **Solve**
   Subtract the length of time it took to fly from Nice to Paris from the time Lee Ann arrived in Paris.
   
   \[
   9:35 \text{ A.M.} - 1 \text{ hour 25 minutes} = 8:10 \text{ A.M.}
   \]
   
   Subtract the length of the layover in Nice.
   
   \[
   8:10 \text{ A.M.} - 1 \text{ hour 12 minutes} = 6:58 \text{ A.M.}
   \]
   
   Subtract the length of the flight from New York to Nice.
   
   \[
   6:58 \text{ A.M.} - 7 \text{ hours 55 minutes} = 11:03 \text{ P.M. Monday}
   \]
   
   Since Paris time is ahead of New York time, subtract the time difference.
   
   \[
   11:03 \text{ P.M., Monday} - 6 \text{ hours} = 5:03 \text{ P.M. Monday}
   \]
   
   Lee Ann’s flight left New York City on Monday at 5:03 P.M.

4. **Look Back**
   Work forward to check your answer.
   
   \[
   5:03 \text{ P.M. Monday} + 6 \text{ h} + 7 \text{ h 55 min} + 1 \text{ h 12 min} + 1 \text{ h 25 min}
   = 5:03 \text{ P.M. Monday} + 16 \text{ h 32 min}
   = 9:35 \text{ A.M. Tuesday}
   \]
   
   This matches the information given in the problem.

**PRACTICE**

1. A bus arrives in Dallas, Texas, at 10:59 A.M. on Friday. The bus left Atlanta, Georgia, and took 12 hours and 15 minutes to arrive in Shreveport, Louisiana, where there was a 45-minute layover. From Shreveport it took 4 hours and 29 minutes to get to Dallas. Dallas time is 1 hour behind Atlanta time. What time did the bus leave Atlanta?

2. Carolina bought a DVD player that was on sale for 90% of the original price. The total amount she paid was $135.72, which included a sales tax of $5.22. What was the original price of the DVD player?
Find a Pattern
If a problem involves a sequence of numbers or figures, it is often necessary to find a pattern to solve the problem.

**EXAMPLE**

Darian created the following sequence of stars:

```
* *  
* * * *  
* * * * * *  
* * * * * * * * *  
* * * * * * * * * * * *  
```

How many stars will be in the 6th figure?

1. **Understand the Problem**
You need to find the number of stars in the 6th figure. You can find the number in the first four figures by counting.

2. **Make a Plan**
Count the number of stars in each of the first four figures. Use the information to find a pattern and determine a general rule.

3. **Solve**
Look for a pattern between the position of each figure in the sequence and the number of stars in that figure.

<table>
<thead>
<tr>
<th>Position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stars</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

The number of stars is the square of the position number plus the position number. This rule written algebraically is \( n^2 + n \).

Evaluate the expression for \( n = 6 \):

\[
6^2 + 6 = 36 + 6 = 42
\]

There will be 42 stars in the 6th figure.

4. **Look Back**
Look for another pattern. The number of stars in each position increases by 4, then by 6, then by 8. That is, the amount of increase always increases by 2. So the number of stars in the 5th position will be 20 + 10, or 30, and the number of stars in the 6th position will be 30 + 12, or 42.

**PRACTICE**

1. The first three figures of a pattern are shown. How many circles will be in the 10th figure?

2. Lily drew the first four figures of a pattern. How many squares will be in the 7th figure?
Make a Table

You can make a table to solve problems because the rows and columns can help you arrange information. Sometimes this also allows you to discover relationships that might otherwise be hard to notice.

EXAMPLE

A scientist begins a culture with 500 bacteria. The number of bacteria triples every 30 minutes. How many bacteria are in the culture after 2 1/2 hours?

1. Understand the Problem

You are asked to find the number of bacteria in the culture after 2 1/2 hours. You know the initial number of bacteria, and you know that the population triples every half hour.

2. Make a Plan

Make a table with rows for time and number of bacteria. Start with the initial number in the culture. Increase the time in 30-minute increments and triple the number of bacteria with each increase. Keep extending the table until the time is 2 1/2 hours (150 minutes).

3. Solve

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bacteria</td>
<td>500</td>
<td>1500</td>
<td>4500</td>
<td>13,500</td>
<td>40,500</td>
<td>121,500</td>
</tr>
</tbody>
</table>

There are 121,500 bacteria in the culture after 2 1/2 hours.

4. Look Back

Check your answer by solving a simpler problem. The number of bacteria in the culture triples five times (150 min ÷ 30 min = 5). Start with 5 instead of 500 and triple the number five times.

- 5 × 3 = 15
- 15 × 3 = 45
- 45 × 3 = 135
- 135 × 3 = 405
- 405 × 3 = 1215

Multiply by 100 to find the total if you had started with 500; 1215 × 100 = 121,500

PRACTICE

1. A dietician’s report states that a 125-pound woman needs to eat about 1750 Calories a day to maintain her weight. It also states that a 132-pound woman needs 1848 Calories and a 139-pound woman needs 1946 Calories a day. Based on these values, how many Calories does a 160-pound woman need to eat each day to maintain her weight?

2. Simon opened a savings account with an initial deposit of $200. At the end of each year, the account earns 4% interest. If he does not deposit or withdraw any additional money, what would his balance be at the end of 6 years?
**Solve a Simpler Problem**

Sometimes a problem contains numbers that make it seem difficult to solve. You can solve a simpler problem by rewriting the numbers so they are easier to compute.

---

**EXAMPLE**

During a skating competition, Jules skated around the track 35 times. One lap is 0.9 mile. If Jules finished in 1 hour 30 minutes, what was his average speed?

1. **Understand the Problem**
   
   You are asked to find Jules’ s average speed for 35 laps. You know the distance of each lap and the amount of time it took him to finish the competition.

2. **Make a Plan**
   
   Solve a simpler problem by using easier numbers to do the computations.

3. **Solve**
   
   Find the total distance skated.

   \[ 35(0.9) \]
   \[ 35(1 - 0.1) \]
   \[ 35(1) - 3.5(0.1) \]
   \[ 35 - 3.5 \]
   \[ 31.5 \]

   Use the distance formula to find the average speed.

   \[ d = rt \]
   \[ 31.5 = r \times 1.5 \]
   \[ 31.5 = 1.5r \]
   \[ 31.5 \div 1.5 = r \]
   \[ 21 = r \]

   Jules skated at an average speed of 21 miles per hour.

4. **Look Back**
   
   Each lap is a little less than 1 mile, so 35 laps is a little less than 35 miles. Round this distance to 30 miles and use \( d = rt \) to find the rate when the time is 1.5 hours:

   \[ 30 \text{ mi} = (1.5 \text{ h})r \rightarrow r = 20 \text{ mi/h}. \] This is close to 21 mi/h.

---

**PRACTICE**

1. Diana swam 24 laps in the pool today. One lap is 200 feet. She swam at an average rate of 4 feet per second. How many minutes did Diana swim?
Use Logical Reasoning

Use logical reasoning to solve problems when you are given several facts and can use common sense to find a missing fact.

Example 1

Five players on a baseball team wear the numbers 2, 12, 15, 34, and 42. Their positions are pitcher, catcher, first base, left field, and center field. The pitcher's number is less than the left fielder's number. The center fielder's number is greater than 25, and the left fielder wears an even number. The catcher wears number 34. What is the pitcher's number?

1. Understand the Problem
   You want to find the jersey number of the pitcher. You know there are five positions and five jersey numbers. Some information about who wears which number is given.

2. Make a Plan
   Organize the information in a table. Start with the fact that the catcher wears number 34 and use logical reasoning to determine the numbers of the other positions.

3. Solve
   The catcher wears number 34. No other player wears 34, and the catcher wears no other number. Enter Y's and N's in the chart as shown.

   The center fielder's number is greater than 25, so he must wear number 42. The left fielder cannot wear number 15 (because it is odd), and he cannot have the least number (the pitcher's number is less than his). The left fielder must wear 12.

   The pitcher's number is less than 12 (the left fielder's), so he must wear number 2.

   The pitcher wears number 2.

4. Look Back
   Complete the chart if needed. Read the problem while looking at the chart to make sure there are no contradictions.

Practice

1. Rose, Jill, Gaby, and Chloe bowled the scores 110, 125, 144, and 150. Jill did not bowl the 110. The person who bowled the 150 is Rose's sister and Jill's aunt. Chloe bowled the 125. What score did Jill bowl?
Use a Venn Diagram

You can use a Venn diagram to display relationships among sets of numbers. Circles are used to represent the individual sets.

**Example**

At a local supermarket, 194 people were given samples of two brands of orange juice. Their opinions were as follows: 120 people liked brand A, 101 people liked brand B, and 15 people did not like either brand. How many people liked only brand A?

1. **Understand the Problem**
   
The total number of people was 194, and 15 of them did not like either brand. The statement “120 people liked brand A” means some of the 120 people liked only brand A and some liked brand A and brand B. The statement “101 people liked brand B” means some of the 101 people liked only brand B and some liked brand A and brand B.

2. **Make a Plan**
   
   Use a Venn diagram to show the relationship among the groups of people.

3. **Solve**
   
   Draw and label two intersecting circles to show the sets of people who liked brand A and brand B. Write 15 in the area labeled “Neither.”
   
   Out of 194 people, 15 liked neither brand. Subtract 15 from 194 to find how many people liked at least one brand: 194 − 15 = 179.
   
   Add the number of people who liked brand A to the number of people who liked brand B: 120 + 101 = 221. You know there are only 179 people who liked at least one brand, so subtract 179 from 221: 221 − 179 = 42. This means 42 people were counted twice, and that 42 people liked both brands. Write 42 in the area labeled both.
   
   Out of 120 people who liked brand A, 42 also liked brand B. Subtract 42 from 120 to find the number of people who liked only brand A: 120 − 42 = 78.
   
   So 78 people liked only brand A.

4. **Look Back**
   
   Find the number of people who liked brand B only: 101 − 42 = 59. Add all the numbers in the Venn diagram. The sum of the number who liked only brand A, the number who liked only brand B, the number who liked both brands, and the number who liked neither brand should be the total number of people surveyed: 78 + 59 + 42 + 15 = 194.

**Practice**

In a group of 138 people, 55 own a cat, 27 own a cat and a dog, and 42 own neither pet.

1. How many people own only a cat?
2. How many people own a dog?
Make an Organized List

If a problem asks you to find all the possible ways in which something can happen, you can make an organized list to keep track of the outcomes.

**Example**

A fair coin is tossed 4 times. What is the probability that it lands heads up at least 3 times?

1. **Understand the Problem**
   You need to find the probability that a coin tossed 4 times lands heads up 3 or 4 times.

2. **Make a Plan**
   The formula for probability is:
   \[
   \text{probability} = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}
   \]
   The total number of outcomes is the number of items in the list. The number of favorable outcomes is the number of times the coin lands heads up 3 or 4 times. Make an organized list of the coin tosses to find the total number of outcomes.

3. **Solve**
   Start with heads for all 4 tosses, then heads for the first 3 tosses, then heads for the first 2 tosses, and then heads for the first toss. Repeat the pattern for tails.

<table>
<thead>
<tr>
<th>HHHH</th>
<th>HTHH</th>
<th>TTHT</th>
<th>THTH</th>
</tr>
</thead>
<tbody>
<tr>
<td>HHTH</td>
<td>HTHT</td>
<td>TTHH</td>
<td>THHT</td>
</tr>
<tr>
<td>HHTH</td>
<td>HHTT</td>
<td>TTHH</td>
<td>THHH</td>
</tr>
</tbody>
</table>

   There are 16 total outcomes. There are 5 favorable outcomes.

4. **Look Back**
   Double-check that each combination is listed and that no combination is written more than once. You can also use the Fundamental Counting Principle to check the total number of outcomes. For each of the 4 coin tosses, there are 2 possible outcomes, so the total number of outcomes is \(2 \times 2 \times 2 \times 2 = 16\).

**Practice**

1. A beagle, a fox terrier, an Afghan hound, and a golden retriever are competing in the finals of a dog show. How many ways can the dogs finish in first, second, and third place?
2. Two number cubes are rolled. What is the probability that the sum of the numbers rolled is an odd number?
### ENGLISH

**absolute value**  The absolute value of \( x \) is the distance from zero to \( x \) on a number line, denoted \( |x| \).

\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
- x & \text{if } x < 0 
\end{cases}
\]

**absolute-value equation**  An equation that contains absolute-value expressions.

**absolute-value function**  A function whose rule contains absolute-value expressions.

**absolute-value inequality**  An inequality that contains absolute-value expressions.

**accuracy**  The closeness of a given measurement or value to the actual measurement or value.

**acute angle**  An angle that measures greater than 0° and less than 90°.

**acute triangle**  A triangle with three acute angles.

**Addition Property of Equality**  For real numbers \( a \), \( b \), and \( c \), if \( a = b \), then \( a + c = b + c \).

\[
x - 6 = 8 \\
+ 6 \\
\hline
x = 14
\]

**Addition Property of Inequality**  For real numbers \( a \), \( b \), and \( c \), if \( a < b \), then \( a + c < b + c \). Also holds true for \( >, \leq, \geq, \text{ and } \neq \).

\[
x - 6 < 8 \\
+ 6 \\
\hline
x < 14
\]

**additive inverse**  The opposite of a number. Two numbers are additive inverses if their sum is zero.

**SPANISH**

**valor absoluto**  El valor absoluto de \( x \) es la distancia de cero a \( x \) en una recta numérica, y se expresa \( |x| \).

\[
|x| = \begin{cases} 
  x & \text{si } x \geq 0 \\
- x & \text{si } x < 0 
\end{cases}
\]

**ecuación de valor absoluto**  Ecuación que contiene expresiones de valor absoluto.

**función de valor absoluto**  Función cuya regla contiene expresiones de valor absoluto.

**desigualdad de valor absoluto**  Desigualdad que contiene expresiones de valor absoluto.

**exactitud**  Cercanía de una medida o un valor a la medida o el valor real.

**ángulo agudo**  Ángulo que mide más de 0° y menos de 90°.

**triángulo acutángulo**  Triángulo con tres ángulos agudos.

**Propiedad de igualdad de la suma**  Dados los números reales \( a \), \( b \) y \( c \), si \( a = b \), entonces \( a + c = b + c \).

**Propiedad de desigualdad de la suma**  Dados los números reales \( a \), \( b \) y \( c \), si \( a < b \), entonces \( a + c < b + c \). Es válido también para \( >, \leq, \geq \), y \( \neq \).

**inverso aditivo**  El opuesto de un número. Dos números son inversos aditivos si su suma es cero.

**EXAMPLES**

\[
|3| = 3 \\
|-3| = 3
\]

\[
|x + 4| = 7
\]

\[
y = |x + 4|
\]

\[
|x + 4| > 7
\]

The additive inverse of 5 is −5.

The additive inverse of −5 is 5.
<table>
<thead>
<tr>
<th><strong>ENGLISH</strong></th>
<th><strong>SPANISH</strong></th>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AND</strong> A logical operator representing the intersection of two sets.</td>
<td><strong>Y</strong> Operador lógico que representa la intersección de dos conjuntos.</td>
<td>$A = {2, 3, 4, 5}$ $B = {1, 3, 5, 7}$ The set of values that are in $A$ AND $B$ is $A \cap B = {3, 5}$.</td>
</tr>
<tr>
<td><strong>angle</strong> A figure formed by two rays with a common endpoint.</td>
<td><strong>ángulo</strong> Figura formada por dos rayos con un extremo común.</td>
<td></td>
</tr>
<tr>
<td><strong>area</strong> The number of nonoverlapping unit squares of a given size that will exactly cover the interior of a plane figure.</td>
<td><strong>área</strong> Cantidad de cuadrados unitarios de un determinado tamaño no superpuestos que cubren exactamente el interior de una figura plana.</td>
<td></td>
</tr>
<tr>
<td><strong>arithmetic sequence</strong> A sequence whose successive terms differ by the same nonzero number $d$, called the common difference.</td>
<td><strong>sucesión aritmética</strong> Sucesión cuyos términos sucesivos difieren en el mismo número distinto de cero $d$, denominado diferencia común.</td>
<td>$4, 7, 10, 13, 16, \ldots$ $\frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{3}{3} = 3$ $(5 + 3) + 7 = 5 + (3 + 7)$ $(5 \cdot 3) \cdot 7 = 5 \cdot (3 \cdot 7)$</td>
</tr>
<tr>
<td><strong>Associative Property of Addition</strong> For all numbers $a$, $b$, and $c$, $(a + b) + c = a + (b + c)$.</td>
<td><strong>Propiedad asociativa de la suma</strong> Dados tres números cualesquiera $a$, $b$ y $c$, $(a + b) + c = a + (b + c)$.</td>
<td></td>
</tr>
<tr>
<td><strong>Associative Property of Multiplication</strong> For all numbers $a$, $b$, and $c$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.</td>
<td><strong>Propiedad asociativa de la multiplicación</strong> Dados tres números cualesquiera $a$, $b$ y $c$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.</td>
<td></td>
</tr>
<tr>
<td><strong>asymptote</strong> A line that a graph gets closer to as the value of a variable becomes extremely large or small.</td>
<td><strong>asíntota</strong> Línea recta a la cual se aproxima una gráfica a medida que el valor de una variable se hace sumamente grande o pequeño.</td>
<td></td>
</tr>
<tr>
<td><strong>average</strong> See mean.</td>
<td><strong>promedio</strong> Ver media.</td>
<td></td>
</tr>
<tr>
<td><strong>axis of a coordinate plane</strong> One of two perpendicular number lines, called the $x$-axis and the $y$-axis, used to define the location of a point in a coordinate plane.</td>
<td><strong>eje de un plano cartesiano</strong> Una de las dos rectas numéricas perpendiculares, denominadas eje $x$ y eje $y$, utilizadas para definir la ubicación de un punto en un plano cartesiano.</td>
<td></td>
</tr>
<tr>
<td><strong>axis of symmetry</strong> A line that divides a plane figure or a graph into two congruent reflected halves.</td>
<td><strong>eje de simetría</strong> Línea que divide una figura plana o una gráfica en dos mitades reflejadas congruentes.</td>
<td></td>
</tr>
</tbody>
</table>
**ENGLISH**

**back-to-back stem-and-leaf plot**  (A graph used to organize and compare two sets of data so that the frequencies can be compared. *See also* stem-and-leaf plot.

**bar graph**  A graph that uses vertical or horizontal bars to display data.

**base of a power**  The number in a power that is used as a factor.

**base of an exponential function**  The value of \( b \) in a function of the form \( f(x) = ab^x \), where \( a \) and \( b \) are real numbers with \( a \neq 0 \), \( b > 0 \), and \( b \neq 1 \).

**biased sample**  A sample that does not fairly represent the population.

**binomial**  A polynomial with two terms.

**boundary line**  A line that divides a coordinate plane into two half-planes.

---

**SPANISH**

**diagrama doble de tallo y hojas**  Gráfica utilizada para organizar y comparar dos conjuntos de datos para poder comparar las frecuencias. *Ver también* diagrama de tallo y hojas.

**gráfica de barras**  Gráfica con barras horizontales o verticales para mostrar datos.

**base de una potencia**  Número de una potencia que se utiliza como factor.

**base de una función exponencial**  Valor de \( b \) en una función del tipo \( f(x) = ab^x \), donde \( a \) y \( b \) son números reales con \( a \neq 0 \), \( b > 0 \) y \( b \neq 1 \).

**muestra no representativa**  Muestra que no representa adecuadamente una población.

**binomio**  Polinomio con dos términos.

**línea de limite**  Línea que divide un plano cartesiano en dos semiplanos.

---

**EXAMPLES**

**Data set A:** 9, 12, 14, 16, 23, 27  
**Data set B:** 6, 8, 10, 13, 15, 16, 21

<table>
<thead>
<tr>
<th>Set A</th>
<th>Set B</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>6 4 2</td>
<td>1 0 3 5 6</td>
</tr>
<tr>
<td>7 3</td>
<td>2 1</td>
</tr>
</tbody>
</table>

*Key:*  \( |2| \) means 21  
\( |7| \) means 27

\[ 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81 \]

3 is the base.

In the function \( f(x) = 5(2)^x \), the base is 2.

To find out about the exercise habits of average Americans, a fitness magazine surveyed its readers about how often they exercise. The population is all Americans and the sample is readers of the fitness magazine. This sample will likely be biased because readers of fitness magazines may exercise more often than other people do.

\[
\begin{align*}
x + y &= 2a^2 + 3 \\
4m^3n^2 + 6mn^4 &= 0
\end{align*}
\]
**ENGLISH**

**box-and-whisker plot**  A method of showing how data are distributed by using the median, quartiles, and minimum and maximum values; also called a *box plot*.

**SPANISH**

**gráfica de mediana y rango**  Método para mostrar la distribución de datos utilizando la mediana, los cuartiles y los valores mínimo y máximo; también llamado *gráfica de caja*.

---

**Cartesian coordinate system**  See coordinate plane.

**sistema de coordenadas cartesianas**  Ver plano cartesiano.

**center of a circle**  The point inside a circle that is the same distance from every point on the circle.

**centro de un círculo**  Punto dentro de un círculo que se encuentra a la misma distancia de todos los puntos del círculo.

**central angle of a circle**  An angle whose vertex is the center of a circle.

**ángulo central de un círculo**  Ángulo cuyo vértice es el centro de un círculo.

**circle**  The set of points in a plane that are a fixed distance from a given point called the center of the circle.

**círculo**  Conjunto de puntos en un plano que se encuentran a una distancia fija de un punto determinado denominado centro del círculo.

**circle graph**  A way to display data by using a circle divided into non-overlapping sectors.

**gráfica circular**  Forma de mostrar datos mediante un círculo dividido en sectores no superpuestos.

**circumference**  The distance around a circle.

**circunferencia**  Distancia alrededor de un círculo.

**closure**  A set of numbers is said to be closed, or to have closure, under a given operation if the result of the operation on any two numbers in the set is also in the set.

**cerradura**  Se dice que un conjunto de números es cerrado, o tiene cerradura, respecto de una operación determinada, si el resultado de la operación entre dos números cualesquiera del conjunto también está en el conjunto.

The natural numbers are closed under addition because the sum of two natural numbers is always a natural number.

**coefficient**  A number that is multiplied by a variable.

**coeficiente**  Número que se multiplica por una variable.

In the expression $2x + 3y$, 2 is the coefficient of $x$ and 3 is the coefficient of $y$. 
ENGLISH | SPANISH | EXAMPLES
--- | --- | ---
**common difference** In an arithmetic sequence, the nonzero constant difference of any term and the previous term. | **diferencia común** En una sucesión aritmética, diferencia constante distinta de cero entre cualquier término y el término anterior. | In the arithmetic sequence 3, 5, 7, 9, 11, …, the common difference is 2.

**common factor** A factor that is common to all terms of an expression or to two or more expressions. | **factor común** Factor que es común a todos los términos de una expresión o a dos o más expresiones. | Expression: $4x^2 + 16x^3 - 8x$
Common factor: 4x
Expressions: 12 and 18
Common factors: 2, 3, and 6

**common ratio** In a geometric sequence, the constant ratio of any term and the previous term. | **razón común** En una sucesión geométrica, la razón constante entre cualquier término y el término anterior. | In the geometric sequence 32, 16, 8, 4, 2, …, the common ratio is $\frac{1}{2}$.

**Commutative Property of Addition** For any two numbers $a$ and $b$, $a + b = b + a$. | **Propiedad conmutativa de la suma** Dados dos números cualesquiera $a$ y $b$, $a + b = b + a$. | $3 + 4 = 4 + 3 = 7$

**Commutative Property of Multiplication** For any two numbers $a$ and $b$, $a \cdot b = b \cdot a$. | **Propiedad conmutativa de la multiplicación** Dados dos números cualesquiera $a$ y $b$, $a \cdot b = b \cdot a$. | $3 \cdot 4 = 4 \cdot 3 = 12$

**complement of an event** The set of all outcomes that are not the event. | **complemento de un suceso** Todos los resultados que no están en el suceso. | In the experiment of rolling a number cube, the complement of rolling a 3 is rolling a 1, 2, 4, 5, or 6.

**complementary angles** Two angles whose measures have a sum of 90°. | **ángulos complementarios** Dos ángulos cuyas medidas suman 90°. | \[ \begin{align*}
\angle A &= 53° \\
\angle B &= 37°
\end{align*} \]

**completing the square** A process used to form a perfect-square trinomial. To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$. | **completar el cuadrado** Proceso utilizado para formar un trinomio cuadrado perfecto. Para completar el cuadrado de $x^2 + bx$, hay que sumar $\left(\frac{b}{2}\right)^2$. | $x^2 + 6x + \square$
Add $\left(\frac{6}{2}\right)^2 = 9$.
x$^2 + 6x + 9$

**complex fraction** A fraction that contains one or more fractions in the numerator, the denominator, or both. | **fracción compleja** Fracción que contiene una o más fracciones en el numerador, en el denominador, o en ambos. | $\frac{1}{2}$

$\frac{1}{1 + \frac{2}{3}}$

**composite figure** A plane figure made up of triangles, rectangles, trapezoids, circles, and other simple shapes, or a three-dimensional figure made up of prisms, cones, pyramids, cylinders, and other simple three-dimensional figures. | **figura compuesta** Figura plana compuesta por triángulos, rectángulos, trapezios, círculos y otras figuras simples, o figura tridimensional compuesta por prismas, conos, pirámides, cilindros y otras figuras tridimensionales simples. |
ENGLISH | SPANISH | EXAMPLES
--- | --- | ---
**compound event** An event made up of two or more simple events. | **sceso compuesto** Suceso formado por dos o más sucesos simples. | In the experiment of tossing a coin and rolling a number cube, the event of the coin landing heads and the number cube landing on 3.

**compound inequality** Two inequalities that are combined into one statement by the word and or or. | **desigualdad compuesta** Dos desigualdades unidas en un enunciado por la palabra y o a. | $x \geq 2$ AND $x < 7$ (also written $2 \leq x < 7$)

$x < 2$ OR $x > 6$

If $100 is put into an account with an interest rate of 5% compounded monthly, then after 2 years, the account will have $100\left(1 + \frac{0.05}{12}\right)^{12\cdot2} = $110.49.

**compound interest** Interest earned or paid on both the principal and previously earned interest. The formula for compound interest is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where $A$ is the final amount, $P$ is the principal, $r$ is the interest rate expressed as a decimal, $n$ is the number of times interest is compounded, and $t$ is the time. | **interés compuesto** Intereses ganados o pagados sobre el capital y los intereses ya devengados. La fórmula de interés compuesto es $A = P\left(1 + \frac{r}{n}\right)^{nt}$, donde $A$ es la cantidad final, $P$ es el capital, $r$ es la tasa de interés expresada como un decimal, $n$ es la cantidad de veces que se capitaliza el interés y $t$ es el tiempo. | The sky is blue and the grass is green.

I will drive to school or I will take the bus.

**compound statement** Two statements that are connected by the word and or or. | **enunciado compuesto** Dos enunciados unidos por la palabra y o a. | The conjugate of $1 + \sqrt{2}$ is $1 - \sqrt{2}$.

**constant** A value that does not change. | **constante** Valor que no cambia. | $3, 0, \pi$

---

**cone** A three-dimensional figure with a circular base and a curved surface that connects the base to a point called the vertex. | **cono** Figura tridimensional con una base circular y una superficie lateral curva que conecta la base con un punto denominado vértice.

---

**consistent system** A system of equations or inequalities that has at least one solution. | **sistema consistente** Sistema de ecuaciones o desigualdades que tiene por lo menos una solución. | $\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$

solution: $(5, 1)$
<table>
<thead>
<tr>
<th><strong>ENGLISH</strong></th>
<th><strong>SPANISH</strong></th>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>constant of variation</strong> The constant ( k ) in direct and inverse variation equations.</td>
<td><strong>constante de variación</strong> La constante ( k ) en ecuaciones de variación directa e inversa.</td>
<td>( y = 5x )</td>
</tr>
<tr>
<td><strong>continuous graph</strong> A graph made up of connected lines or curves.</td>
<td><strong>gráfica continua</strong> Gráfica compuesta por líneas rectas o curvas conectadas.</td>
<td></td>
</tr>
</tbody>
</table>

| **convenience sample** A sample based on members of the population that are readily available. | **muestra de conveniencia** Una muestra basada en miembros de la población que están fácilmente disponibles. | A reporter surveys people he personally knows. |
| **conversion factor** The ratio of two equal quantities, each measured in different units. | **factor de conversión** Razón entre dos cantidades iguales, cada una medida en unidades diferentes. | \( \frac{12 \text{ inches}}{1 \text{ foot}} \) |
| **coordinate plane** A plane that is divided into four regions by a horizontal line called the \( x \)-axis and a vertical line called the \( y \)-axis. | **plano cartesiano** Plano dividido en cuatro regiones por una línea horizontal denominada eje \( x \) y una línea vertical denominada eje \( y \). | 

| **correlation** A measure of the strength and direction of the relationship between two variables or data sets. | **correlación** Medida de la fuerza y dirección de la relación entre dos variables o conjuntos de datos. | 

| **correlation coefficient** A number \( r \), where \(-1 \leq r \leq 1\), that describes how closely the points in a scatter plot cluster around the least-squares line. | **coeficiente de correlación** Número \( r \), donde \(-1 \leq r \leq 1\), que describe a qué distancia de la recta de mínimos cuadrados se agrupan los puntos de un diagrama de dispersión. | An \( r \)-value close to 1 describes a strong positive correlation. An \( r \)-value close to 0 describes a weak correlation or no correlation. An \( r \)-value close to \(-1\) describes a strong negative correlation. |
| **corresponding angles of polygons** Angles in the same relative position in polygons with an equal number of angles. | **ángulos correspondientes de los polígonos** Ángulos que se ubican en la misma posición relativa en polígonos que tienen el mismo número de ángulos. | \( \angle A \) and \( \angle D \) are corresponding angles. |
corresponding sides of polygons  
Sides in the same relative position in polygons with an equal number of sides.

cosine  
In a right triangle, the cosine of angle \( A \) is the ratio of the length of the leg adjacent to angle \( A \) to the length of the hypotenuse.

cross products  
In the statement \( \frac{a}{b} = \frac{c}{d} \), \( bc \) and \( ad \) are the cross products.

cube  
A prism with six square faces.

cube in numeration  
The third power of a number.

cube root  
A number, written as \( \sqrt[3]{x} \), whose cube is \( x \).

cubic equation  
An equation that can be written in the form \( ax^3 + bx^2 + cx + d = 0 \), where \( a, b, c, \) and \( d \) are real numbers and \( a \neq 0 \).

cubic function  
A function that can be written in the form \( f(x) = ax^3 + bx^2 + cx + d \), where \( a, b, c, \) and \( d \) are real numbers and \( a \neq 0 \).

cubic polynomial  
A polynomial of degree 3.

cumulative frequency  
The frequency of all data values that are less than or equal to a given value.

**Examples**

\( A \) \( B \) \( C \) \( D \) \( E \) \( F \)

\( \overline{AB} \) and \( \overline{DE} \) are corresponding sides.

coseno  
En un triángulo rectángulo, el coseno del ángulo \( A \) es la razón entre la longitud del cateto adyacente al ángulo \( A \) y la longitud de la hipotenusa.

products cruzados  
En el enunciado \( \frac{a}{b} = \frac{c}{d} \), \( bc \) y \( ad \) son productos cruzados.

Propiedad de productos cruzados  
Dados los números reales \( a, b, c, \) y \( d \), donde \( b \neq 0 \) y \( d \neq 0 \), si \( \frac{a}{b} = \frac{c}{d} \), entonces \( ad = bc \).

If \( \frac{4}{6} = \frac{10}{x} \), then \( 4x = 60 \), so \( x = 15 \).

8 is the cube of 2.

\( \sqrt[3]{64} = 4 \), because \( 4^3 = 64 \); 4 is the cube root of 64.

\( 4x^3 + x^2 - 3x - 1 = 0 \)

\( f(x) = x^3 + 2x^2 - 6x + 8 \)

\( x^3 + 4x^2 - 6x + 2 \)

<table>
<thead>
<tr>
<th>Data</th>
<th>Frequency</th>
<th>Cumulative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>ENGLISH</td>
<td>SPANISH</td>
<td>EXAMPLES</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td><strong>cylinder</strong> A three-dimensional figure with two parallel congruent</td>
<td><strong>cilindro</strong> Figura tridimensional con dos bases circulares congruentes</td>
<td></td>
</tr>
<tr>
<td>circular bases and a curved surface that connects the bases.</td>
<td>paralelas y una superficie lateral curva que conecta las bases.</td>
<td></td>
</tr>
<tr>
<td><strong>data</strong> Information gathered from a survey or experiment.</td>
<td><strong>datos</strong> Información reunida en una encuesta o experimento.</td>
<td></td>
</tr>
<tr>
<td><strong>degree measure of an angle</strong> A unit of angle measure; one degree is</td>
<td><strong>medida en grados de un ángulo</strong> Unidad de medida de los ángulos; un</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{360} ) of a circle.</td>
<td>grado es ( \frac{1}{360} ) de un círculo.</td>
<td></td>
</tr>
<tr>
<td><strong>degree of a monomial</strong> The sum of the exponents of the variables in</td>
<td><strong>grado de un monomio</strong> Suma de los exponentes de las variables del</td>
<td></td>
</tr>
<tr>
<td>the monomial.</td>
<td>monomio.</td>
<td></td>
</tr>
<tr>
<td><strong>degree of a polynomial</strong> The degree of the term of the polynomial</td>
<td><strong>grado de un polinomio</strong> Grado del término del polinomio con el grado</td>
<td></td>
</tr>
<tr>
<td>with the greatest degree.</td>
<td>máximo.</td>
<td></td>
</tr>
<tr>
<td><strong>dependent events</strong> Events for which the occurrence or nonoccurrence</td>
<td><strong>sucesos dependientes</strong> Dos sucesos son dependientes si el hecho de</td>
<td></td>
</tr>
<tr>
<td>of one event affects the probability of the other event.</td>
<td>que uno de ellos ocurra o no afecta la probabilidad del otro suceso.</td>
<td></td>
</tr>
<tr>
<td><strong>dependent system</strong> A system of equations that has infinitely many</td>
<td><strong>sistema dependiente</strong> Sistema de ecuaciones que tiene infinitamente</td>
<td></td>
</tr>
<tr>
<td>solutions.</td>
<td>muchas soluciones.</td>
<td></td>
</tr>
<tr>
<td><strong>dependent variable</strong> The output of a function; a variable whose value</td>
<td><strong>variable dependiente</strong> Salida de una función; variable cuyo valor</td>
<td></td>
</tr>
<tr>
<td>depends on the value of the input, or independent variable.</td>
<td>depende del valor de la entrada, o variable independiente.</td>
<td></td>
</tr>
<tr>
<td><strong>diameter</strong> A segment that has endpoints on the circle and that</td>
<td><strong>diámetro</strong> Segmento que atraviesa el centro de un círculo y cuyos</td>
<td></td>
</tr>
<tr>
<td>passes through the center of the circle; also the length of that</td>
<td>extremos están sobre la circunferencia; longitud de dicho segmento.</td>
<td></td>
</tr>
<tr>
<td>segment.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>difference of two cubes</strong> A polynomial of the form ( a^3 - b^3 ),</td>
<td><strong>diferencia de dos cubos</strong> Polinomio del tipo ( a^3 - b^3 ), que</td>
<td></td>
</tr>
<tr>
<td>which may be written as the product ( (a - b)(a^2 + ab + b^2) ).</td>
<td>se puede expresar como el producto ( (a - b)(a^2 + ab + b^2) ).</td>
<td></td>
</tr>
<tr>
<td><strong>difference of two squares</strong> A polynomial of the form ( a^2 - b^2 ),</td>
<td><strong>diferencia de dos cuadrados</strong> Polinomio del tipo ( a^2 - b^2 ), que</td>
<td></td>
</tr>
<tr>
<td>which may be written as the product ( (a + b)(a - b) ).</td>
<td>se puede expresar como el producto ( (a + b)(a - b) ).</td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\text{data} & \quad \text{Information gathered from a survey or experiment.} \\
\text{dependence} & \quad \text{The relationship of one variable to another variable.} \\
\text{dependent} & \quad \text{Related to or determined by another variable.} \\
\text{dependent variable} & \quad \text{The output of a function; a variable whose value depends on the value of the input, or independent variable.} \\
\text{degree of a polynomial} & \quad \text{The degree of the term of the polynomial with the greatest degree.} \\
\text{degree measure of an angle} & \quad \text{A unit of angle measure; one degree is } \frac{1}{360} \text{ of a circle.} \\
\text{dependent events} & \quad \text{Events for which the occurrence or nonoccurrence of one event affects the probability of the other event.} \\
\text{dependent system} & \quad \text{A system of equations that has infinitely many solutions.} \\
\text{difference of two cubes} & \quad \text{A polynomial of the form } a^3 - b^3, \text{ which may be written as the product } (a - b)(a^2 + ab + b^2). \\
\text{difference of two squares} & \quad \text{A polynomial of the form } a^2 - b^2, \text{ which may be written as the product } (a + b)(a - b). \\
\end{align*}
\]
### ENGLISH

**Dimensional Analysis** A process that uses rates to convert measurements from one unit to another.

**Direct Variation** A linear relationship between two variables, \( x \) and \( y \), that can be written in the form \( y = kx \), where \( k \) is a nonzero constant.

**Discontinuous Function** A function whose graph has one or more jumps, breaks, or holes.

**Discount** An amount by which an original price is reduced.

**Discrete Graph** A graph made up of unconnected points.

**Discriminant** The discriminant of the quadratic equation \( ax^2 + bx + c = 0 \) is \( b^2 - 4ac \).

**Distance Formula** In a coordinate plane, the distance from \((x_1, y_1)\) to \((x_2, y_2)\) is \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

**Distributive Property** For all real numbers \( a, b \), and \( c \), \( a(b + c) = ab + ac \), and \((b + c)a = ba + ca\).

**Division Property of Equality** For real numbers \( a, b \), and \( c \), where \( c \neq 0 \), if \( a = b \), then \( \frac{a}{c} = \frac{b}{c} \).

### SPANISH

**Análisis dimensional** Un proceso que utiliza tasas para convertir medidas de unidad a otra.

**Variación Directa** Relación lineal entre dos variables, \( x \) y \( y \), que puede expresarse en la forma \( y = kx \), donde \( k \) es una constante distinta de cero.

**Función Discontinua** Función cuya gráfica tiene uno o más saltos, interrupciones u hoyos.

**Descuento** Cantidad por la que se reduce un precio original.

**Gráfica Discreta** Gráfica compuesta de puntos no conectados.

**Discriminante** El discriminante de la ecuación cuadrática \( ax^2 + bx + c = 0 \) es \( b^2 - 4ac \).

**Fórmula de distancia** En un plano cartesiano, la distancia desde \((x_1, y_1)\) hasta \((x_2, y_2)\) es \( d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

**Propiedad distributiva** Dados los números reales \( a, b \) y \( c \), \( a(b + c) = ab + ac \), y \( (b + c)a = ba + ca \).

**Propiedad de igualdad de la división** Dados los números reales \( a, b \) y \( c \), donde \( c \neq 0 \), si \( a = b \), entonces \( \frac{a}{c} = \frac{b}{c} \).

### EXAMPLES

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENGLISH</td>
<td>SPANISH</td>
</tr>
<tr>
<td>Dimensional Analysis</td>
<td>Análisis dimensional</td>
</tr>
<tr>
<td>Direct Variation</td>
<td>Variación directa</td>
</tr>
<tr>
<td>Discontinuous Function</td>
<td>Función discontinua</td>
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<tr>
<td>Discount</td>
<td>Descuento</td>
</tr>
<tr>
<td>Discrete Graph</td>
<td>Gráfica discreta</td>
</tr>
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<td>Discriminant</td>
<td>Discriminante</td>
</tr>
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<td>Distance Formula</td>
<td>Fórmula de distancia</td>
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<td>Propiedad distributiva</td>
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<td>Division Property of Equality</td>
<td>Propiedad de igualdad de la división</td>
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<th>Spanish Example</th>
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<td>( 12 \text{ pt} \cdot \frac{1 \text{ qt}}{2 \text{ pt}} = 6 \text{ qt} )</td>
<td>El discriminante de ( 2x^2 - 5x - 3 = 0 ) es ((-5)^2 - 4(2)(-3)) o 49.</td>
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</table>

**Theme Park Attendance**

<table>
<thead>
<tr>
<th>People</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (2, 5) )</td>
<td>( (1, 1) )</td>
</tr>
</tbody>
</table>

The distance from \((2, 5)\) to \((1, 1)\) is

\[
d = \sqrt{(-1 - 2)^2 + (1 - 5)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5.
\]

### Dimensional Analysis Examples

<table>
<thead>
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\]
**ENGLISH**

**Division Property of Inequality**
If both sides of an inequality are divided by the same positive quantity, the new inequality will have the same solution set. If both sides of an inequality are divided by the same negative quantity, the new inequality will have the same solution set if the inequality symbol is reversed.

**SPANISH**

**Propiedad de desigualdad de la división**
Cuando ambos lados de una desigualdad se dividen entre el mismo número positivo, la nueva desigualdad tiene el mismo conjunto solución. Cuando ambos lados de una desigualdad se dividen entre el mismo número negativo, la nueva desigualdad tiene el mismo conjunto solución si se invierte el símbolo de desigualdad.

**EXAMPLES**

\[
x + 4 = 7 \quad \text{and} \quad \frac{1}{2} \quad \text{are equivalent ratios.}
\]

**Glossary/Glosario**

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>domain</strong></td>
<td><strong>dominio</strong></td>
<td>The domain of the function {(−5, 3), (−3, −2), (−1, −1), (1, 0)} is {−5, −3, −1, 1}.</td>
</tr>
<tr>
<td><strong>element</strong></td>
<td><strong>elemento</strong></td>
<td></td>
</tr>
<tr>
<td><strong>equation</strong></td>
<td><strong>ecuación</strong></td>
<td></td>
</tr>
<tr>
<td><strong>equally likely outcomes</strong></td>
<td><strong>resultados igualmente probables</strong></td>
<td>If a fair coin is tossed, then (P(\text{heads}) = P(\text{tails}) = \frac{1}{2}). So the outcome “heads” and the outcome “tails” are equally likely.</td>
</tr>
<tr>
<td><strong>equilateral triangle</strong></td>
<td><strong>triángulo equilátero</strong></td>
<td></td>
</tr>
<tr>
<td><strong>equivalent ratios</strong></td>
<td><strong>razones equivalentes</strong></td>
<td></td>
</tr>
</tbody>
</table>

**G11**
### ENGLISH

**evaluate** To find the value of an algebraic expression by substituting a number for each variable and simplifying by using the order of operations.

**EXAMPLES**

Evaluate $2x + 7$ for $x = 3$.

$2x + 7$

$2(3) + 7$

$6 + 7$

$13$

**event** An outcome or set of outcomes of an experiment.

**EXAMPLES**

In the experiment of rolling a number cube, the event “an odd number” consists of the outcomes 1, 3, and 5.

**excluded values** Values of $x$ for which a function or expression is not defined.

**EXAMPLES**

The excluded values of $\frac{x + 2}{(x - 1)(x + 4)}$ are $x = 1$ and $x = -4$, which would make the denominator equal to 0.

**experiment** An operation, process, or activity in which outcomes can be used to estimate probability.

**EXAMPLES**

Tossing a coin 10 times and noting the number of heads.

**experimental probability** The ratio of the number of times an event occurs to the number of trials, or times, that an activity is performed.

**EXAMPLES**

Kendra attempted 27 free throws and made 16 of them. The experimental probability that she will make her next free throw is $P(\text{free throw}) = \frac{\text{number made}}{\text{number attempted}} = \frac{16}{27} \approx 0.59$.

**exponent** The number that indicates how many times the base in a power is used as a factor.

**EXAMPLES**

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

$4$ is the exponent.

**exponential decay** An exponential function of the form $f(x) = ab^x$ in which $0 < b < 1$. If $r$ is the rate of decay, then the function can be written $y = a(1 - r)^t$, where $a$ is the initial amount and $t$ is the time.

**EXAMPLES**

$f(x) = 3 \left(\frac{1}{2}\right)^x$

**exponential expression** An algebraic expression in which the variable is in an exponent with a fixed number as the base.

**EXAMPLES**

$2^{x+1}$

$2x + b$

$3 \cdot 4^x$

**exponential function** A function of the form $f(x) = ab^x$, where $a$ and $b$ are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.

**EXAMPLES**

$2x + b$

$3 \cdot 4^x$
**ENGLISH**

**exponential growth**  An exponential function of the form \( f(x) = ab^x \) in which \( b > 1 \). If \( r \) is the rate of growth, then the function can be written \( y = a(1 + r)^t \), where \( a \) is the initial amount and \( t \) is the time.

**expression**  A mathematical phrase that contains operations, numbers, and/or variables.

**extraneous solution**  A solution of a derived equation that is not a solution of the original equation.

**factor**  A number or expression that is multiplied by another number or expression to get a product. See also factoring.

**factorial**  If \( n \) is a positive integer, then \( n \) factorial, written \( n! \), is \( n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot 2 \cdot 1 \). The factorial of 0 is defined to be 1.

**factoring**  The process of writing a number or algebraic expression as a product.

**fair**  When all outcomes of an experiment are equally likely.

**family of functions**  A set of functions whose graphs have basic characteristics in common. Functions in the same family are transformations of their parent function.

---

**SPANISH**

**crecimiento exponencial**  Función exponencial del tipo \( f(x) = ab^x \) en la que \( b > 1 \). Si \( r \) es la tasa de crecimiento, entonces la función se puede expresar como \( y = a(1 + r)^t \), donde \( a \) es la cantidad inicial y \( t \) es el tiempo.

**expresión**  Frase matemática que contiene operaciones, números y/o variables.

**solución extraña**  Solución de una ecuación derivada que no es una solución de la ecuación original.

**factor**  Número o expresión que se multiplica por otro número o expresión para obtener un producto. Ver también factoreo.

**factorial**  Si \( n \) es un entero positivo, entonces el factorial de \( n \), expresado como \( n! \), es \( n \cdot (n - 1) \cdot (n - 2) \cdot \ldots \cdot 2 \cdot 1 \). Por definición, el factorial de 0 será 1.

**factorización**  Proceso por el que se expresa un número o expresión algebraica como un producto.

**justo**  Cuando todos los resultados de un experimento son igualmente probables.

**familia de funciones**  Conjunto de funciones cuyas gráficas tienen características básicas en común. Las funciones de la misma familia son transformaciones de su función madre.

---

**EXAMPLES**

\[
\begin{align*}
f(x) & = 2^x \\
6x + 1 & \quad \text{To solve } \sqrt{x} = -2, \text{ square both sides; } x = 4. \\
7! & = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040 \\
x^2 - 4x - 21 & = (x - 7)(x + 3) \\
\end{align*}
\]

When tossing a fair coin, heads and tails are equally likely. Each has a probability of \( \frac{1}{2} \).
**ENGLISH**

**first differences** The differences between y-values of a function for evenly spaced x-values.

**SPANISH**

**primeras diferencias** Diferencias entre los valores de y de una función para valores de x espaciados uniformemente.

**first quartile** The median of the lower half of a data set, denoted \( Q_1 \). Also called **lower quartile**.

**primer cuartil** Mediana de la mitad inferior de un conjunto de datos, expresada como \( Q_1 \). También se llama cuartil inferior.

**FOIL** A mnemonic (memory) device for a method of multiplying two binomials:
- Multiply the **First** terms.
- Multiply the **Outer** terms.
- Multiply the **Inner** terms.
- Multiply the **Last** terms.

**formula** A literal equation that states a rule for a relationship among quantities.

**exponente fraccionario** Ver exponente racional.

**frequency** The number of times the value appears in the data set.

**frecuencia** Cantidad de veces que aparece el valor en un conjunto de datos.

**frequency table** A table that lists the number of times, or frequency, that each data value occurs.

**tabla de frecuencia** Tabla que enumera la cantidad de veces que ocurre cada valor de datos, o la frecuencia.

**function** A relation in which every domain value is paired with exactly one range value.

**función** Relación en la que a cada valor de dominio corresponde exactamente un valor de rango.
function notation  If \( x \) is the independent variable and \( y \) is the dependent variable, then the function notation for \( y = f(x) \), read “\( f \) of \( x \),” where \( f \) names the function.

function rule  An algebraic expression that defines a function.

Fundamental Counting Principle  If one event has \( m \) possible outcomes and a second event has \( n \) possible outcomes after the first event has occurred, then there are \( mn \) total possible outcomes for the two events.

geometric sequence  A sequence in which the ratio of successive terms is a constant \( r \), called the common ratio, where \( r \neq 0 \) and \( r \neq 1 \).

equation: \( y = 2x \)

function notation: \( f(x) = 2x \)

If there are 4 colors of shirts, 3 colors of pants, and 2 colors of shoes, then there are \( 4 \cdot 3 \cdot 2 = 24 \) possible outfits.

graph of a function  The set of points in a coordinate plane with coordinates \((x, y)\), where \( x \) is in the domain of the function \( f \) and \( y = f(x) \).

graph of a system of linear inequalities  The region in a coordinate plane consisting of points whose coordinates are solutions to all of the inequalities in the system.

graph of an inequality in one variable  The set of points on a number line that are solutions of the inequality.

graph of an inequality in two variables  The set of points in a coordinate plane whose coordinates \((x, y)\) are solutions of the inequality.

Glossary/Glosario
**ENGLISH**

**graph of an ordered pair** For the ordered pair \((x, y)\), the point in a coordinate plane that is a horizontal distance of \(x\) units from the origin and a vertical distance of \(y\) units from the origin.

**greatest common factor (monomials) (GCF)** The product of the greatest integer and the greatest power of each variable that divide evenly into each monomial.

**greatest common factor (numbers) (GCF)** The largest common factor of two or more given numbers.

**grouping symbols** Symbols such as parentheses ( ), brackets [ ], and braces { } that separate part of an expression. A fraction bar, absolute-value symbols, and radical symbols may also be used as grouping symbols.

**half-life** The half-life of a substance is the time it takes for one-half of the substance to decay into another substance.

**half-plane** The part of the coordinate plane on one side of a line, which may include the line.

**Heron’s Formula** A triangle with side lengths \(a\), \(b\), and \(c\) has area 

\[ A = \sqrt{s(s-a)(s-b)(s-c)}, \]

where \(s\) is one-half the perimeter, or \(s = \frac{1}{2}(a + b + c)\).

**SPANISH**

**gráfica de un par ordenado** Dado el par ordenado \((x, y)\), punto en un plano cartesiano que está a una distancia horizontal de \(x\) unidades desde el origen y a una distancia vertical de \(y\) unidades desde el origen.

**máximo común divisor (monomios) (MCD)** Producto del entero mayor y la potencia mayor de cada variable que divide exactamente cada monomio.

**máximo común divisor (números) (MCD)** El mayor de los factores comunes compartidos por dos o más números dados.

**símbolos de agrupación** Símbolos tales como paréntesis ( ), corchetes [ ] y llaves { } que separan parte de una expresión. La barra de fracciones, los símbolos de valor absoluto y los símbolos de radical también se pueden utilizar como símbolos de agrupación.

**vida media** La vida media de una sustancia es el tiempo que tarda la mitad de la sustancia en desintegrarse y transformarse en otra sustancia.

**semiplano** La parte del plano cartesiano de un lado de una línea, que puede incluir la línea.

**fórmula de Herón** Un triángulo con longitudes de lado \(a\), \(b\) y \(c\) tiene un área 

\[ A = \sqrt{s(s-a)(s-b)(s-c)}, \]

donde \(s\) es la mitad del perímetro \(s = \frac{1}{2}(a + b + c)\).
**ENGLISH**

**histogram** A bar graph used to display data grouped in intervals.

**SPANISH**

**histograma** Gráfica de barras utilizada para mostrar datos agrupados en intervalos de clases.

**EXAMPLES**

<table>
<thead>
<tr>
<th>Salary range (thousand $)</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>20–29</td>
<td>3</td>
</tr>
<tr>
<td>30–39</td>
<td>2</td>
</tr>
<tr>
<td>40–49</td>
<td>4</td>
</tr>
<tr>
<td>50–59</td>
<td>2</td>
</tr>
</tbody>
</table>

**horizontal line** A line described by the equation $y = b$, where $b$ is the $y$-intercept.

**línea horizontal** Línea descrita por la ecuación $y = b$, donde $b$ es la intersección con el eje $y$.

**hypotenuse** The side opposite the right angle in a right triangle.

**hipotenusa** Lado opuesto al ángulo recto de un triángulo rectángulo.

**identity** An equation that is true for all values of the variables.

**identidad** Ecuación verdadera para todos los valores de las variables.

$3 = 3$

$2(x - 1) = 2x - 2$

**inclusive events** Events that have one or more outcomes in common.

**sucesos inclusivos** Sucesos que tienen uno o más resultados en común.

In the experiment of rolling a number cube, rolling an even number and rolling a number less than 3 are inclusive events because both contain the outcome 2.

**inconsistent system** A system of equations or inequalities that has no solution.

**sistema inconsistente** Sistema de ecuaciones o desigualdades que no tiene solución.

\[
\begin{align*}
\begin{cases}
 x + y &= 0 \\
 x + y &= 1
\end{cases}
\end{align*}
\]

**independent events** Events for which the occurrence or nonoccurrence of one event does not affect the probability of the other event.

**sucesos independientes** Dos sucesos son independientes si el hecho de que se produzca o no uno de ellos no afecta la probabilidad del otro suceso.

From a bag containing 3 red marbles and 2 blue marbles, draw a red marble, replace it, and then draw a blue marble.

**independent system** A system of equations that has exactly one solution.

**sistema independiente** Sistema de ecuaciones que tiene sólo una solución.

\[
\begin{align*}
\begin{cases}
 x + y &= 7 \\
 x - y &= 1
\end{cases}
\end{align*}
\]

Solution: $(4, 3)$

**independent variable** The input of a function; a variable whose value determines the value of the output, or dependent variable.

**variable independiente** Entrada de una función; variable cuyo valor determina el valor de la salida, o variable dependiente.

For $y = 2x + 1$, $x$ is the independent variable.
**index** In the radical $\sqrt[n]{x}$, which represents the $n$th root of $x$, $n$ is the index. In the radical $\sqrt[n]{x}$, the index is understood to be 2.

**inequality** A statement that compares two expressions by using one of the following signs: $<, >, \leq, \geq$, or $\neq$.

**input** A value that is substituted for the independent variable in a relation or function.

**input-output table** A table that displays input values of a function or expression together with the corresponding outputs.

**integer** A member of the set of whole numbers and their opposites.

**intercept** See $x$-intercept and $y$-intercept.

**interest** The amount of money charged for borrowing money or the amount of money earned when saving or investing money. See also compound interest, simple interest.

**interquartile range (IQR)** The difference of the third (upper) and first (lower) quartiles in a data set, representing the middle half of the data.

**intersection** The intersection of two sets is the set of all elements that are common to both sets, denoted by $\cap$.

**inverse operations** Operations that undo each other.

---

**indice** En el radical $\sqrt[n]{x}$, que representa la enésima raíz de $x$, $n$ es el índice. En el radical $\sqrt[n]{x}$, se da por sentado que el índice es 2.

**medición indirecta** Método de medición en el que se usan fórmulas, figuras semejantes y/o proporiciones.

**desigualdad** Enunciado que compara dos expresiones utilizando uno de los siguientes signos: $<, >, \leq, \geq$, o $\neq$.

**entrada** Valor que sustituye a la variable independiente en una relación o función.

**tabla de entrada y salida** Tabla que muestra los valores de entrada de una función o expresión junto con las correspondientes salidas.

**entero** Miembro del conjunto de números cabales y sus opuestos.

**intersección** Ver intersección con el eje $x$ e intersección con el eje $y$.

**interés** Cantidad de dinero que se cobra por prestar dinero o cantidad de dinero que se gana cuando se ahorra o invierte dinero. Ver también interés compuesto, interés simple.

**rango entre cuartiles** Diferencia entre el tercer cuartil (superior) y el primer cuartil (inferior) de un conjunto de datos, que representa la mitad central de los datos.

**intersección de conjuntos** La intersección de dos conjuntos es el conjunto de todos los elementos que son comunes a ambos conjuntos, expresado por $\cap$.

**operaciones inversas** Operaciones que se anulan entre sí.

---

For the function $f(x) = x + 5$, the input 3 produces an output of 8.

<table>
<thead>
<tr>
<th>Input</th>
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<tbody>
<tr>
<td>$x$</td>
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<tr>
<td></td>
<td>3</td>
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</tr>
<tr>
<td>$y$</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>10</td>
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The radical $\sqrt[3]{8}$ has an index of 3.

For the sets $A = \{1, 2, 3, 4\}$ and $B = \{1, 3, 5, 7, 9\}$, we have $A \cap B = \{1, 3\}$.

Addition and subtraction of the same quantity are inverse operations: $5 + 3 = 8$, $8 - 3 = 5$.

Multiplication and division by the same quantity are inverse operations: $2 \cdot 3 = 6$, $6 ÷ 3 = 2$. 

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**A** = \{1, 2, 3, 4\}

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---

**A** = \{1, 2, 3, 4\}

**B** = \{1, 3, 5, 7, 9\}

**$A \cap B$** = \{1, 3\}
inverse variation  A relationship between two variables, \(x\) and \(y\), that can be written in the form \(y = \frac{k}{x}\), where \(k\) is a nonzero constant and \(x \neq 0\).

irrational number  A real number that cannot be expressed as the ratio of two integers.

isolate the variable  To isolate a variable in an equation, use inverse operations on both sides until the variable appears by itself on one side of the equation and does not appear on the other side.

isosceles triangle  A triangle with at least two congruent sides.

leading coefficient  The coefficient of the first term of a polynomial in standard form.

least common denominator (LCD)  The least common multiple of the denominators of two or more given fractions or rational expressions.

least common multiple (monomials) (LCM)  The product of the smallest positive number and the lowest power of each variable that divide evenly into each monomial.

least common multiple (numbers) (LCM)  The smallest whole number, other than zero, that is a multiple of two or more given numbers.

least-squares line  The line of fit for which the sum of the squares of the residuals is as small as possible.

variaci\'on inversa  Relaci\'on entre dos variables, \(x\) e \(y\), que puede expresarse en la forma \(y = \frac{k}{x}\), donde \(k\) es una constante distinta de cero y \(x \neq 0\).

número irracional  Número real que no se puede expresar como una raz\'on de enteros.

despejar la variable  Para despejar la variable de una ecuación, utiliza operaciones inversas en ambos lados hasta que la variable aparezca sola en uno de los lados de la ecuación y no aparezca en el otro lado.

triángulo isósceles  Triángulo que tiene al menos dos lados congruentes.

Línea de mínimos cuadrados  La línea de ajuste en que la suma de cuadrados de los residuos es la menor.
### English

**line graph** A graph that uses line segments to show how data changes.

<table>
<thead>
<tr>
<th>Score</th>
<th>Game number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>1</td>
</tr>
<tr>
<td>800</td>
<td>2</td>
</tr>
<tr>
<td>400</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
</tbody>
</table>

### Spanish

**gráfica lineal** Gráfica que se vale de segmentos de recta para mostrar cambios en los datos.

### Examples

- **Marlon’s Video Game Scores**

- **Line of best fit** The line that comes closest to all of the points in a data set.

- **Linear equation in one variable** An equation that can be written in the form $ax = b$ where $a$ and $b$ are constants and $a \neq 0$.

- **Linear equation in two variables** An equation that can be written in the form $Ax + By = C$ where $A$, $B$, and $C$ are constants and $A$ and $B$ are not both 0.

- **Linear function** A function that can be written in the form $y = mx + b$, where $x$ is the independent variable and $m$ and $b$ are real numbers. Its graph is a line.

- **Linear inequality in one variable** An inequality that can be written in one of the following forms: $ax < b$, $ax > b$, $ax \leq b$, $ax \geq b$, or $ax \neq b$, where $a$ and $b$ are constants and $a \neq 0$.

- **Linear regression** A statistical method used to fit a linear model to a given data set.

- **Linear inequality in two variables** An inequality that can be written in one of the following forms: $Ax + By < C$, $Ax + By > C$, $Ax + By \leq C$, $Ax + By \geq C$, or $Ax + By \neq C$, where $A$, $B$, and $C$ are constants and $A$ and $B$ are not both 0.

### Spanish Examples

- **Gráfica lineal** Gráfica que se vale de segmentos de recta para mostrar cambios en los datos.

- **Línea de mejor ajuste** Línea que más se acerca a todos los puntos de un conjunto de datos.

- **Ecuación lineal en una variable** Ecuación que puede expresarse en la forma $ax = b$ donde $a$ y $b$ son constantes y $a \neq 0$.

- **Ecuación lineal en dos variables** Ecuación que puede expresarse en la forma $Ax + By = C$ donde $A$, $B$ y $C$ son constantes y $A$ y $B$ no son ambas 0.

- **Función lineal** Función que puede expresarse en la forma $y = mx + b$, donde $x$ es la variable independiente y $m$ y $b$ son números reales. Su gráfica es una línea.

- **Desigualdad lineal en una variable** Desigualdad que puede expresarse de una de las siguientes formas: $ax < b$, $ax > b$, $ax \leq b$, $ax \geq b$ o $ax \neq b$, donde $a$ y $b$ son constantes y $a \neq 0$.

- **Desigualdad lineal en dos variables** Desigualdad que puede expresarse de una de las siguientes formas: $Ax + By < C$, $Ax + By > C$, $Ax + By \leq C$, $Ax + By \geq C$ o $Ax + By \neq C$, donde $A$, $B$ y $C$ son constantes y $A$ y $B$ no son ambas 0.

- **Regresión lineal** Método estadístico utilizado para ajustar un modelo lineal a un conjunto de datos determinado.
### Glossary/Glosario

#### ENGLISH

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>SPANISH</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>literal equation</td>
<td>An equation that contains two or more variables.</td>
<td>ecuación literal</td>
<td>( d = rt )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ecuación que contiene dos o más variables.</td>
<td>( A = \frac{1}{2} h(b_1 + b_2) )</td>
</tr>
<tr>
<td>lower quartile</td>
<td>See first quartile.</td>
<td>cuartil inferior</td>
<td>Ver primer cuartil.</td>
</tr>
<tr>
<td>maximum of a function</td>
<td>The ( y )-value of the highest point on the graph of the function.</td>
<td>máximo de una función</td>
<td>Valor de ( y ) del punto más alto en la gráfica de la función.</td>
</tr>
<tr>
<td>mean</td>
<td>The sum of all the values in a data set divided by the number of data values. Also called the average.</td>
<td>media</td>
<td>Suma de todos los valores de un conjunto de datos dividida entre el número de valores de datos. También llamada promedio.</td>
</tr>
<tr>
<td>measure of an angle</td>
<td>Angles are measured in degrees. A degree is ( \frac{1}{360} ) of a complete circle.</td>
<td>medida de un ángulo</td>
<td>Los ángulos se miden en grados. Un grado es ( \frac{1}{360} ) de un círculo completo.</td>
</tr>
<tr>
<td>measure of central tendency</td>
<td>A measure that describes the center of a data set.</td>
<td>medida de tendencia dominante</td>
<td>Medida que describe el centro de un conjunto de datos.</td>
</tr>
<tr>
<td>median</td>
<td>For an ordered data set with an odd number of values, the median is the middle value. For an ordered data set with an even number of values, the median is the average of the two middle values.</td>
<td>mediana</td>
<td>Dado un conjunto de datos ordenado con un número impar de valores, la mediana es el valor medio. Dado un conjunto de datos con un número par de valores, la mediana es el promedio de los dos valores medios.</td>
</tr>
<tr>
<td>midpoint</td>
<td>The point that divides a segment into two congruent segments.</td>
<td>punto medio</td>
<td>Punto que divide un segmento en dos segmentos congruentes.</td>
</tr>
</tbody>
</table>

#### Diagrams

- **Mapping Diagram**
  - **Domain**
  - **Range**
  - Diagrama que muestra la relación entre los elementos del dominio y los elementos del rango de una función.

- **Mapping Diagram**
  - **Domain**
  - **Range**
  - Diagrama de correspondencia

- **Markup**
  - The amount by which a wholesale cost is increased.
  - margen de ganancia

- **Matrix**
  - A rectangular array of numbers.
  - matriz

- **Maximum of a Function**
  - The \( y \)-value of the highest point on the graph of the function.

- **Mean**
  - The sum of all the values in a data set divided by the number of data values. Also called the average.

- **Measure of an Angle**
  - Angles are measured in degrees. A degree is \( \frac{1}{360} \) of a complete circle.

- **Measure of Central Tendency**
  - A measure that describes the center of a data set.

- **Median**
  - For an ordered data set with an odd number of values, the median is the middle value. For an ordered data set with an even number of values, the median is the average of the two middle values.

- **Midpoint**
  - The point that divides a segment into two congruent segments.

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  - The amount by which a wholesale cost is increased.

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  - A measure that describes the center of a data set.

- **Median**
  - For an ordered data set with an odd number of values, the median is the middle value. For an ordered data set with an even number of values, the median is the average of the two middle values.

- **Midpoint**
  - The point that divides a segment into two congruent segments.
**ENGLISH**

**minimum of a function** The y-value of the lowest point on the graph of the function.

**SPANISH**

**mínimo de una función** Valor de y del punto más bajo en la gráfica de la función.

**EXAMPLES**

The minimum of the function is \(-2\).

---

**mode** The value or values that occur most frequently in a data set; if all values occur with the same frequency, the data set is said to have no mode.

**moda** El valor o los valores que se presentan con mayor frecuencia en un conjunto de datos. Si todos los valores se presentan con la misma frecuencia, se dice que el conjunto de datos no tiene moda.

Data set: 3, 6, 8, 8, 10 Mode: 8
Data set: 2, 5, 5, 7, 7 Modes: 5 and 7
Data set: 2, 3, 6, 9, 11 No mode

---

**monomial** A number or a product of numbers and variables with whole-number exponents, or a polynomial with one term.

**monómio** Número o producto de números y variables con exponentes de números cabales, o polinomio con un término.

3x^2y^4

---

**Multiplication Property of Equality** If \(a\), \(b\), and \(c\) are real numbers and \(a = b\), then \(ac = bc\).

**Propiedad de igualdad de la multiplicación** Si \(a\), \(b\) y \(c\) son números reales y \(a = b\), entonces \(ac = bc\).

\[
\frac{1}{3}x = 7 \\
(3)\left(\frac{1}{3}x\right) = (3)(7) \\
x = 21
\]

---

**Multiplication Property of Inequality** If both sides of an inequality are multiplied by the same positive quantity, the new inequality will have the same solution set. If both sides of an inequality are multiplied by the same negative quantity, the new inequality will have the same solution set if the inequality symbol is reversed.

**Propiedad de desigualdad de la multiplicación** Si ambos lados de una desigualdad se multipican por el mismo número positivo, la nueva desigualdad tendrá el mismo conjunto solución. Si ambos lados de una desigualdad se multipican por el mismo número negativo, la nueva desigualdad tendrá el mismo conjunto solución si se invierte el símbolo de desigualdad.

\[
\frac{1}{3}x > 7 \\
(3)\left(\frac{1}{3}x\right) > (3)(7) \\
x > 21 \\
-x \leq 2 \\
(-1)(-x) \geq (-1)(2) \\
x \geq -2
\]

---

**mutually exclusive events** Two events are mutually exclusive if they cannot both occur in the same trial of an experiment.

**sucesos mutuamente excluyentes** Dos sucesos son mutuamente excluyentes si ambos no pueden ocurrir en la misma prueba de un experimento.

In the experiment of rolling a number cube, rolling a 3 and rolling an even number are mutually exclusive events.

---

**natural number** A counting number.

**número natural** Número que se utiliza para contar.

1, 2, 3, 4, 5, 6, ...

---

**negative correlation** Two data sets have a negative correlation if one set of data values increases as the other set decreases.

**correlación negativa** Dos conjuntos de datos tienen una correlación negativa si un conjunto de valores de datos aumenta a medida que el otro conjunto disminuye.
negative exponent  For any nonzero real number \( x \) and any integer \( n \), \( x^{-n} = \frac{1}{x^n} \).

exponente negativo  Para cualquier número real distinto de cero \( x \) y cualquier entero \( n \), \( x^{-n} = \frac{1}{x^n} \).

negative number  A number that is less than zero. Negative numbers lie to the left of zero on a number line.

número negativo  Número menor que cero. Los números negativos se ubican a la izquierda del cero en una recta numérica.

net  A diagram of the faces of a three-dimensional figure arranged in such a way that the diagram can be folded to form the three-dimensional figure.

plantilla  Diagrama de las caras de una figura tridimensional que se puede plegar para formar la figura tridimensional.

no correlation  Two data sets have no correlation if there is no relationship between the sets of values.

sin correlación  Dos conjuntos de datos no tienen correlación si no existe una relación entre los conjuntos de valores.

nonlinear system of equations  A system in which at least one of the equations is not linear.

sistema no lineal de ecuaciones  Sistema en el cual por lo menos una de las ecuaciones no es lineal.

A system that contains one quadratic equation and one linear equation is a nonlinear system.

nth root  The \( n \)th root of a number \( a \), written as \( \sqrt[n]{a} \) or \( a^{\frac{1}{n}} \), is a number that is equal to \( a \) when it is raised to the \( n \)th power.

enésima raíz  La enésima raíz de un número \( a \), que se escribe \( \sqrt[n]{a} \) o \( a^{\frac{1}{n}} \), es un número igual a \( a \) cuando se eleva a la enésima potencia.

number line  A line used to represent the real numbers.

recta numérica  Línea utilizada para representar los números reales.

obtuse angle  An angle that measures greater than 90° and less than 180°.

ángulo obtuso  Ángulo que mide más de 90° y menos de 180°.

obtuse triangle  A triangle with one obtuse angle.

triángulo obtusángulo  Triángulo con un ángulo obtuso.

odds  A comparison of favorable and unfavorable outcomes. The odds in favor of an event are the ratio of the number of favorable outcomes to the number of unfavorable outcomes. The odds against an event are the ratio of the number of unfavorable outcomes to the number of favorable outcomes.

probabilidades a favor y en contra  Comparación de los resultados favorables y desfavorables. Las probabilidades a favor de un suceso son la razón entre la cantidad de resultados favorables y la cantidad de resultados desfavorables. Las probabilidades en contra de un suceso son la razón entre la cantidad de resultados desfavorables y la cantidad de resultados favorables.

The odds in favor of rolling a 3 on a number cube are 1:5.  The odds against rolling a 3 on a number cube are 5:1.
**ENGLISH**

**opposite** The opposite of a number $a$, denoted $-a$, is the number that is the same distance from zero as $a$, on the opposite side of the number line. The sum of opposites is 0.

**opposite reciprocal** The opposite of the reciprocal of a number. The opposite reciprocal of any nonzero number $a$ is $-\frac{1}{a}$.

**OR** A logical operator representing the union of two sets.

**order of operations** A process for evaluating expressions:
- First, perform operations in parentheses or other grouping symbols.
- Second, simplify powers and roots.
- Third, perform all multiplication and division from left to right.
- Fourth, perform all addition and subtraction from left to right.

**ordered pair** A pair of numbers $(x, y)$ that can be used to locate a point on a coordinate plane. The first number $x$ indicates the distance to the left or right of the origin, and the second number $y$ indicates the distance above or below the origin.

**origin** The intersection of the $x$- and $y$-axes in a coordinate plane. The coordinates of the origin are $(0, 0)$.

**outcome** A possible result of a probability experiment.

**outlier** A data value that is far removed from the rest of the data.

**output** The result of substituting a value for a variable in a function.

---

**SPANISH**

**opuesto** El opuesto de un número $a$, expresado $-a$, es el número que se encuentra a la misma distancia de cero que $a$, del lado opuesto de la recta numérica. La suma de los opuestos es 0.

**reciprocó opuesto** Opuesto del recíproco de un número. El recíproco opuesto de $a$ es $-\frac{1}{a}$.

**OR** Operador lógico que representa la unión de dos conjuntos.

**orden de las operaciones** Regla para evaluar las expresiones:
- Primero, realizar las operaciones entre paréntesis u otros símbolos de agrupación.
- Segundo, simplificar las potencias y las raíces.
- Tercero, realizar todas las multiplicaciones y divisiones de izquierda a derecha.
- Cuarto, realizar todas las sumas y restas de izquierda a derecha.

**par ordenado** Par de números $(x, y)$ que se pueden utilizar para ubicar un punto en un plano cartesiano. El primer número, $x$, indica la distancia a la izquierda o derecha del origen y el segundo número, $y$, indica la distancia hacia arriba o hacia abajo del origen.

**origen** Intersección de los ejes $x$ e $y$ en un plano cartesiano. Las coordenadas de origen son $(0, 0)$.

**resultado** Resultado posible de un experimento de probabilidad.

**valor extremo** Valor de datos que está muy alejado del resto de los datos.

**salida** Resultado de la sustitución de una variable por un valor en una función.

---

**EXAMPLES**

- $2 + 3^2 - (7 + 5) ÷ 4 · 3$
- $2 + 3^2 - 12 ÷ 4 · 3$

Add inside parentheses. Simplify the power.

- $2 + 9 - 12 ÷ 4 · 3$
- $2 + 9 - 3 · 3$
- $2 + 9 - 9$
- $11 - 9$
- $2$


- In the experiment of rolling a number cube, the possible outcomes are 1, 2, 3, 4, 5, and 6.

- For the function $f(x) = x^2 + 1$, the input 3 produces an output of 10.
<table>
<thead>
<tr>
<th>ENGLISH</th>
<th>SPANISH</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>parabola</strong></td>
<td>The shape of the graph of a quadratic function.</td>
<td><strong>parábola</strong></td>
</tr>
<tr>
<td><strong>parallel lines</strong></td>
<td>Lines in the same plane that do not intersect.</td>
<td><strong>líneas paralelas</strong></td>
</tr>
<tr>
<td><strong>parallelogram</strong></td>
<td>A quadrilateral with two pairs of parallel sides.</td>
<td><strong>paralelogramo</strong></td>
</tr>
<tr>
<td><strong>parent function</strong></td>
<td>The simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent function.</td>
<td><strong>función madre</strong></td>
</tr>
<tr>
<td><strong>Pascal's triangle</strong></td>
<td>A triangular arrangement of numbers in which every row starts and ends with 1 and each other number is the sum of the two numbers above it.</td>
<td><strong>triángulo de Pascal</strong></td>
</tr>
<tr>
<td><strong>percent</strong></td>
<td>A ratio that compares a number to 100.</td>
<td><strong>porcentaje</strong></td>
</tr>
<tr>
<td><strong>percent change</strong></td>
<td>An increase or decrease given as a percent of the original amount. See also percent decrease, percent increase.</td>
<td><strong>porcentaje de cambio</strong></td>
</tr>
<tr>
<td><strong>percent decrease</strong></td>
<td>A decrease given as a percent of the original amount.</td>
<td><strong>porcentaje de disminución</strong></td>
</tr>
<tr>
<td><strong>percent increase</strong></td>
<td>An increase given as a percent of the original amount.</td>
<td><strong>porcentaje de incremento</strong></td>
</tr>
<tr>
<td><strong>perfect square</strong></td>
<td>A number whose positive square root is a whole number.</td>
<td><strong>cuadrado perfecto</strong></td>
</tr>
</tbody>
</table>

- **Pascal's triangle** example:
  
  1 1 1
  1 2 1
  1 3 3 1
  1 4 6 4 1

- **Percent change** example:
  
  If an item that costs $8.00 is marked down to $6.00, the amount of the decrease is $2.00, so the percent decrease is \( \frac{2.00}{8.00} = 0.25 = 25\% \).

- **Percent increase** example:
  
  If an item’s wholesale cost of $8.00 is marked up to $12.00, the amount of the increase is $4.00, so the percent increase is \( \frac{4.00}{8.00} = 0.5 = 50\% \).
### English

**perfect-square trinomial** A trinomial whose factored form is the square of a binomial. A perfect-square trinomial has the form \(a^2 - 2ab + b^2 = (a - b)^2\) or \(a^2 + 2ab + b^2 = (a + b)^2\).

### Examples

\[
x^2 + 6x + 9 \text{ is a perfect-square trinomial, because } x^2 + 6x + 9 = (x + 3)^2.
\]

<table>
<thead>
<tr>
<th><strong>Perimeter</strong></th>
<th><strong>Suma de las longitudes de los lados de una figura plana cerrada.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>18 ft + 6 ft + 18 ft + 6 ft = 48 ft</td>
<td><strong>Perimeter</strong> = 18 + 6 + 18 + 6 = 48 ft</td>
</tr>
</tbody>
</table>

### Spanish

**trinomio cuadrado perfecto** Trinomio cuya forma factorizada es el cuadrado de un binomio. Un trinomio cuadrado perfecto tiene la forma \(a^2 - 2ab + b^2 = (a - b)^2\) o \(a^2 + 2ab + b^2 = (a + b)^2\).

### Examples

- \(x^2 + 6x + 9\) is a perfect-square trinomial, because \(x^2 + 6x + 9 = (x + 3)^2\).
### Polynomial Long Division

A method of dividing one polynomial by another.

### Population

The entire group of objects or individuals considered for a survey.

### Positive Correlation

Two data sets have a positive correlation if both sets of data values increase.

### Positive Number

A number greater than zero.

### Power of a Power Property

If \( a \) is any nonzero real number and \( m \) and \( n \) are integers, then \( (a^m)^n = a^{mn} \).

### Power of a Product Property

If \( a \) and \( b \) are any nonzero real numbers and \( n \) is any integer, then \( (ab)^n = a^n b^n \).

### Power of a Quotient Property

If \( a \) and \( b \) are any nonzero real numbers and \( n \) is an integer, then \( (\frac{a}{b})^n = \frac{a^n}{b^n} \).

### Precision

The level of detail of a measurement, determined by the unit of measure.

### Prediction

An estimate or guess about something that has not yet happened.

### Prime Factorization

A representation of a number or a polynomial as a product of primes.

### Examples

\[
\begin{align*}
\frac{x + 1}{x + 2} & = \frac{x^2 + 3x + 5}{-(x^2 + 2x)} \\
& = \frac{x + 5}{-(x + 2)} \\
& = \frac{x^2 + 3x + 5}{x + 2} = x + 1 + \frac{3}{x + 2}
\end{align*}
\]

- **Polynomial Long Division**

- **Population**

- **Positive Correlation**

- **Positive Number**

- **Power of a Power Property**

- **Power of a Product Property**

- **Power of a Quotient Property**

- **Precision**

- **Prediction**

- **Prime Factorization**

- **Examples**
### ENGLISH

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>prime number</td>
<td>A whole number greater than 1 that has exactly two positive factors, itself and 1.</td>
<td>5 is prime because its only positive factors are 5 and 1.</td>
</tr>
<tr>
<td>principal</td>
<td>An amount of money borrowed or invested.</td>
<td></td>
</tr>
<tr>
<td>prism</td>
<td>A polyhedron formed by two parallel congruent polygonal bases connected by faces that are parallelograms.</td>
<td></td>
</tr>
<tr>
<td>probability</td>
<td>A number from 0 to 1 (or 0% to 100%) that is the measure of how likely an event is to occur.</td>
<td></td>
</tr>
<tr>
<td>Product of Powers Property</td>
<td>If $a$ is any nonzero real number and $m$ and $n$ are integers, then $a^m \cdot a^n = a^{m+n}$.</td>
<td></td>
</tr>
<tr>
<td>Product Property of Square</td>
<td>For $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.</td>
<td></td>
</tr>
<tr>
<td>proportion</td>
<td>A statement that two ratios are equal; $\frac{a}{b} = \frac{c}{d}$.</td>
<td>$\frac{2}{3} = \frac{4}{6}$</td>
</tr>
<tr>
<td>pyramid</td>
<td>A polyhedron formed by a polygonal base and triangular lateral faces that meet at a common vertex.</td>
<td></td>
</tr>
<tr>
<td>Pythagorean Theorem</td>
<td>If a right triangle has legs of lengths $a$ and $b$ and a hypotenuse of length $c$, then $a^2 + b^2 = c^2$.</td>
<td>The numbers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.</td>
</tr>
<tr>
<td>Pythagorean triple</td>
<td>A set of three positive integers $a$, $b$, and $c$ such that $a^2 + b^2 = c^2$.</td>
<td></td>
</tr>
<tr>
<td>quadrant</td>
<td>One of the four regions into which the $x$- and $y$-axes divide the coordinate plane.</td>
<td></td>
</tr>
</tbody>
</table>

### SPANISH

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>número primo</td>
<td>Número cabal mayor que 1 que es divisible únicamente entre sí mismo y entre 1.</td>
<td></td>
</tr>
<tr>
<td>capital</td>
<td>Cantidad de dinero que se pide prestado o se invierte.</td>
<td></td>
</tr>
<tr>
<td>prisma</td>
<td>Poliedro formado por dos bases poligonales congruentes y paralelas conectadas por caras laterales que son paralelogramos.</td>
<td></td>
</tr>
<tr>
<td>probabilidad</td>
<td>Número entre 0 y 1 (o entre 0% y 100%) que describe cuán probable es que ocurra un suceso.</td>
<td>A bag contains 3 red marbles and 4 blue marbles. The probability of randomly choosing a red marble is $\frac{3}{7}$.</td>
</tr>
<tr>
<td>Propiedad del producto de potencias</td>
<td>Dado un número real $a$ distinto de cero y los números enteros $m$ y $n$, entonces $a^m \cdot a^n = a^{m+n}$.</td>
<td>$6^7 \cdot 6^4 = 6^{7+4} = 6^{11}$</td>
</tr>
<tr>
<td>Propiedad del producto de raíces cuadradas</td>
<td>Dados $a \geq 0$ y $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.</td>
<td>$\sqrt{9} \cdot 25 = \sqrt{9} \cdot \sqrt{25} = 3 \cdot 5 = 15$</td>
</tr>
<tr>
<td>proporción</td>
<td>Ecuación que establece que dos razones son iguales; $\frac{a}{b} = \frac{c}{d}$.</td>
<td>$\frac{2}{3} = \frac{4}{6}$</td>
</tr>
<tr>
<td>pirámide</td>
<td>Poliedro formado por una base poligonal y caras laterales triangulares que se encuentran en un vértice común.</td>
<td></td>
</tr>
<tr>
<td>Teorema de Pitágoras</td>
<td>Dado un triángulo rectángulo con catetos de longitudes $a$ y $b$ y una hipotenusa de longitud $c$, entonces $a^2 + b^2 = c^2$.</td>
<td>The numbers 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$.</td>
</tr>
<tr>
<td>Tripletas de Pitágoras</td>
<td>Conjunto de tres enteros positivos $a$, $b$ y $c$ tal que $a^2 + b^2 = c^2$.</td>
<td></td>
</tr>
</tbody>
</table>

### Diagram

- Quadrant I
- Quadrant II
- Quadrant III
- Quadrant IV

- 13 cm
- 5 cm
- 12 cm
- $5^2 + 12^2 = 13^2$
- $25 + 144 = 169$
<table>
<thead>
<tr>
<th>ENGLISH</th>
<th>SPANISH</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>quadratic equation An equation that can be written in the form $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.</td>
<td>ecuación cuadrática Ecuación que se puede expresar como $ax^2 + bx + c = 0$, donde $a$, $b$ y $c$ son números reales y $a \neq 0$.</td>
<td>$x^2 + 3x - 4 = 0$ $x^2 - 9 = 0$</td>
</tr>
<tr>
<td>Quadratic Formula The formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, which gives solutions, or roots, of equations in the form $ax^2 + bx + c = 0$, where $a \neq 0$.</td>
<td>fórmula cuadrática La fórmula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, que da soluciones, o raíces, para las ecuaciones del tipo $ax^2 + bx + c = 0$, donde $a \neq 0$.</td>
<td>The solutions of $2x^2 - 5x - 3 = 0$ are given by $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)} = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5 \pm 7}{4}$ $x = 3$ or $x = -\frac{1}{2}$</td>
</tr>
<tr>
<td>quadratic function A function that can be written in the form $f(x) = ax^2 + bx + c$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.</td>
<td>función cuadrática Función que se puede expresar como $f(x) = ax^2 + bx + c$, donde $a$, $b$ y $c$ son números reales y $a \neq 0$.</td>
<td>$f(x) = x^2 - 6x + 8$</td>
</tr>
<tr>
<td>quadratic polynomial A polynomial of degree 2.</td>
<td>polinomio cuadrático Polinomio de grado 2.</td>
<td>$x^2 - 6x + 8$</td>
</tr>
<tr>
<td>quartile The median of the upper or lower half of a data set. See also first quartile, third quartile.</td>
<td>cuartil La mediana de la mitad superior o inferior de un conjunto de datos. Ver también primer cuartil, tercer cuartil.</td>
<td></td>
</tr>
<tr>
<td>Quotient of Powers Property If $a$ is a nonzero real number and $m$ and $n$ are integers, then $\frac{a^m}{a^n} = a^{m-n}$.</td>
<td>Propiedad del cociente de potencias Dado un número real $a$ distinto de cero y los números enteros $m$ y $n$, entonces $\frac{a^m}{a^n} = a^{m-n}$.</td>
<td>$\frac{6^7}{6^4} = 6^{7-4} = 6^3$</td>
</tr>
<tr>
<td>Quotient Property of Square Roots For $a \geq 0$ and $b &gt; 0$, $\sqrt[2]{\frac{a}{b}} = \frac{\sqrt[2]{a}}{\sqrt[2]{b}}$.</td>
<td>Propiedad del cociente de raíces cuadradas Dados $a \geq 0$ y $b &gt; 0$, $\sqrt[2]{\frac{a}{b}} = \frac{\sqrt[2]{a}}{\sqrt[2]{b}}$.</td>
<td>$\sqrt[5]{\frac{9}{25}} = \frac{\sqrt[5]{9}}{\sqrt[5]{25}} = \frac{3}{5}$</td>
</tr>
<tr>
<td>radical equation An equation that contains a variable within a radical.</td>
<td>ecuación radical Ecuación que contiene una variable dentro de un radical.</td>
<td>$\sqrt{x} + 3 + 4 = 7$</td>
</tr>
<tr>
<td>radical expression An expression that contains a radical sign.</td>
<td>expresión radical Expresión que contiene un signo de radical.</td>
<td>$\sqrt{x} + 3 + 4$</td>
</tr>
<tr>
<td>radical symbol The symbol $\sqrt{\phantom{x}}$ used to denote a root. The symbol is used alone to indicate a square root or with an index, $\sqrt[2]{\phantom{x}}$, to indicate the $n$th root.</td>
<td>símbolo de radical Símbolo $\sqrt{\phantom{x}}$ que se utiliza para expresar una raíz. Puede utilizarse solo para indicar una raíz cuadrada, o con un índice, $\sqrt[2]{\phantom{x}}$, para indicar la enésima raíz.</td>
<td>$\sqrt[3]{36} = 6$ $\sqrt[3]{27} = 3$</td>
</tr>
<tr>
<td><strong>ENGLISH</strong></td>
<td><strong>SPANISH</strong></td>
<td><strong>EXAMPLES</strong></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
| **radicand** The expression under a radical sign. | **radicando** Número o expresión debajo del signo de radical. | Expression: $\sqrt{x + 3}$  
Radicand: $x + 3$ |
| **radius** A segment whose endpoints are the center of a circle and a point on the circle; the distance from the center of a circle to any point on the circle. | **radio** Segmento cuyos extremos son el centro de un círculo y un punto en la circunferencia; distancia desde el centro de un círculo hasta cualquier punto de la circunferencia. |  
Mr. Hansen chose a random sample of the class by writing each student’s name on a slip of paper, mixing up the slips, and drawing five slips without looking. |
| **random sample** A sample selected from a population so that each member of the population has an equal chance of being selected. | **muestra aleatoria** Muestra seleccionada de una población tal que cada miembro de ésta tenga igual probabilidad de ser seleccionada. |  
The data set $\{3, 3, 5, 7, 8, 10, 11, 11, 12\}$ has a range of $12 - 3 = 9$. |
| **range of a data set** The difference of the greatest and least values in the data set. | **rango de un conjunto de datos** La diferencia del mayor y menor valor en un conjunto de datos. | The range of the function $\{(−5, 3), (−3, −2), (−1, −1), (1, 0)\}$ is $\{-2, −1, 0, 3\}$. |
| **range of a function or relation** The set of all second coordinates (or y-values) of a function or relation. | **rango de una función o relación** Conjunto de todos los valores de la segunda coordenada (o valores de $y$) de una función o relación. |  
The range of the function $\{(−5, 3), (−3, −2), (−1, −1), (1, 0)\}$ is $\{-2, −1, 0, 3\}$. |
| **rate** A ratio that compares two quantities measured in different units. | **tasa** Razón que compara dos cantidades medidas en diferentes unidades. |  
55 miles $\div$ 1 hour $= 55 \text{ mi/h}$ |
| **rate of change** A ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable. | **tasa de cambio** Razón que compara la cantidad de cambio de la variable dependiente con la cantidad de cambio de la variable independiente. | The cost of mailing a letter increased from 22 cents in 1985 to 25 cents in 1988. During this period, the rate of change was  
$$ \frac{\text{change in cost}}{\text{change in year}} = \frac{25 - 22}{1988 - 1985} = \frac{3}{3} = 1 \text{ cent per year.}$$ |
| **ratio** A comparison of two quantities by division. | **razón** Comparación de dos cantidades mediante una división. | $\frac{1}{2}$ or 1:2 |
| **rational equation** An equation that contains one or more rational expressions. | **ecuación racional** Ecuación que contiene una o más expresiones racionales. | $\frac{x + 2}{x^2 + 3x - 1} = 6$ |
| **rational exponent** An exponent that can be expressed as $\frac{m}{n}$ such that if $m$ and $n$ are integers, then $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$. | **exponente racional** Exponente que se puede expresar como $\frac{m}{n}$ tal que si $m$ y $n$ son números enteros, entonces $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$. | $64^{\frac{1}{6}} = \sqrt[6]{64}$ |
| **rational expression** An algebraic expression whose numerator and denominator are polynomials and whose denominator has a degree $\geq 1$. | **expresión racional** Expresión algebraica cuyo numerador y denominador son polinomios y cuyo denominador tiene un grado $\geq 1$. | $\frac{x + 2}{x^2 + 3x - 1}$ |
**ENGLISH**

**rational function** A function whose rule can be written as a rational expression.

**rational number** A number that can be written in the form $\frac{a}{b}$, where $a$ and $b$ are integers and $b \neq 0$.

**rationalizing the denominator** A method of rewriting a fraction by multiplying by another fraction that is equivalent to 1 in order to remove radical terms from the denominator.

**ray** A part of a line that starts at an endpoint and extends forever in one direction.

**real number** A rational or irrational number. Every point on the number line represents a real number.

**reciprocal** For a real number $a \neq 0$, the reciprocal of $a$ is $\frac{1}{a}$. The product of reciprocals is 1.

**rectangle** A quadrilateral with four right angles.

**rectangular prism** A prism whose bases are rectangles.

**rectangular pyramid** A pyramid whose base is a rectangle.

**reflection** A transformation that reflects, or “flips,” a graph or figure across a line, called the line of reflection.

**regular polygon** A polygon that is both equilateral and equiangular.

**SPANISH**

**función racional** Función cuya regla se puede expresar como una expresión racional.

**número racional** Número que se puede expresar como $\frac{a}{b}$, donde $a$ y $b$ son números enteros y $b \neq 0$.

**racionalizar el denominador** Método que consiste en escribir nuevamente una fracción multiplicándola por otra fracción equivalente a 1 a fin de eliminar los términos radicales del denominador.

**rayo** Parte de una recta que comienza en un extremo y se extiende infinitamente en una dirección.

**número real** Número racional o irracional. Cada punto de la recta numérica representa un número real.

**recíproco** Dado el número real $a \neq 0$, el recíproco de $a$ es $\frac{1}{a}$. El producto de los recíprocos es 1.

**rectángulo** Cuadrilátero con cuatro ángulos rectos.

**prisma rectangular** Prisma cuyas bases son rectángulos.

**pirámide rectangular** Pirámide cuya base es un rectángulo.

**reflexión** Transformación en la que una gráfica o figura se refleja o se invierte sobre una línea, denominada la línea de reflexión.

**polígono regular** Polígono equilátero de ángulos iguales.

**EXAMPLES**

\[ f(x) = \frac{x + 2}{x^2 + 3x - 1} \]

3, 1.75, $0.3\overline{3}$, $-\frac{2}{3}$, 0

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

<table>
<thead>
<tr>
<th>Number</th>
<th>Reciprocal</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>0</td>
<td>No reciprocal</td>
</tr>
</tbody>
</table>

$D$
<table>
<thead>
<tr>
<th><strong>ENGLISH</strong></th>
<th><strong>SPANISH</strong></th>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>relation A set of ordered pairs.</td>
<td>relación Conjunto de pares ordenados.</td>
<td>{(0, 5), (0, 4), (2, 3), (4, 0)}</td>
</tr>
<tr>
<td>repeating decimal A rational number in decimal form that has a nonzero block of one or more digits that repeat continuously.</td>
<td>decimal periódico Número racional en forma decimal que tiene un bloque de uno o más dígitos que se repite continuamente.</td>
<td>1.3, 0.6, 2.14, 6.773</td>
</tr>
<tr>
<td>replacement set A set of numbers that can be substituted for a variable.</td>
<td>conjunto de reemplazo Conjunto de números que pueden sustituir una variable.</td>
<td></td>
</tr>
<tr>
<td>residual The signed vertical distance between a data point and a line of fit.</td>
<td>residuo La diferencia vertical entre un dato y una línea de ajuste.</td>
<td></td>
</tr>
<tr>
<td>rhombus A quadrilateral with four congruent sides.</td>
<td>rombo Cuadrilátero con cuatro lados congruentes.</td>
<td></td>
</tr>
<tr>
<td>right angle An angle that measures 90°.</td>
<td>ángulo recto Ángulo que mide 90°.</td>
<td></td>
</tr>
<tr>
<td>rise The difference in the y-values of two points on a line.</td>
<td>distancia vertical Diferencia entre los valores de y de dos puntos de una línea.</td>
<td>For the points (3, -1) and (6, 5), the rise is 5 - (-1) = 6.</td>
</tr>
<tr>
<td>rotation A transformation that rotates or turns a figure about a point called the center of rotation.</td>
<td>rotación Transformación que rota o gira una figura sobre un punto llamado centro de rotación.</td>
<td>For the points (3, -1) and (6, 5), the run is 6 - 3 = 3.</td>
</tr>
<tr>
<td>run The difference in the x-values of two points on a line.</td>
<td>distancia horizontal Diferencia entre los valores de x de dos puntos de una línea.</td>
<td></td>
</tr>
<tr>
<td>sample A part of the population.</td>
<td>muestra Una parte de la población.</td>
<td>In a survey about the study habits of high school students, a sample is a survey of 100 students.</td>
</tr>
<tr>
<td>sample space The set of all possible outcomes of a probability experiment.</td>
<td>espacio muestral Conjunto de todos los resultados posibles de un experimento de probabilidad.</td>
<td>In the experiment of rolling a number cube, the sample space is {1, 2, 3, 4, 5, 6}.</td>
</tr>
<tr>
<td>scale The ratio between two corresponding measurements.</td>
<td>escala Razón entre dos medidas correspondientes.</td>
<td>1 cm : 5 mi</td>
</tr>
</tbody>
</table>
ENGLISH

**scale drawing**  A drawing that uses a scale to represent an object as smaller or larger than the actual object.

**scale factor**  The multiplier used on each dimension to change one figure into a similar figure.

**scale model**  A three-dimensional model that uses a scale to represent an object as smaller or larger than the actual object.

**scalene triangle**  A triangle with no congruent sides.

**scatter plot**  A graph with points plotted to show a possible relationship between two sets of data.

**second differences**  Differences between first differences of a function.

**sequence**  A list of numbers that often form a pattern.

**set**  A collection of items called elements.

**set-builder notation**  A notation for a set that uses a rule to describe the properties of the elements of the set.

**SPANISH**

**dibujo a escala**  Dibujo que utiliza una escala para representar un objeto como más pequeño o más grande que el objeto original.

**factor de escala**  El multiplicador utilizado en cada dimensión para transformar una figura en una figura semejante.

**modelo a escala**  Modelo tridimensional que utiliza una escala para representar un objeto como más pequeño o más grande que el objeto real.

**triángulo escaleno**  Triángulo sin lados congruentes.

**diagrama de dispersión**  Gráfica con puntos que se usa para demostrar una relación posible entre dos conjuntos de datos.

**segundas diferencias**  Diferencias entre las primeras diferencias de una función.

**sucesión**  Lista de números que generalmente forman un patrón.

**conjunto**  Grupo de componentes denominados elementos.

**notación de conjuntos**  Notación para un conjunto que se vale de una regla para describir las propiedades de los elementos del conjunto.

<table>
<thead>
<tr>
<th><strong>ENGLISH</strong></th>
<th><strong>SPANISH</strong></th>
<th><strong>EXAMPLES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>scale drawing</td>
<td>dibujo a escala</td>
<td>A blueprint is an example of a scale drawing.</td>
</tr>
<tr>
<td>scale factor</td>
<td>factor de escala</td>
<td>Scale factor: $\frac{3}{2} = 1.5$</td>
</tr>
<tr>
<td>scale model</td>
<td>modelo a escala</td>
<td></td>
</tr>
<tr>
<td>scalene triangle</td>
<td>triángulo escaleno</td>
<td></td>
</tr>
<tr>
<td>scatter plot</td>
<td>diagrama de dispersión</td>
<td></td>
</tr>
<tr>
<td>second differences</td>
<td>segundas diferencias</td>
<td></td>
</tr>
<tr>
<td>sequence</td>
<td>sucesión</td>
<td>1, 2, 4, 8, 16, ...</td>
</tr>
<tr>
<td>set</td>
<td>conjunto</td>
<td>{1, 2, 3}</td>
</tr>
<tr>
<td>set-builder notation</td>
<td>notación de conjuntos</td>
<td>{x</td>
</tr>
</tbody>
</table>
**ENGLISH**

**similar** Two figures are similar if they have the same shape but not necessarily the same size.

**semi**

**similarity statement** A statement that indicates that two polygons are similar by listing the vertices in the order of correspondence.

**enunciado de semejanza** Enunciado que indica que dos polígonos son semejantes enumerando los vértices en orden de correspondencia.

**simple event** An event consisting of only one outcome.

**suceso simple** Suceso que tiene sólo un resultado.

**simple interest** A fixed percent of the principal. For principal $P$, interest rate $r$, and time $t$ in years, the simple interest is $I = Prt$.

**simplest form of a square root expression** A square root expression is in simplest form if it meets the following criteria:

1. No perfect squares are in the radicand.
2. No fractions are in the radicand.
3. No square roots appear in the denominator of a fraction.

**simplest form of a rational expression** A rational expression is in simplest form if the numerator and denominator have no common factors.

**SPANISH**

**semejantes** Dos figuras con la misma forma pero no necesariamente del mismo tamaño.

**enunciado de semejanza** Enunciado que indica que dos polígonos son semejantes enumerando los vértices en orden de correspondencia.

**simple event** Suceso que tiene sólo un resultado.

**interés simple** Porcentaje fijo del capital. Dado el capital $P$, la tasa de interés $r$ y el tiempo $t$ expresado en años, el interés simple es $I = Prt$.

**simplest form of a square root expression** Una expresión de raíz cuadrada está en forma simplificada si reúne los siguientes requisitos:

1. No hay cuadrados perfectos en el radicando.
2. No hay fracciones en el radicando.
3. No aparecen raíces cuadradas en el denominador de una fracción.

**simplest form of a rational expression** Una expresión racional está en forma simplificada cuando el numerador y el denominador no tienen factores comunes.

**EXAMPLES**

<table>
<thead>
<tr>
<th>Not Simplest Form</th>
<th>Simplest Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{180}$</td>
<td>$6\sqrt{5}$</td>
</tr>
<tr>
<td>$\sqrt{216a^2b^2}$</td>
<td>$6ab\sqrt{6}$</td>
</tr>
<tr>
<td>$\frac{\sqrt{7}}{\sqrt{2}}$</td>
<td>$\frac{\sqrt{14}}{2}$</td>
</tr>
</tbody>
</table>

\[
\frac{x^2 - 1}{x^2 + x - 2} = \frac{(x - 1)(x + 1)}{(x - 1)(x + 2)} = \frac{x + 1}{x + 2}
\]
**ENGLISH**

**simply form of an exponential expression** An exponential expression is in simplest form if it meets the following criteria:
1. There are no negative exponents.
2. The same base does not appear more than once in a product or quotient.
3. No powers, products, or quotients are raised to powers.
4. Numerical coefficients in a quotient do not have any common factor other than 1.

**ENGLISH**

**simulate** To perform all indicated operations.

**ENGLISH**

**simulation** A model of an experiment, often one that would be too difficult or time-consuming to actually perform.

**ENGLISH**

**sine** In a right triangle, the ratio of the length of the leg opposite \( \angle A \) to the length of the hypotenuse.

**ENGLISH**

**slope** A measure of the steepness of a line. If \((x_1, y_1)\) and \((x_2, y_2)\) are any two points on the line, the slope of the line, known as \( m \), is represented by the equation \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

**ENGLISH**

**slope-intercept form** The slope-intercept form of a linear equation is \( y = mx + b \), where \( m \) is the slope and \( b \) is the \( y \)-intercept.

**ENGLISH**

**solution of a linear equation in two variables** An ordered pair or ordered pairs that make the equation true.

---

**SPANISH**

**forma simplificada de una expresión exponencial** Una expresión exponencial está en forma simplificada si reúne los siguientes requisitos:
1. No hay exponentes negativos.
2. La misma base no aparece más de una vez en un producto o cociente.
3. No se elevan a potencias productos, cocientes ni potencias.
4. Los coeficientes numéricos en un cociente no tienen ningún factor común que no sea 1.

**SPANISH**

**simplificar** Realizar todas las operaciones indicadas.

**SPANISH**

**simulación** Modelo de un experimento; generalmente se recurre a la simulación cuando realizar dicho experimento sería demasiado difícil o llevaría mucho tiempo.

**SPANISH**

**seno** En un triángulo rectángulo, razón entre la longitud del cateto opuesto a \( \angle A \) y la longitud de la hipotenusa.

**SPANISH**

**pendiente** Medida de la inclinación de una línea. Dados dos puntos \((x_1, y_1)\) y \((x_2, y_2)\) en una línea, la pendiente de la línea, denominada \( m \), se representa con la ecuación \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

**SPANISH**

**forma de pendiente-intersección** La forma de pendiente-intersección de una ecuación lineal es \( y = mx + b \), donde \( m \) es la pendiente y \( b \) es la intersección con el eje \( y \).

**SPANISH**

**solución de una ecuación lineal en dos variables** Un par ordenado o pares ordenados que hacen que la ecuación sea verdadera.
<table>
<thead>
<tr>
<th>ENGLISH</th>
<th>SPANISH</th>
<th>EXAMPLES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>solution of a linear inequality in two variables</strong> An ordered pair or ordered pairs that make the inequality true.</td>
<td><strong>solución de una desigualdad lineal en dos variables</strong> Un par ordenado o pares ordenados que hacen que la desigualdad sea verdadera.</td>
<td><em>(3, 1) is a solution of $x + y &lt; 6.$</em></td>
</tr>
</tbody>
</table>
| **solution of a system of linear equations** Any ordered pair that satisfies all the equations in a system. | **solución de un sistema de ecuaciones lineales** Cualquier par ordenado que resuelva todas las ecuaciones de un sistema. | \[
\begin{align*}
x + y &= -1 \\
-x + y &= -3 \\
\text{Solution: } (1, -2)
\end{align*}
\] |
| **solution of a system of linear inequalities** Any ordered pair that satisfies all the inequalities in a system. | **solución de un sistema de desigualdades lineales** Cualquier par ordenado que resuelva todas las desigualdades de un sistema. | *(2, 1) is in the overlapping shaded regions, so it is a solution.* |
| **solution of an equation in one variable** A value or values that make the equation true. | **solución de una ecuación en una variable** Valor o valores que hacen que la ecuación sea verdadera. | Equation: $x + 2 = 6$  
Solution: $x = 4$ |
| **solution of an inequality in one variable** A value or values that make the inequality true. | **solución de una desigualdad en una variable** Valor o valores que hacen que la desigualdad sea verdadera. | Inequality: $x + 2 < 6$  
Solution: $x < 4$ |
| **solution set** The set of values that make a statement true. | **conjunto solución** Conjunto de valores que hacen verdadero un enunciado. | Inequality: $x + 3 \geq 5$  
Solution set: $\{x | x \geq 2\}$ |
| **square** A quadrilateral with four congruent sides and four right angles. | **cuadrado** Cuadrilátero con cuatro lados congruentes y cuatro ángulos rectos. | 16 is the square of 4. |
| **square in numeration** The second power of a number. | **cuadrado en numeración** La segunda potencia de un número. | 16 is the square of 4. |
| **standard form of a linear equation** $Ax + By = C$, where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both 0. | **forma estándar de una ecuación lineal** $Ax + By = C$, donde $A$, $B$ y $C$ son números reales y $A$ y $B$ no son ambos cero. | $2x + 3y = 6$ |
| **standard form of a polynomial** A polynomial in one variable is written in standard form when the terms are in order from greatest degree to least degree. | **forma estándar de un polinomio** Un polinomio de una variable se expresa en forma estándar cuando los términos se ordenan de mayor a menor grado. | $4x^5 - 2x^4 + x^2 - x + 1$ |
ENGLISH

standard form of a quadratic equation $ax^2 + bx + c = 0$, where $a$, $b$, and $c$ are real numbers and $a \neq 0$.

stem-and-leaf plot A graph used to organize and display data by dividing each data value into two parts, a stem and a leaf.

stratified random sample A sample in which a population is divided into distinct groups and members are selected at random from each group.

subset A set that is contained entirely within another set. Set $B$ is a subset of set $A$ if every element of $B$ is contained in $A$, denoted $B \subset A$.

substitution method A method used to solve systems of equations by solving an equation for one variable and substituting the resulting expression into the other equation(s).

Subtraction Property of Equality If $a$, $b$, and $c$ are real numbers and $a = b$, then $a - c = b - c$.

Subtraction Property of Inequality For real numbers $a$, $b$, and $c$, if $a < b$, then $a - c < b - c$. Also holds true for $>$, $\leq$, $\geq$, and $\neq$.

supplementary angles Two angles whose measures have a sum of 180°.

surface area The total area of all faces and curved surfaces of a three-dimensional figure.

SPANISH

forma estándar de una ecuación cuadrática $ax^2 + bx + c = 0$, donde $a$, $b$ y $c$ son números reales y $a \neq 0$.

diagrama de tallo y hojas Gráfica utilizada para organizar y mostrar datos dividiendo cada valor de datos en dos partes, un tallo y una hoja.

muestra aleatoria estratificada Muestra en la que la población está dividida en grupos diferenciados y los miembros de cada grupo se seleccionan al azar.

subconjunto Conjunto que se encuentra dentro de otro conjunto. El conjunto $B$ es un subconjunto del conjunto $A$ si todos los elementos de $B$ son elementos de $A$; se expresa $B \subset A$.

sustitución Método utilizado para resolver sistemas de ecuaciones resolviendo una ecuación para una variable y sustituyendo la expresión resultante en las demás ecuaciones.

Propiedad de igualdad de la resta Si $a$, $b$ y $c$ son números reales y $a = b$, entonces $a - c = b - c$.

Propiedad de desigualdad de la resta Dados los números reales $a$, $b$ y $c$, si $a < b$, entonces $a - c < b - c$. Es válido también para $>$, $\leq$, $\geq$ y $\neq$.

ángulos suplementarios Dos ángulos cuyas medidas suman 180°.

área total Área total de todas las caras y superficies curvas de una figura tridimensional.

EXAMPLES

$2x^2 + 3x - 1 = 0$

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaves</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2 3 4 4 7 9</td>
</tr>
<tr>
<td>4</td>
<td>0 1 5 7 7 7 8</td>
</tr>
<tr>
<td>5</td>
<td>1 2 2 3</td>
</tr>
</tbody>
</table>

Key: 3|2 means 3.2

Ms. Carter chose a stratified random sample of her school's student population by randomly selecting 30 students from each grade level.

The set of integers is a subset of the set of rational numbers.

Surface area $= 2(8)(12) + 2(8)(6) + 2(12)(6) = 432 \text{ cm}^2$
<table>
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| **system of linear equations**                                         | **sistema de ecuaciones lineales**                                     | \[
\begin{align*}
2x + 3y &= -1 \\
x - 3y &= 4
\end{align*}
|                                                                        |                                                                        |                                                                        |
| **system of linear inequalities**                                       | **sistema de desigualdades lineales**                                   | \[
\begin{align*}
2x + 3y &> -1 \\
x - 3y &\leq 4
\end{align*}
|                                                                        |                                                                        |                                                                        |
| **systematic random sample**                                           | **muestra sistemática**                                                | Mr. Martin chose a systematic random sample of customers visiting a store by selecting one customer at random and then selecting every tenth customer after that. |
|                                                                        |                                                                        |                                                                        |
| **tangent**                                                            | **tangente**                                                           |                                                                        |
| In a right triangle, the ratio of the length of the leg opposite \(\angle A\) to the length of the leg adjacent to \(\angle A\). | En un triángulo rectángulo, razón entre la longitud del cateto opuesto a \(\angle A\) y la longitud del cateto adyacente a \(\angle A\). |                                                                        |
|                                                                        |                                                                        |                                                                        |
| **term of an expression**                                              | **término de una expresión**                                           |                                                                        |
| The parts of the expression that are added or subtracted.              | Parte de una expresión que debe sumarse o restarse.                    |                                                                        |
|                                                                        |                                                                         |                                                                        |
| **term of a sequence**                                                 | **término de una sucesión**                                            |                                                                        |
| An element or number in the sequence.                                  | Elemento o número de una sucesión.                                     | 5 is the third term in the sequence 1, 3, 5, 7, ...                    |
|                                                                        |                                                                        |                                                                        |
| **terminating decimal**                                                | **decimal finito**                                                     |                                                                        |
| A decimal that ends, or terminates.                                    | Decimal con un número determinados de posiciones decimales.            | 1.5, 2.75, 4.0                                                          |
|                                                                        |                                                                        |                                                                        |
| **theoretical probability**                                            | **probabilidad teórica**                                               |                                                                        |
| The ratio of the number of equally likely outcomes in an event to the total number of possible outcomes. | Razón entre el número de resultados igualmente probables de un suceso y el número total de resultados posibles. | In the experiment of rolling a number cube, the theoretical probability of rolling an odd number is \(\frac{3}{6} = \frac{1}{2}\). |
|                                                                        |                                                                        |                                                                        |
| **third quartile**                                                     | **tercer cuartil**                                                     |                                                                        |
| The median of the upper half of a data set. Also called upper quartile. | La mediana de la mitad superior de un conjunto de datos. También se llama cuartil superior. | Lower half  \[18, 23, 28, 29, \underline{36}, 42\]  Upper half  \[ \underline{36}, 42 \]  Third quartile |
|                                                                        |                                                                        |                                                                        |
| **tolerance**                                                          | **tolerancia**                                                         |                                                                        |
| The amount by which a measurement is permitted to vary from a specified value. | La cantidad por que una medida se permite variar de un valor especificado. |                                                                        |
|                                                                        |                                                                        |                                                                        |
| **transformation**                                                    | **transformación**                                                     |                                                                        |
| A change in the position, size, or shape of a figure or graph.         | Cambio en la posición, tamaño o forma de una figura o gráfica.          |                                                                        |
|                                                                        |                                                                        |                                                                        |
ENGLISH

translation  A transformation that shifts or slides every point of a figure or graph the same distance in the same direction.

SPANISH

traslación  Transformación en la que todos los puntos de una figura o gráfica se mueven la misma distancia en la misma dirección.

EXAMPLES

<table>
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<tr>
<td>C</td>
<td>D</td>
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</table>

trapezoid  A quadrilateral with exactly one pair of parallel sides.

trapezio  Cuadrilátero con sólo un par de lados paralelos.

tree diagram  A branching diagram that shows all possible combinations or outcomes of an experiment.

diagrama de árbol  Diagrama con ramificaciones que muestra todas las combinaciones o resultados posibles de un experimento.

trapezoid  A quadrilateral with exactly one pair of parallel sides.

trend line  A line on a scatter plot that helps show the correlation between data sets more clearly.

línea de tendencia  Línea en un diagrama de dispersión que sirve para mostrar la correlación entre conjuntos de datos más claramente.

ENGLISH

The tree diagram shows the possible outcomes when tossing a coin and rolling a number cube.

spanish

El diagrama de árbol muestra los posibles resultados al lanzar una moneda y rodar un dado.

trial  Each repetition or observation of an experiment.

prueba  Una sola repetición u observación de un experimento.

In the experiment of rolling a number cube, each roll is one trial.

triangular prism  A prism whose bases are triangles.

prisma triangular  Prisma cuyas bases son triángulos.

ENGLISH

In the experiment of rolling a number cube, each roll is one trial.

triangle  A three-sided polygon.

triángulo  Polígono de tres lados.

triangular pyramid  A pyramid whose base is a triangle.

pirámide triangular  Pirámide cuya base es un triángulo.

trigonometric ratio  Ratio of the lengths of two sides of a right triangle.

razón trigonométrica  Razón entre dos lados de un triángulo rectángulo.

$\sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\tan A = \frac{a}{b}$

ENGLISH

trinomial  A polynomial with three terms.

trinomio  Polinomio con tres términos.

$4x^2 + 3xy - 5y^2$
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</table>
| **union** The union of two sets is the set of all elements that are in either set, denoted by ∪. | **unión** La unión de dos conjuntos es el conjunto de todos los elementos que se encuentran en ambos conjuntos, expresado por ∪. | \[ A = \{1, 2, 3, 4\} \]
\[ B = \{1, 3, 5, 7, 9\} \]
\[ A \cup B = \{1, 2, 3, 4, 5, 7, 9\} \] |
| **unit rate** A rate in which the second quantity in the comparison is one unit. | **tasa unitaria** Tasa en la que la segunda cantidad de la comparación es una unidad. | \[ \frac{30 \text{ mi}}{1 \text{ h}} = 30 \text{ mi/h} \] |
| **unlike radicals** Radicals with a different quantity under the radical. | **radicales distintos** Radicales con cantidades diferentes debajo del signo de radical. | \[ 2\sqrt{2} \text{ and } 2\sqrt{3} \] |
| **unlike terms** Terms with different variables or the same variables raised to different powers. | **términos distintos** Términos con variables diferentes o las mismas variables elevadas a potencias diferentes. | \[ 4xy^2 \text{ and } 6x^2y \] |
| **upper quartile** See third quartile. | **cuartil superior** Ver tercer cuartil. | |
| **value of a function** The result of replacing the independent variable with a number and simplifying. | **valor de una función** Resultado de reemplazar la variable independiente por un número y luego simplificar. | The value of the function \( f(x) = x + 1 \) for \( x = 3 \) is 4. |
| **value of a variable** A number used to replace a variable to make an equation true. | **valor de una variable** Número utilizado para reemplazar una variable y hacer que una ecuación sea verdadera. | In the equation \( x + 1 = 4 \), the value of \( x \) is 3. |
| **value of an expression** The result of replacing the variables in an expression with numbers and simplifying. | **valor de una expresión** Resultado de reemplazar las variables de una expresión por un número y luego simplificar. | The value of the expression \( x + 1 \) for \( x = 3 \) is 4. |
| **variable** A symbol used to represent a quantity that can change. | **variable** Símbolo utilizado para representar una cantidad que puede cambiar. | In the expression \( 2x + 3 \), \( x \) is the variable. |
| **Venn diagram** A diagram used to show relationships between sets. | **diagrama de Venn** Diagrama utilizado para mostrar la relación entre conjuntos. | |
| **vertex of a parabola** The highest or lowest point on the parabola. | **vértice de una parábola** Punto más alto o más bajo de una parábola. | The vertex is \( (0, -2) \). |
**ENGLISH**

vertex of an absolute-value graph The point on the axis of symmetry of the graph.

vertical angles The nonadjacent angles formed by two intersecting lines.

vertical line A line whose equation is \( x = a \), where \( a \) is the \( x \)-intercept.

vertical-line test A test used to determine whether a relation is a function. If any vertical line crosses the graph of a relation more than once, the relation is not a function.

volume The number of nonoverlapping unit cubes of a given size that will exactly fill the interior of a three-dimensional figure.

voluntary response sample A sample in which members choose to be in the sample.

whole number A member of the set of natural numbers and zero.

**SPANISH**

vértice de una gráfica de valor absoluto Punto en el eje de simetría de la gráfica.

ángulos opuestos por el vértice Ángulos no adyacentes formados por dos líneas que se cruzan.

línea vertical Línea cuya ecuación es \( x = a \), donde \( a \) es la intersección con el eje \( x \).

prueba de la línea vertical Prueba utilizada para determinar si una relación es una función. Si una línea vertical corta la gráfica de una relación más de una vez, la relación no es una función.

volumen Cantidad de cubos unitarios no superpuestos de un determinado tamaño que llenan exactamente el interior de una figura tridimensional.

muestra de respuesta voluntaria Una muestra en la que los miembros eligen participar.

número cabal Miembro del conjunto de los números naturales y cero.

**EXAMPLES**

\[ \angle 1 \text{ and } \angle 3 \text{ are vertical angles. } \]
\[ \angle 2 \text{ and } \angle 4 \text{ are vertical angles. } \]

\[ \text{Volume } = (3)(4)(12) = 144 \text{ ft}^3 \]
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<tr>
<td><strong>x-coordinate</strong> The first number in an ordered pair, which indicates the horizontal distance of a point from the origin on the coordinate plane.</td>
<td><strong>coordenada x</strong> Primer número de un par ordenado, que indica la distancia horizontal de un punto desde el origen en un plano cartesiano.</td>
<td><img src="image1" alt="Graph" /> The x-intercept is 2.</td>
</tr>
<tr>
<td><strong>x-intercept</strong> The x-coordinate(s) of the point(s) where a graph intersects the x-axis.</td>
<td><strong>intersección con el eje x</strong> Coordenada(s) x de uno o más puntos donde una gráfica corta el eje x.</td>
<td><img src="image2" alt="Graph" /> The x-intercept is 2.</td>
</tr>
<tr>
<td><strong>y-axis</strong> The vertical axis in a coordinate plane.</td>
<td><strong>eje y</strong> Eje vertical en un plano cartesiano.</td>
<td><img src="image3" alt="Graph" /></td>
</tr>
<tr>
<td><strong>y-coordinate</strong> The second number in an ordered pair, which indicates the vertical distance of a point from the origin on the coordinate plane.</td>
<td><strong>coordenada y</strong> Segundo número de un par ordenado, que indica la distancia vertical de un punto desde el origen en un plano cartesiano.</td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td><strong>y-intercept</strong> The y-coordinate(s) of the point(s) where a graph intersects the y-axis.</td>
<td><strong>intersección con el eje y</strong> Coordenada(s) y de uno o más puntos donde una gráfica corta el eje y.</td>
<td><img src="image5" alt="Graph" /> The y-intercept is 2.</td>
</tr>
<tr>
<td><strong>zero exponent</strong> For any nonzero real number $x$, $x^0 = 1$.</td>
<td><strong>exponente cero</strong> Dado un número real distinto de cero $x$, $x^0 = 1$.</td>
<td>$5^0 = 1$</td>
</tr>
<tr>
<td><strong>zero of a function</strong> For the function $f$, any number $x$ such that $f(x) = 0$.</td>
<td><strong>cero de una función</strong> Dada la función $f$, todo número $x$ tal que $f(x) = 0$.</td>
<td><img src="image6" alt="Graph" /> The zeros are $-3$ and $1$.</td>
</tr>
<tr>
<td><strong>Zero Product Property</strong> For real numbers $p$ and $q$, if $pq = 0$, then $p = 0$ or $q = 0$.</td>
<td><strong>Propiedad del producto cero</strong> Dados los números reales $p$ y $q$, si $pq = 0$, entonces $p = 0$ o $q = 0$.</td>
<td>If $(x - 1)(x + 2) = 0$, then $x = 1$ or $x = -2$.</td>
</tr>
</tbody>
</table>